

ELECTROMAGNETIC INDUCTION

Important Points:

1. The magnetic lines of force passing through a normal plane is called magnetic flux.

$$\phi = \vec{B} \cdot \vec{A} = BA \cos \theta \quad \text{Where } \theta \text{ is the angle between } \vec{B} \text{ and } \vec{A}$$

(Vector \vec{A} is perpendicular to its plane)

2. Faraday Laws:

a. Whenever there is changing in magnetic flux linked with a coil, emf is induced in it.

b. The induced emf is proportional to the negative rate of change of magnetic flux linked with the coil.

$$e = -\frac{d\phi}{dt} \quad (\text{Or}) \quad e = \frac{-d\phi}{dt}$$

$$\text{For } N \text{ turns } e = -\frac{d(N\phi)}{dt} \quad (\text{Proportionality constant } K = 1)$$

$$e = -\frac{d}{dt}(NBA) \quad (\text{Neumann's Law})$$

$$\text{c. Induced current } (i) = e = -N \frac{d\phi}{dt} = -N \frac{(\phi_2 - \phi_1)}{t}$$

$$\text{d. Induced charge } (q) = \frac{-N(\phi_2 - \phi_1)}{R} = (\text{Induced charge is independent of time})$$

3. Lenz's Law:

Induced emf always opposes the change that produces it. Lenz's law obeys the law of conservation of energy.

4. Motional EMF:

If a straight conductor of length l moves with a constant speed v perpendicular to the uniform magnetic field of induction B , then a voltage is developed across ends of the conductor ends which is given by $e = Blv$

$$\text{5. Induced current } (I) = \frac{e}{R} = \frac{Blv}{R}$$

6. Self Inductance:

If the current in a coil changes, the magnetic flux around the coil changes. Hence emf is induced in the coil called self inductance.

If i is the current through the coil and ϕ is the flux lines around the coil then

$\phi = Li$. Where L is the coefficient of self induction

$$e = -\frac{d\phi}{dt} = -L \frac{di}{dt}$$

Unit of L - Henry or wb/amp or volt-sec/amp

7. Energy stored in an Inductor $U = \frac{1}{2} Li^2$

8. Mutual Inductance:

a. When current in a coil changes, the magnetic flux linked with other coil changes and an emf is induced in the secondary coil called mutual induction.

b. If I_p is the current through the primary and ϕ_s is the flux linked with secondary,

$\phi_s \propto I_p \Rightarrow \phi_s = MI_p$ Where M is mutual inductance of the coil

Unit of M : Henry (or) volt - sec. amp (or) weber/ amp.

Induced emf $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI_p) = -M \cdot \frac{di_p}{dt}$

Very Short Answer Questions

1. What did the experiments of Faraday and Henry show?

- A 1) whenever the magnetic flux linked with a closed circuit changes, an induced emf and hence an induced current is set up in it.
- 2) The higher the rate of change of magnetic flux linked with the closed circuit, the greater is the induced emf (or) current.

2. Define magnetic flux?

- A. The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.

$$\text{Magnetic flux } \phi = \vec{B} \cdot \vec{A} = BA \cos \theta$$

Units: Weber Dimensional formula: $ML^2T^{-2}A^{-1}$

3. State Faraday's laws of electromagnetic induction.

A. Faraday's Laws:

- a) Whenever magnetic flux linked with a coil changes, emf is induced in it.
- b) The induced emf is proportional to the negative rate of change of magnetic flux linked with the coil.

$$e = -\frac{d\phi}{dt} \quad (\text{or}) \quad e = \frac{-d\phi}{dt} \quad (\text{Proportionality constant } K = 1)$$

$$\text{For } N \text{ turns} \quad e = -\frac{d(N\phi)}{dt}$$

$$\text{Or} \quad e = -\frac{d}{dt}(NBA) \quad \text{Or} \quad e = -N \frac{d\phi}{dt} = -N \frac{(\phi_2 - \phi_1)}{t}$$

4. State Lenz's law.

A. Lenz's Law:

The direction of an induced emf is always opposes the change in the magnetic flux that causes it. Lenz's law obeys the law of conservation of energy.

5. What happens to the mechanical energy (of motion) when a conductor is moved in a uniform magnetic field?

- A. The mechanical energy spent in moving the conductor is converted into electrical energy and then into thermal energy.

6. What are eddy currents?

A. Eddy Currents:

When the magnetic field in a metal changes with time, induced electric field is produced which applies force and makes the free electrons of metal to move in closed paths. These are eddy currents.

These currents were discovered by Foucault, so they are also known as Foucault currents. The direction of eddy currents is given by Lenz's law.

7. Define Inductance?

A. Inductance (L) is the ratio of the flux-linkage to current, it is equal to $N\Phi / I$.

Units: Henry Dimensional formula: $ML^2T^{-2}A^{-2}$.

8. What do you understand by 'self inductance'?

A. The magnetic flux ($N\phi$) linked with the coil is found to be proportional to the strength of the current (I)

$$\text{i.e. } N\phi \propto I \Rightarrow N\phi = LI$$

Where constant of proportionality 'L' is called co-efficient of self induction (or) self inductance.

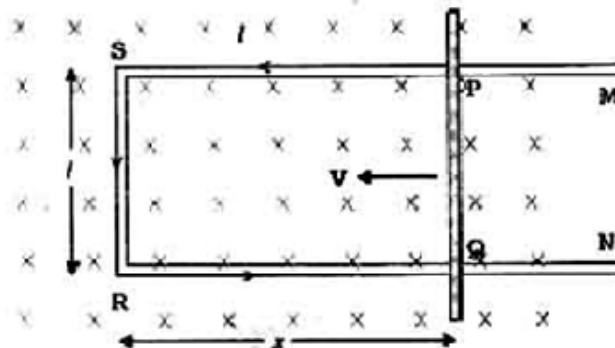
$$\text{If } I = 1A, N\phi = L$$

Thus co-efficient of self induction of a coil is defined as the magnetic flux linked with a coil through which a unit current flows.

Short Answer Questions

1. Obtain an expression for the emf induced across a conductor which is moved in a uniform magnetic field which is perpendicular to the plane of motion?

A. Consider a straight conductor moving in a uniform and time independent magnetic field. Figure shows a rectangular conductor PQRS in which the conductor PQ is free to move.



The rod PQ is moved towards the left with a constant velocity as shown in the figure.

Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field B which is perpendicular to the plane of this system.

If the length RQ = x and RS = l, the magnetic flux Φ_B enclosed by the loop PQRS will be

$$\Phi_B = Blx$$

Since x is changing with time, the rate of change of flux will induce an emf given by:

$$\begin{aligned}\epsilon &= \frac{-d\Phi_B}{dt} = -\frac{d}{dt}(Blx) \\ &= -Bl \frac{dx}{dt} = Blv\end{aligned}$$

Where we have used $dx/dt = -v$ which is the speed of the conductor PQ.

The induced emf Blv is called motional emf.

2. Describe the ways in which Eddy currents are used to advantage?

A. i) **Magnetic Braking In Trains:**

In some electric trains electromagnets are situated above the rails. When these are activated, the eddy currents induced in the rails oppose the motion of the train.

ii) **Electromagnetic Damping:**

In some galvanometers core is made of nonmagnetic metallic material. When the coil oscillates, the eddy currents induced in the core oppose the motion of the coil and bring it rest quickly.

iii) **Induction Furnace:**

In an induction, a metallic block to be melted is placed in a high frequency changing magnetic field. Strong eddy currents are induced in the block. Due to the high resistance of the metal, a large amount of heat is produced in it. This heat ultimately melts the metallic block.

iv) Electric Power Meters:

The shiny metal disc in the electric power meter rotates due to eddy currents. Electric currents induced in the disc by magnetic fields produced by sinusoidally varying currents in the coil.

3. Obtain an expression for the mutual inductance of two long co-axial solenoids?

A. Mutual Inductance of Two Long Solenoids:

Consider two long co-axial solenoids s_1 and s_2 , with s_2 wound over s_1 .

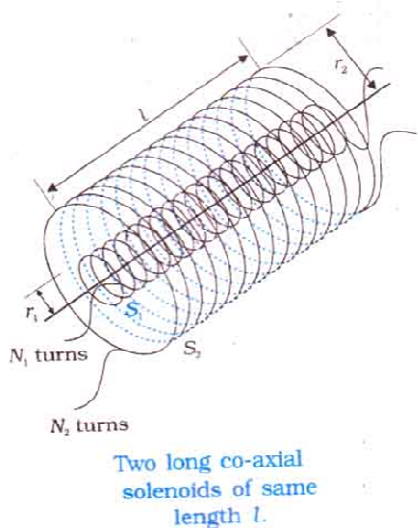
Let l = Length of each solenoid

r_1, r_2 = radii of inner and outer solenoids

$$A = \pi r_1^2$$

= area of cross section of inner solenoids s_1

N_1, N_2 = number of turns in the two solenoids



First we pass a time varying current I_2 through S_2 . The magnet field set up inside S_2 due to I_2 is

$$B_2 = \mu_0 n_2 I_2$$

Where $n_2 = N_2/l$ = the number of turns per unit length of S_2 .

Total magnetic flux linked with inner solenoid S_1 is

$$\phi_2 = B_2 A N_1 = \mu_0 n_2 I_2 \cdot A N_1$$

Mutual inductance of coil 1 with respect to coil 2 is

$$M_{12} = \frac{\phi_1}{I_2} = \mu_0 n_2 A N_1 = \frac{\mu_0 N_1 N_2 A}{l}$$

Now we consider the flux linked with the outer solenoid S_2 due to the current I_1 in the inner solenoid S_1 . The field B_1 due to I_1 is constant inside S_1 but zero in the annular region between the two solenoids. Hence

$$B_1 = \mu_0 n_1 I_1$$

Where $n_1 = N_1/l$ = the no of turns per unit length of S_1

Total flux linked with the outer solenoid S_2 is

$$\phi_2 = B_1 A N_2 = \mu_0 n_1 I_1 \cdot A N_2 = \frac{\mu_0 N_1 N_2 A I_1}{l}$$

\therefore Mutual inductance of coil 2 with respect to coil 1 is

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Clearly $M_{12} = M_{21} = M$ (say)

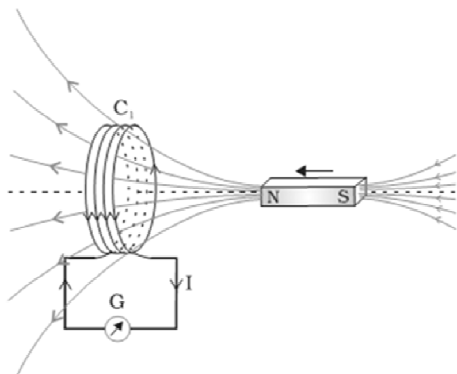
$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l = \mu_0 n_1 n_2 \pi r_1^2 l$$

Thus, the mutual inductance of two coils is the property of their combination. It does not matter which one of them functions as the primary or the secondary coil.

Long Answer Questions

1. Outline the path-breaking experiments of Faraday and Henry and highlight the contributions of these experiments to our understanding of electromagnetism?

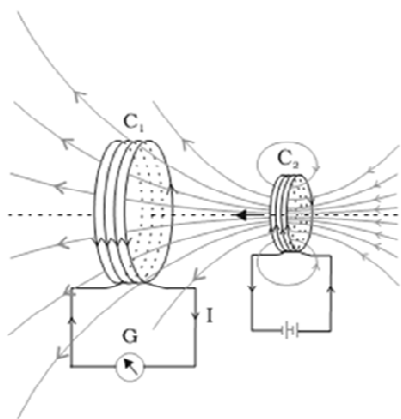
A. Whenever, there is a change in magnetic flux through a closed circuit, an induced emf is set up in the circuit. This phenomenon is known as electromagnetic induction.



I. Magnet - Coil Experiment:

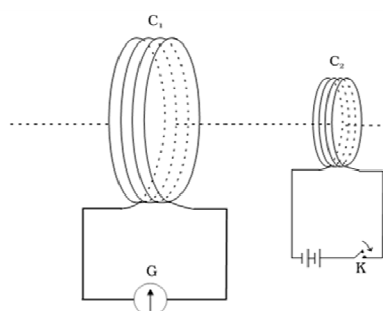
- 1) The circuit consists a coil C_1 connected in series with a galvanometer.
- 2) When the N-pole of magnet is moved towards the coil C_1 , the galvanometer shows deflection. The deflection shows that there is current in the circuit.
- 3) When the N-pole of magnet is moved away from coil C_1 , the galvanometer shows deflection in other direction.
- 4) When the N-pole of magnet comes to rest, the galvanometer shows no deflection.
- 5) The similar results are obtained when S-pole of a magnet is moved towards or away from coil C_1 .
- 6) When the magnet is moved fast, the deflection in the galvanometer is large and when it is moved slowly the deflection is small.
- 7) The induced current may be also produced by moving the coil C_1 to a stationary (next) magnet.
- 8) This experiment clearly demonstrates the EMI. Whenever there is a relative motion between magnet and coil, the magnetic flux linked with coil changes and hence an induced current is produced due to which the galvanometer shows a deflection.

II. Coil-Coil Experiment - Current induced by Current:



- 1) As shown in figure coil C_1 is connected with galvanometer and coil C_2 is connected with battery.
- 2) When the coil C_2 is moved towards or away from C_1 , the galvanometer shows deflection due to current is induced in coil C_1 .
- 3) When the coil C_2 is held fixed and C_1 is moved, the galvanometer shows deflection due to current induced in coil C_1 .
- 4) When two coils C_1 and C_2 are in relative motion that induces electric current.

III. Coil-Coil experiment - current induced by changing current:



- 1) Two coils C_1 and C_2 are held stationary. Coil C_1 is connected to galvanometer G and coil C_2 is connected to a battery through a tapping key K.
- 2) First, the battery circuit is closed by pressing the tap key K, and then broken, the galvanometer shows deflection in one direction and then in the other.
- 3) When the key K is kept pressed continuously, there is no deflection in the galvanometer.
- 4) The deflection is produced in the galvanometer only at make or break of the current in the coil C_1 .

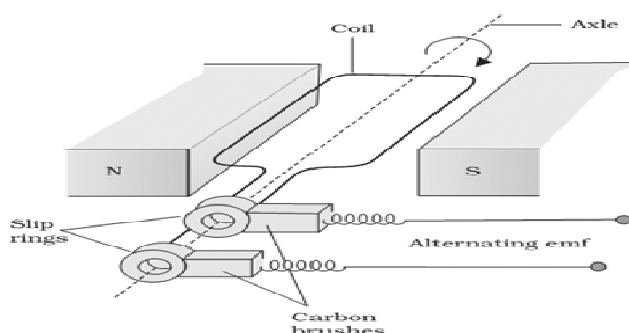
2. Describe the working of a AC generator with the aid of a simple diagram and necessary expressions?

A. AC Generator:

An electric generator or dynamo is a device which converts mechanical energy into electrical energy. The simplest practical generator consists of a rectangular coil rotating in a uniform magnetic field.

Description:

AC generator consists of coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (armature) is rotated mechanically in the magnetic field. The ends of the coil are connected to the external circuit by means of slip rings and brushes.



Working:

When the coil rotates, the magnetic flux cutting the coil changes and hence emf is induced in it. At any instant let the normal to the plane of the coil makes an angle with the field direction where is the angular speed of the coil. The flux linked with the coil is given by, Where N is the number of turns of the coil.

$$\text{But } \varepsilon = \frac{-d\phi}{dt} = +BAN\omega \sin \omega t$$

$$\therefore \varepsilon = \varepsilon_m \sin \omega t$$

Where $\varepsilon_m = BAN\omega$ is known as peak value of alternating emf

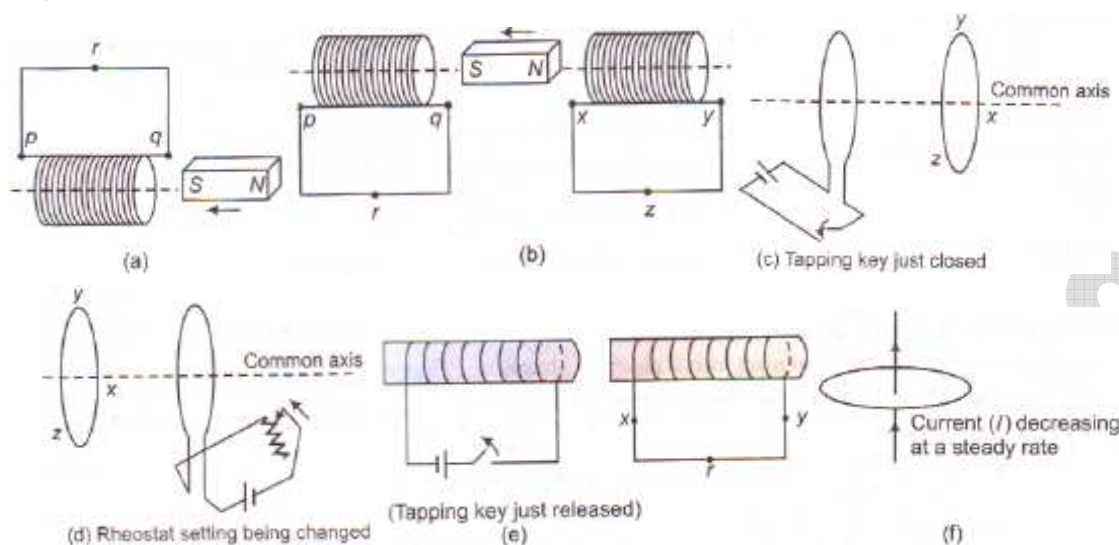
The direction of current changes periodically and hence the current is called alternating current.

$$\text{Also, } \varepsilon = \varepsilon_m \sin 2\pi vt$$

Where v is the frequency of revolution of the coil.

PROBLEMS

1. Predict the direction of induced current in the situation described by the following figures:



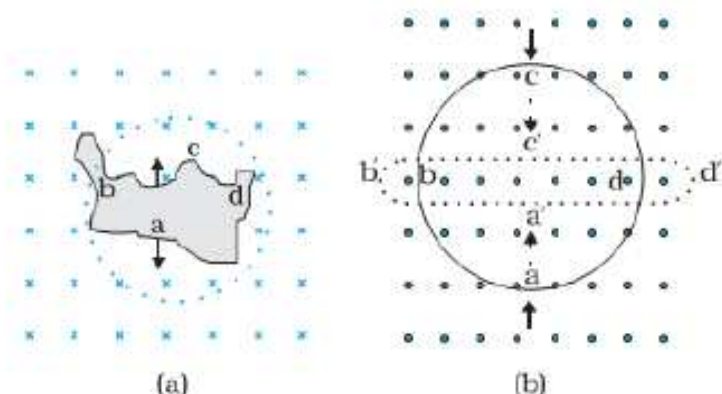
According to Lenz's law, the direction of induced current is always in such a way that it opposes the cause due to which it is produced.

- A. (a) Here, south pole is moving towards the coil, so according to Lenz's law this end becomes S-pole. (To oppose the motion of South Pole by repelling it). Hence, the direction of current is clockwise (by using clock rule) and the current flows along p to q or qrpq.
- (b) In coil p-q, at end q \Rightarrow S-pole is moving towards end q, so it behaves like a south pole (by Lenz's law). The direction of current is clockwise (by clock rule), i.e., from p to q or along prq. North Pole is moving away so this end will behave like South pole (To oppose its away motion by attracting it). In coil x-y, S-pole is induced (by Lenz's law) and the direction of current is clockwise i.e., x to y or along yzx.
- (c) As the tapping key is just closed, the current in coil increases. So, the magnetic flux and field increases. According to Maxwell's right hand grip rule, the direction of magnetic field is leftwards. Thus, the direction of induced current in the neighbouring coil is such that it try to decrease the field, thus the direction of field in the neighbouring coil should be rightwards, i.e., according to Maxwell's right hand rule the direction of induced current is anticlockwise, i.e., along xyz or yzx.
- (d) As the Rheostat setting is changed, the current is changed. The direction of field due to the coil is leftwards according to Maxwell's right hand grip rule. The direction of induced current in the left coil is such that the magnetic field produced by it in rightwards, thus the direction of current in left coil is anticlockwise i.e., from zyx.
- (e) As the key is just released, the current which is flowing anticlockwise goes on decreasing. Thus, the induced current developed in such a sense the magnetic field due to left coil increases (which is towards right). So, the magnetic field due to the right coil should also towards right and hence the induced current is in anticlockwise, i.e., along xry.
- (f) The magnetic field lines due to the current carrying wire are in the plane of the loop. Hence, no induced current is produced in the loop (because no flux lines cross the area of loop).

2. Use Lenz's law to determine the direction of induced current in the situations described by figure.

(a) A wire of irregular shape turning into a circular shape.

(b) A circular loop being deformed into a narrow straight wire.



A. (a) Here, the direction of magnetic field is perpendicularly inwards to the plane of paper. If a wire of irregular shape turns into a circular shape then its area increases (The circular loop has greater area than the loop of irregular shape) so that the magnetic flux linked also increases. Now, the induced current is produced in a direction such that it decreases the magnetic field [i.e., the current will flow in such a direction so that the wire forming the loop is pulled inward in all directions (to decrease the area)], i.e., current is in anticlockwise direction, i.e., $adcba$.

(b) When a circular loop deforms into a narrow straight wire, the magnetic flux linked with it also decreases. The current induced due to change in flux will flow in such a direction that it will oppose the decrease in magnetic flux so it will flow anticlockwise, i.e., along $ab'c'b'a'$, due to which the magnetic field produced will be out of the plane of paper.

3. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

A. Given, number of turns $n = 15 \text{ per cm} = 1500 \text{ per meter}$

Area of small loop $A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

Change in current $\frac{dI}{dt} = \frac{4-2}{0.1} = \frac{2}{0.1} = 20 \text{ A/s}$

Let e be the induced emf,

According to Faraday's law

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) \quad (\phi = BA)$$

Or
$$e = A \frac{dB}{dt} = A \frac{d}{dt}(\mu_0 nI)$$

(Magnetic field inside the solenoid $B = \mu_0 n I$)

Or
$$e = A \mu_0 n \frac{dI}{dt}$$

$$e = 2 \times 10^{-4} \times 4 \times 3.14 \times 10^{-7} \times 1500 \times 20 \quad (\because \mu_0 = 4\pi \times 10^{-7})$$

$$e = 7.5 \times 10^{-6} \text{ V}$$

Thus, the induced emf in the loop is $7.5 \times 10^{-6} \text{ V}$.

4. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm/s in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

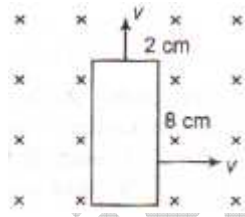
Use the concept of motional emf as wire loop is moving normal to the magnetic field.

A. Given, length of the loop $l = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$.

Width of the loop $b = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

Velocity of the loop $= 1 \text{ cm/s} = 0.01 \text{ m/s}$

Magnitude of magnetic field $B = 0.3 \text{ T}$



(a) When velocity is normal to the longer side

$$(l = 8 \text{ cm} = 8 \times 10^{-2} \text{ m})$$

In this case, motional emf

$$e = B l v = 0.3 \times 8 \times 10^{-2} \times 0.01 = 2.4 \times 10^{-4} \text{ V}.$$

$$\text{Time} = \frac{\text{Distance}}{\text{velocity}} = \frac{\text{Shorter side (width)}}{\text{Velocity}}$$

$$t = \frac{2 \times 10^{-2}}{0.01} = 2 \text{ s}.$$

(b) When velocity is normal to the shorter side

$$(l = 2 \text{ cm} = 2 \times 10^{-2} \text{ m})$$

In this case, the developed emf

$$e = B l v = 0.3 \times 2 \times 10^{-2} \times 0.01$$

$$e = 0.6 \times 10^{-4} \text{ V}$$

$$\text{Time} = \frac{\text{Longer side (length)}}{\text{Velocity}} = \frac{8 \times 10^{-2}}{0.01}$$

$$t = 8 \text{ s.}$$

5. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring?

A. $L = 1.0 \text{ m}, \omega = 400 \text{ rad s}^{-1}, B = 0.5 \text{ T}$

The emf developed between the centre and the ring is

$$\varepsilon = \frac{1}{2} B L^2 \omega = \frac{1}{2} \times 0.5 \times (1.0)^2 \times 400 = 100 \text{ V}$$

6. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of magnitude $3.0 \times 10^{-2} \text{ T}$. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω , calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

A. Radius of coil = 8 cm = 0.08 m

Number of turns = 20

Resistance of closed loop = 10Ω

Angular speed = 50 rad/s

Magnitude of magnetic field $B = 3 \times 10^{-2} \text{ T}$.

Induced emf produced in the coil $E = NBA \sin \omega t$

For maximum emf $\sin \omega t = 1$

\therefore Maximum emf $e_0 = NBA \omega = 20 \times 3 \times 10^{-2} \times 3.14 \times (0.08)^2 \times 50$

$$e_0 = 0.603 \text{ V}$$

Maximum current in the coil $I_0 = \frac{e_0}{R} = \frac{0.603}{10} = 0.0603 \text{ A.}$

Average induced emf

$$e_{av} = \frac{1}{T} \int_0^{2\pi} e dt = \frac{1}{T} \int_0^{2\pi} NBA\omega \sin \omega t dt$$

$$e_{av} = \frac{1}{T} NAB\omega \left[\frac{\cos \omega t}{\omega} \right]_0^{2\pi}$$

$$= \frac{NBA}{T} [\cos 2\pi - \cos 0^\circ]$$

$$e_{av} = \frac{NBA}{T} [1 - 1] = 0$$

For full cycle average emf, $e_{av} = 0$

Average power loss due to heating

$$= \frac{E_0 I_0}{2} = \frac{0.603 \times 0.0603}{2} = 0.018 \text{ W.}$$

The source of power dissipated as heat in the coil is the external rotar. The current induced in the coil causes a torque which opposes the rotation of the coil, so the external agent rotar counter this torque to keep the coil rotating uniformly.

7. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1} at right angles to the horizontal component of the Earth's magnetic field, $0.30 \times 10^{-4} \text{ Wb m}^{-2}$.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

- A. (a) $l = 10 \text{ m}, v = 50 \text{ m s}^{-1}, B_H = 0.30 \times 10^{-4} \text{ Wb m}^{-2}$

$$\mathcal{E} = B_H l v = 0.30 \times 10^{-4} \times 10 \times 5.0 = 1.5 \times 10^{-3} \text{ V.}$$

(b) According to Fleming's right hand rule, the direction of emf is from west to east.

(c) Western end of the wire is at the higher electrical potential.

8. Current in a circuit falls from 5.0 A to 0.0A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit?

A. $I_1 = 5.0A, I_2 = 0.0A, t = 0.1s, \varepsilon = 200V$

$$\text{As } \varepsilon = -L \frac{dI}{dt} = -L \frac{I_2 - I_1}{t}$$

$$\therefore 200 = -L \frac{0.0 - 5.0}{0.1} = +50L$$

$$L = 4H.$$

9. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5s, what is the change of flux linkage with the other coil?

A. $M = 1.5H, I_1 = 0A, I_2 = 20A, t = 0.5s$

$$\text{As } \varepsilon = -\frac{d\phi}{dt} = -L \frac{dI}{dt}$$

$$\therefore d\phi = LdI = 1.5 \times (20 - 0) = 30Wb.$$

10. A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30° ?

A. Speed of jet plane $v = 1800 \text{ km/h} = 1800 \times \frac{5}{18} = 500 \text{ m/s}$

$$l = \text{Distance between the ends of the wings} = 25 \text{ m}$$

$$\text{The magnitude of magnetic field } B = 5 \times 10^{-4} \text{ T.}$$

$$\text{Angle of dip } \delta = 30^\circ$$

Use the formula of motional emf

$$e = B_V v l$$

$$e = B \sin \delta v l$$

(Where, B_V = vertical component of the Earth's magnetic field $B_V = B \sin \delta$)

$$e = 5 \times 10^{-4} \sin 30^\circ \times 500 \times 25 = 3.1 \text{ V.}$$

Thus, the voltage difference developed between the ends is 3.1 V.