ELECTRIC CHARGES AND FIELDS

Important Points:

1. Coulomb's Law:

The force of attraction (or) repulsion between two charges at rest is directly proportional to the product of magnitude of two charges and inversely proportional to the square of the distance between them.

2. Electric Field:

Electric field exists in a region in which a charge experiences a force.

3. Intensity of Electric Field:

The force experienced by a unit positive charge kept in an electric field is called the intensity of the electric field (E). It is a vector quantity.

4. Electric Lines Of Force:

The path traced by a unit positive charge placed in a uniform electric field is called electric line of force.

5. Gauss's Law:

The total flux linked with a closed surface is $(1/\in_0)$ times the charge enclosed by the closed surface. $\phi = \oint \vec{E}.\vec{d}s = \frac{1}{\in_0} q$.

6. The electric intensity at a point due to an infinitely long charged wire is, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

Here 'λ' is the uniform linear charge density, 'r' is the radial distance of the point from the axis of the wire.

7. The electric intensity due to an infinite plane sheet of charge is, $E = \frac{\sigma}{2\varepsilon_0}$

Here 'σ' is the uniform surface charge density.

Very Short Answer Questions

1. What is meant by the statement that ‘charge is quantized’?

A. Charge always exists as an integer multiple of electronic charge (1.6×10^{-19}C). Fractions of electronic charge are not possible. i.e., Q = $\pm ne$ where n is an integer. Hence charge is said to be quantized.

2. Repulsion is the sure test of electrification than attraction. Why?

A. Positively charged body can attract both negatively charged and neutral bodies, but positively charged body can only repel another positively charged body. Hence repulsion is sure test of electrification.

3. How many electrons constitute 1C of charge?

A: Charge q = ne

Given q = 1C

$1 = n\times1.6\times10^{-19}$

$\therefore n = \dfrac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$

4. What happens to the weight of a body when it is charged positively?

A: Every body acquires positive charge due to the loss of electrons. Hence the weight of a body decreases when it is positively charged.

5. What happens to the force between two charges if the distance between them is a) Halved b) Doubled?

A. $F \propto \dfrac{1}{d^2}$

a) $\dfrac{F_2}{F_1} = \left(\dfrac{d_1}{d_2}\right)^2 \Rightarrow \dfrac{F_2}{F_1} = \left(\dfrac{d}{d/2}\right)^2 \Rightarrow F_2 = 4F_1$

The force between the charges increases by four times

b) $\frac{F_2}{F_1} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{d}{2d}\right)^2 \Rightarrow F_2 = \frac{F_1}{4}$

The force is reduced to of its original value.

6. The electric lines of force do not intersect. Why?

A. The tangent drawn to electric line of force gives the direction of electric field at the point. If the electric lines of force intersect then at the point of intersection electric field will act in two different directions, which is not possible. Hence electric lines of force do not intersect.

7. Consider two charges +q and -q placed at B and C of an equilateral triangle ABC. For this system, the total charge is zero. But the electric field (intensity) at A which is equidistant from B and C is not zero. Why?

A: Charge is a scalar, Hence it is added algebraically. But field is a vector, it should be added vectorially. The fields due to the given charges are not in the same direction. Therefore the net field is not zero.

8. Electrostatic field lines of force do not form closed loops. If they form closed loops then the work done in moving a charge along a closed path will not be zero. From the above two statements can you guess the nature of electrostatic force?

A: Electric force is conservative force. If field lines form closed loop, then work done by the conservative force along the closed path is not be zero.

9. State Gauss's law in electrostatics?

A: The total electric flux through any closed Gaussian surface is equal to $\frac{1}{\varepsilon_0}$ times the algebraic sum of charge enclosed by the surface. Here ε_0 is the permittivity of free space.

Mathematically the Gauss law can be stated as $\int \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$

10. When will be the electric flux negative and when is it positive?

A. For a closed surface, inward flux is taken to be negative and outward flux is taken to be positive.

11. Write the expression for electric intensity due to an infinite long charged wire at a radial distance 'r' from the wire?

A: The electric intensity due to an infinitely long charged wire $E = \frac{\lambda}{2\pi\varepsilon_0 r}$. Where λ is the linear charge density and r is the perpendicular distance from wire.

12. Write the expression for electric intensity due to an infinite plane sheet of charge?

A: Electric field intensity due to an infinite non-conducting plane sheet of charge $E = \frac{\sigma}{2\varepsilon_0}$, where σ is surface charge density.

13. Write the expression for electric intensity due to charged conducting spherical shell at points outside and inside the shell?

A. Outside the shell ($r > R$): If 'σ' is the surface charge density $E = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2}$

Inside the shell $(r < R)$: If 'σ' is the surface charge density E = 0.

Short Answer Questions

1. State and explain Coulomb's inverse square law in electricity?

A. Statement:

The force of attraction (or) repulsion between two charges at rest is directly proportional to the product of magnitude of two charges and inversely proportional to the square of the distance between them and acts along the line joining the charges.

Explanation:

Let two point charges q_1 and q_2 at rest are separated by a distance r. The force F between them is given by

$F \propto q_1 q_2$

$\propto \frac{1}{d^2}$

(Or) $F \propto \frac{q_1 q_2}{r^2}$ (Or) $F = \frac{1}{4\pi \in} \frac{q_1 q_2}{r^2}$

Where $\in$ is called to absolute permittivity of the medium.

$\in = \in_0 \in_r$ Where $\in_0$ is called the permittivity of free space and $\in_r$ is the relative permittivity of the medium

$\in_r = 1$ For air or vacuum

Hence, for air (or) vacuum $F_{air} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$

$F_{air} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$ (Or) $\in_o = 8.854 \times 10^{-12}$ Farad / m.

2. Define intensity of electric field at a point. Derive an expression for the intensity due to a point charge?

A. Intensity of Electric Field (E):

The force acting on unit positive charge placed at a point in the electric field is called the Intensity of electric field at that point.

Expression:

Let a point charge Q be placed at point 'O' in air (or) vacuum. Let P be a point at a distance 'r' from the charge Q. Let a unit positive charge is placed at P.

The force acting on this unit charge is given by,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q \times 1}{r^2}$$

Hence from definition, $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

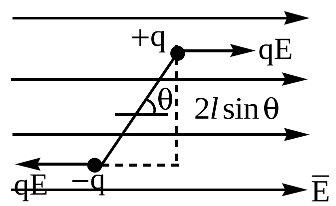
Intensity of electric field in vector form can be given by, $\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^3} \bar{r}$.

3. Derive the equation for the couple acting on an electric dipole in a uniform electric field?

A. Let an electric dipole of dipole moment P placed at an angle θ to the direction of a uniform electric field of intensity E. Each charge of the dipole experiences a force of magnitude qE. Hence the dipole experiences two equal, unlike and non collinear forces which form a couple. The torque acting on the dipole is given by

$\tau = qE \times 2a \sin\theta$ Or $\tau = pE \sin\theta$

In vector form $\bar{\tau} = \bar{p} \times \bar{E}$

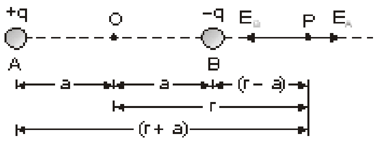


The torque on the dipole tends to align the dipole in the direction of electric field.

4. Derive an expression for the electric intensity of electric field at a point on the axial line of an electric dipole?

A. Electric Field on Axial Line:

Consider an electric dipole consisting of two point charges – q and +q separated by a distance '2a'. Let P be a point on the axial line at a distance 'r' from centre of 'o' the dipole.



Electric field intensity at P due to charge +q is given by,

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

Electric field intensity at P due to charge - q is given by,

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

Net intensity, $E_{axial} = E_B - E_A \qquad (\because E_B > E_A)$

$$E_{axial} = \frac{q}{4\pi\varepsilon_0 (r-a)^2} - \frac{q}{4\pi\varepsilon_0 (r+a)^2}$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2-a^2)^2}$$

Since p = q(2a)

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$$

If, r>>>a then $E_{axial} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$

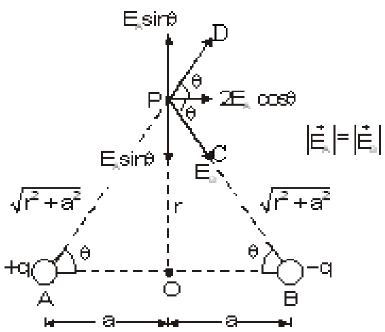
5. Derive an expression for the electric intensity of electric field at a point on the equatorial line of an electric dipole.

A. Electric Field on Equatorial Line:

Consider an electric dipole consisting of two point charges +q and – q separated by a distance '2a'. Let P be a point on the equatorial line at a distance 'r' from centre of 'o' the dipole.

Electric field intensity at P due to charge +q is given by,

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \text{ (Along } \overrightarrow{PD}\text{)}$$



Electric field intensity at P due to charge - q is given by,

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \quad \text{(Along } \overrightarrow{PC}\text{)}$$

Clearly $E_A = E_B$ .Let us resolve and $\overrightarrow{E_B}$ into two components in two mutually perpendicular directions components of $\overrightarrow{E_A}$ and $\overrightarrow{E_B}$ along the equatorial line cancel each other. But the components perpendicular to equatorial line get added up because they act in same direction. So magnitude of resultant intensity $\vec{E}$ at P

$$E = 2E_A \cos\theta = 2\frac{q}{4\pi\varepsilon_0 (r^2+a^2)}\frac{a}{\sqrt{r^2+a^2}}$$

$$E_{equitorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

If r >>> a, then $E_{equitorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$. This intensity is parallel to AB from P.

6. State Gauss's law in electrostatics and explain its importance?

A: The total electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

Here ε_0 is the permittivity of free space, Mathematically, Gauss law can be stated as,

$$\phi_E = \int \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

Here q is the total charge enclosed by the surface.

Importance:

1) Gauss's theorem holds good for any closed surface.

2) It is useful in calculating the electric field and potentials due to Continuous symmetric charge distribution.

3) It is valid for stationary charges as well as for rapidly moving charges.

4) Gauss theorem can help us to calculate electric flux that radiated outwards from one coulomb of charge.

Long Answer Questions

1. Define Electric Flux. Applying Gauss's law derives the expression for electric intensity due to an infinite long straight charged wire. (Assume that the electric field is everywhere radial and depends only on the radial distance r of the point from the wire.)

A. Electric Flux:

The total number of electric lines of force passing through a normal plane inside an electric field is called Electric flux (ϕ). It is a scalar quantity.

$d\phi = \overline{E}.d\overline{s}$.

Gauss Theorem:

"The Electric flux (ϕ) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface".

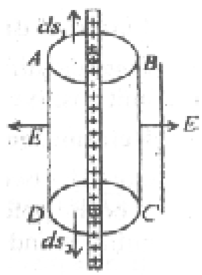
$$\phi = \int \vec{E}.\vec{d}s = \frac{1}{\in_0} q$$

Here ε_0 is the permittivity of free space.

Electric field due to a line charge:

Consider an infinitely long thin straight wire having a uniform charge density λ. Imagine a cylindrical Gaussian surface ABCD of length l and radial distance r with its axis along the wire.

For the top and bottom flat surfaces AB and CD, $\overline{E}$ and $d\overline{s}$ are perpendicular. Hence electric flux through them is zero.



Along the curved surface ADCB, $E\oint ds = \frac{1}{\varepsilon_0} q$

Or $E(2\pi rl) = \frac{1}{\varepsilon_0} q$

$$\therefore E = \frac{q}{2\pi\varepsilon_o r\ell} = \frac{\lambda}{2\pi\varepsilon_o r} \quad \left(\because \lambda = \frac{q}{l}\right)$$

This gives electric intensity due to infinitely long charged wire.

2. State Gauss's law in electrostatics. Applying Gauss's laws derive the expression for electric intensity due to an infinite plane sheet of charge?

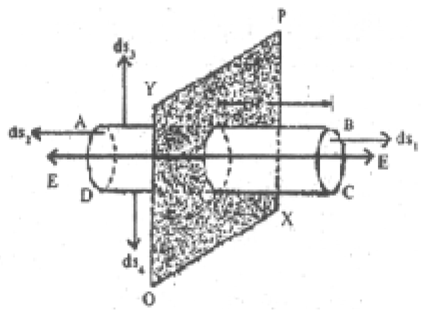
A. Gauss Theorem:

"The Electric flux (ϕ) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface".

$$\phi = \oint \vec{E}.\vec{d}s = \frac{1}{\in_0} q$$

This is the integral form of Gauss law. Here ε_0 is the permittivity of free space.

Electric field due to an infinite plane sheet of charge:



Consider an infinite plane sheet of charge OXPY of uniform surface charge density 'σ'. Imagine a Gaussian surface in the form of horizontal cylinder ABCD, passing through the vertical plane sheet and having length '2r' perpendicular to the plane sheet and symmetrical on either side of the sheet. The areas ds_3 *and* ds_4 are perpendicular to **E**. Hence electric flux through them is zero.

For the areas ds_3 *and* ds_4, $\oint E.ds$ = E $\oint ds$ = E (S + S) = E (2S)

Here S is the area of each flat surface of the cylinder.

From the Gauss's law we have, $\oint E.ds = \frac{q}{\varepsilon_0}$

But $q = \sigma s$

$$E\,(2S) = \frac{S\sigma}{\varepsilon_0}, \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}.$$

Thus the electric intensity due to an infinite plane sheet of charge $E = \frac{\sigma}{2\varepsilon_0}$.

If σ is positive, E will be along the outward drawn normal

If σ is negative, E will be along the inward drawn normal

3. Applying Gauss's law derive the expression for electric intensity due to charged conducting spherical shell at (i) a point outside the shell (ii) a point on the surface of the shell and (iii) a point inside the shell ?

A. Gauss Theorem:

"The Electric flux (ϕ) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface".

$$\phi = \oint \vec{E}.\vec{d}s = \frac{1}{\epsilon_0} q$$

This is the integral form of Gauss law. Here ε_0 is the permittivity of free space

Electric field due to a spherical shell:

Consider a spherical shell of radius R and a charge 'q'. Let us find the electric field at a point P at a distance r from the centre 'O' of the shell.

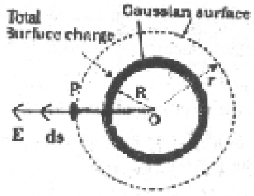
Surface charge density $\sigma = \frac{q}{4\pi R^2} \Rightarrow q = 4\pi R^2 \sigma$

(a) When P is outside the shell:

Consider a spherical Gaussian surface of radius 'r' .The total flux through the surface is given by,

$$\phi = E\oint ds = E(4\pi r^2)$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0} q \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



Charged spherical shell : Point outside

$$\therefore E = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_o r^2} \Rightarrow E = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2}$$

(b) When the point is on the surface (r = R):

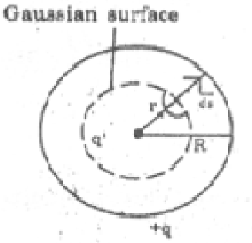
Consider a spherical Gaussian surface of radius 'R' .The total flux through the surface is given by,

$$E(4\pi R^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0} R^2 \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

(c)When P is inside the shell (r < R):

Consider a spherical Gaussian surface of radius 'r' where (r < R). In this case Gaussian surface does not enclose any charge and hence according to Gauss law.



Inside charged spherical shell

$$E(4\pi r^2) = \frac{0}{\varepsilon_0}$$

$$\therefore E = 0$$

PROBLEMS

1. **Two small identical balls, each of mass 0.20 g. carry identical charges and are suspended by two threads of equal lengths. The balls position themselves at equilibrium such that the angle between the threads is 60^0. If the distance between the balls is 0.5 m, find the charge on each ball?**

Sol: m = 0.2 $= gm = 2\times10^{-4}\ kg$

$\theta = 30^0,\ x = 0.5\ m = 5\times10^{-1} m$

$$\frac{1}{4\pi \in_0}\frac{q^2}{x^2} = mg\tan\theta.$$

$$\frac{9\times10^9\times q^2}{25\times10^{-2}} = 2\times10^{-4}\times9.8\times\tan 30^0$$

$q = 1.77\times10^{-7} col.$

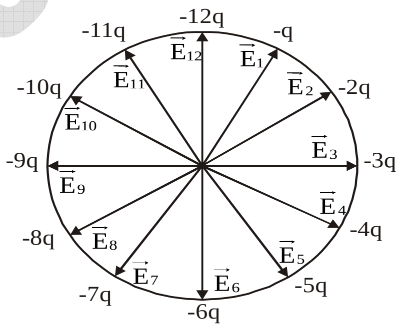
2. **An infinite number of charges each of magnitude q are placed on x- axis at distances of 1, 2, 4, 8 ... Meter from the origin respectively. Find intensity of the electric field at origin.**

Sol: $E_{res} = \frac{q}{4\pi \in_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{8}{8^2}+\frac{1}{16^2}+.....\right]$

$= \frac{q}{4\pi\in_0}\left[\frac{1}{1-\frac{1}{4}}\right] = \frac{q}{3\pi\,\varepsilon_o}$ towards origin

3. **A clock face has negative charges –q, –2q, –3q.......–12q fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to the point charges. At what time does the hour hand point in the direction of the electric field at the centre of the dial?**

A .



The resultant electrified due to all charges lies in between 9 and 10 i.e. at the time 9:30

4. Consider a uniform electric field E = 3×10^3 N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60^0 angle with the x-axis?

Sol: $E = 3\times10^3\,NC^{-1}$

$A = \left(10^{-1}\right)^2 = 10^{-2}\,m^2$

a) $\phi = EA = 3\times10^3\times10^{-2} = 30$

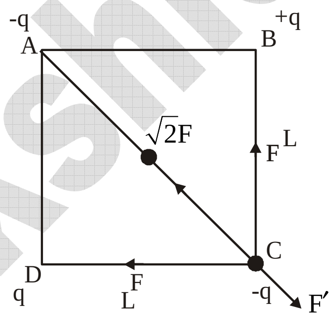
b) $\phi = EA\cos\theta$

$= 3\times10^3\times10^{-2}\times\frac{1}{2} = 15.$ Here $\theta = 60^0$

5. There are four charges, each with a magnitude q. Two are positive and two are negative. The charges are fixed to the corners of a square of side 'L', one to each corner, in such a way that the force on any charge is directed toward the center of the square. Find the magnitude of the net electric force experienced by any charge?

A. The attractive force experienced by charge at 'C' due to the charges at B and D is same

$$F_1 = F_2 = F = \frac{1}{4\pi\epsilon_0}.\frac{q\times q}{L^2} \qquad \text{......... (1)}$$



The resultant of two equal forces is

$$F' = \sqrt{F^2 + F^2} = \sqrt{2}F = \sqrt{2}\times\frac{1}{4\pi\epsilon_0}.\frac{q^2}{L^2} \qquad \text{.......... (2)}$$

The repulsive force experienced by the charge at 'C' due to the charge at A is

$$F' = \frac{1}{4\pi\epsilon_0}\cdot\frac{q\times q}{\left(\sqrt{2}L\right)^2} \qquad \text{.........} (3)$$

$\therefore$ The net force experienced by the charge at C towards the centre is

$$F_R = \sqrt{2}F - F' = \sqrt{2}\times\frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{L^2} - \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{2L^2}$$

$$F_R = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{2L^2}\left[2\sqrt{2}-1\right]$$

6. The electric field in a region is given by $\vec{E} = a\hat{i} + b\hat{j}$. Here a and b are constants. Find the net flux passing through a square area of side L parallel to y-z plane.

Sol: $\vec{A} = L^2\ \hat{i};\ \vec{E} = a\hat{i} + b\hat{j}$

$$\therefore \phi = \vec{E}.\vec{A} = \left(ai + bj\right).\left(L^2\hat{i}\right)$$

$$\Rightarrow \phi = aL^2$$

7. A hollow spherical shell of radius r has a uniform charge density σ. It is kept in a cube of edge 3*r* such that the centre of the cube coincides with the centre of the shell. Calculate the electric flux that comes out of a face of the cube?

Sol: For the spherical shell

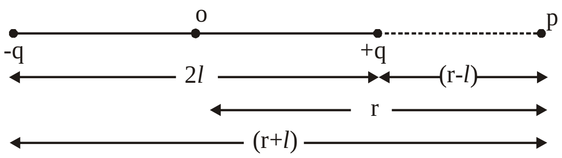
$$\sigma = \frac{q}{4\pi r^2} \Rightarrow q = \sigma.4\pi r^2$$

Electric flux through one face of the cube is given by $E_o = \frac{q}{6E_o} = \frac{\sigma\times 4\pi r^2}{6E_o}$

$$\therefore \phi = \frac{2\pi r^2\sigma}{3E_o}$$

8. An electric dipole consists of two equal and opposite point charges +Q and –Q, separated by a distance 2*l*. P is a point collinear with the charges such that its distance from the positive charge is half of its distance from the negative charge. The electric intensity at P is-

A. Given $r - l = \frac{1}{2}(r + l)$



$\Rightarrow 2r - 2l = r + l \Rightarrow r = 3l$

$\therefore$ The electric intensity at P is $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{\left(r^2 - l^2\right)^2}$

$$E = \frac{1}{4\pi\epsilon_0} \frac{2P(3l)}{\left[(3l)^2 - l^2\right]^2} = \frac{1}{4\pi\epsilon_0} \frac{2P(3l)}{\left(8l^2\right)^2} \quad \left[\because P = (2l)Q\right]$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2(2l \times Q)3l}{64l^4} = \frac{1}{4\pi\epsilon_0} \frac{3Q}{16l^2}$$

9. Two infinitely long thin straight wires having uniform linear charge densities λ and 2λ are arranged parallel to each other at a distance r apart. The intensity of the electric field at a point midway between them is-

Sol: $\lambda_1 = \lambda,\ \lambda_2 = 2\lambda,\ r_1 = r_2 = \frac{r}{2}$

$$E_1 = \frac{\lambda_1}{2\pi \in_0 r_1} = \frac{\lambda}{2\pi \in_0 \left(\frac{r}{2}\right)} = \frac{\lambda}{\pi \in_0 r}$$

$$E_2 = \frac{\lambda_2}{2\pi \in_0 r_2} = \frac{2\lambda}{2\pi \in_0 \left(\frac{r}{2}\right)} = \frac{2\lambda}{\pi \in_0 r}$$

$$E_{net} = E_2 - E_1 = \frac{2\lambda}{\pi \in_0 r} - \frac{\lambda}{\pi \in_0 r} = \frac{\lambda}{\pi \in_0 r}$$

10. Two infinitely long thin straight wires having uniform linear charge densities *ë* and 2*ë* are arranged parallel to each other at a distance *r* apart. The intensity of the electric field at a point midway between them is-

Sol: $\lambda_1 = \lambda,\ \lambda_2 = 2\lambda,\ r_1 = r_2 = \frac{r}{2}$

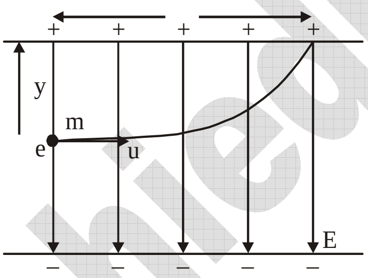
$$E_1 = \frac{\lambda_1}{2\pi \in_0 r_1} = \frac{\lambda}{2\pi \in_0 \left(\frac{r}{2}\right)} = \frac{\lambda}{\pi \in_0 r}$$

$$E_2 = \frac{\lambda_2}{2\pi \in_0 r_2} = \frac{2\lambda}{2\pi \in_0 \left(\frac{r}{2}\right)} = \frac{2\lambda}{\pi \in_0 r}$$

$$E_{net} = E_2 - E_1 = \frac{2\lambda}{\pi \in_0 r} - \frac{\lambda}{\pi \in_0 r} = \frac{\lambda}{\pi \in_0 r}$$

11. An electron of mass 'm' and charge 'e' is fired perpendicular to a uniform electric field of intensity E with an initial velocity 'u'. If the electron traverses a distance 'x' in the field in the direction of firing, find the transverse displacement 'y' it suffers?

A. The transverse distance of electron in the direction of firing x = ut....... (1)



The transverse distance of the electron in the perpendicular direction is $y = \frac{1}{2}at^2$

Since acceleration acquired by the electron by the field is $a = \frac{eE}{m}$

$\therefore$ Deflection of the electron

$$y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{x}{u}\right)^2 = \frac{eEx^2}{2mu^2}$$