

ATOMS

Important Points:

1. Rutherford's α -Particle Scattering Experiment:

- Most of the α -particles were found to pass through the gold- foil without being deviated from their paths.
- Some α -particles were found to be deflected through small angles $\theta < 90^\circ$.
- Few α -particles were found to be scattered at fairly large angles from their initial path $\theta < 90^\circ$
- A very small number of α -particles about in 8000 practically retraced their paths or suffered deflections of nearly 180°
- Most of the portion of the atom is hollow inside.
- The whole positive charge of the atom must be concentrated in a small space which is at the centre of the atom called nucleus.

2. Distance of Closest Approach: $\frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{2} m_\alpha v_\alpha^2$

3. Bohr's Model of Hydrogen like Atoms:

- Electron revolves round the nucleus only in certain allowed orbits called stationary orbits and the Coulomb's force of attraction between electron and the positively charged nucleus provides the necessary centripetal force-

$$\frac{k(Ze)e}{r^2} = \frac{mV^2}{r}$$

- The angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$, where h is the

Planck's constant. $mvr = n \frac{h}{2\pi}$, where $n = 1, 2, 3, 4, \dots$ called principal quantum number.

- An electron in a stationary orbit has a definite amount of energy. It possesses kinetic energy because of its motion and potential energy on account of the attraction of the nucleus.

iv) Energy is radiated or absorbed when an electron jumps from one stationary orbit to another stationary orbit. This energy is equal to the energy difference between these two orbits and emitted or absorbed as one quantum of radiation of frequency ν given by

Planck's equation $E_2 - E_1 = h\nu = \frac{hc}{\lambda}$

4. Radius of Bohr's Orbit:

Radius of the n^{th} orbit $r_n = \frac{h^2}{4\pi^2 ke^2} \cdot \left(\frac{n^2}{mZ}\right)$

In general, the radius of the n^{th} orbit of hydrogen like atom is given by

$$r_n = 0.53 \left(\frac{n^2}{Z}\right) \text{ \AA} \quad \text{where } n = 1, 2, 3, \dots$$

5. Velocity of the Electron in the Orbit:

The velocity of an electron in n^{th} orbit $V_n = \frac{2\pi ke^2}{h} \cdot \left(\frac{Z}{n}\right)$

6. Time Period of Electron in the Orbit:

The time period of rotation of electron in n^{th} orbit $T = \frac{n^3}{2\pi\omega_0 Z^2}$ i.e $T \propto \frac{n^3}{Z^2}$.

7. Energy of the Electron in the Orbit:

Kinetic energy $K_n = \frac{2\pi^2 k^2 e^4}{h^2} \cdot \left(\frac{mZ^2}{n^2}\right)$ and potential energy $U_n = -\frac{4\pi^2 k^2 e^4}{h^2} \cdot \left(\frac{mZ^2}{n^2}\right)$

Total energy of the electron in n^{th} orbit $E_n = -\frac{2\pi^2 k^2 e^4}{h^2} \cdot \left(\frac{mZ^2}{n^2}\right)$

Very Short Answer Questions

1. What is the angular momentum of electron in the second orbit of Bohr's model of hydrogen atom?

A. The angular momentum $L = n \frac{h}{2\pi}$, where $n = 1, 2, 3, 4, \dots$ called principal quantum number.

For second orbit of Bohr's model of hydrogen atom $n = 2$,

$$\therefore L = 2 \times \frac{h}{2\pi} = \frac{h}{\pi}$$

2. What is the expression for fine structure constant and what is its value?

A. The term $\alpha = \frac{e^2}{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right) c}$ is fine structure constant. Where h is Planck's constant, C is speed

of light, e is charge of an electron and ϵ_0 permittivity of free space. The value of

$$\alpha = 7.2973 \times 10^{-3}$$

3. What is the physical meaning of 'Negative Energy of an Electron'?

A. The total energy of the electron is negative implies that the atomic electron bound to the nucleus. To remove the electron from its orbit against the nuclear pull, energy is required.

4. Sharp lines are present in the spectrum of a gas. What does this indicate?

A. When atomic gas or vapour is excited at low pressure by passing electric current through it the emitted radiation has a spectrum which contains specific wavelengths. This indicates emission line spectrum.

5. Name a physical quantity whose dimensions are the same as those of angular momentum.

A. Planck's constant.

6. What is the difference between α -particle and helium atom?

A. Alpha particles consist of two protons and two neutrons bound together into a particle identical to a helium nucleus. This is produced in the process of alpha decay. Helium atom is composed of two electrons in orbit around a nucleus containing two protons along with either one or two neutrons.

7. How is impact parameter related to angle of scattering?

A. Impact Parameter (b):

The impact parameter is defined as the perpendicular distance of the initial velocity vector of the α -particle from the center of the nucleus.

In case of head-on collision impact parameter is minimum and the α -particle rebounds back ($\theta \cong \pi$). For a large impact parameter, the α -particle goes nearly undeviated and has a small deflection ($\theta \cong 0$).

$$\text{Impact parameter } b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2\right)} \Rightarrow b \propto \cot(\theta/2)$$

For large b, particles will go un-deviated and for small b the α -particle will suffer large scattering.

8. Among alpha, beta and gamma radiations, which get affected by the electric field?

A. Alpha and beta radiations are affected by the electric field because they are charged particles.

9. What do you understand by the phrase 'ground state atom'?

A. Lowest energy state of the atom is called ground state atom in which electron is in most stable condition.

10. Why does the mass of the nucleus not have any, significance in scattering in Rutherford's experiment?

A. The cause of scattering in Rutherford's experiment is the charge on the nucleus. The scattering occurs due to the electric field of the charge on the nucleus (but not due to gravitational field). So, scattering is independent of the mass of the nucleus.

11. The Lyman series of hydrogen spectrum lies in the ultraviolet region. Why?

A. The Lyman series of hydrogen spectrum lies in UV region is because of the large energies involved in the transitions. As an illustration, the energy radiated corresponding to the first line of Lyman series is

$[-3.4 - (-13.6)]$ eV i.e., 10.2eV, such high energy photons are in the UV region of the spectrum.

12. Write down a table giving longest and shortest wavelengths of different spectral series.

A. Wavelength of spectral line in Hydrogen spectrum for any series is $\lambda = \frac{912A^{\circ}}{\left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]}$

Name of the spectral series	Lower state (n_1)	Upper state (n_2)	Wavelength (λ) in A°
Lyman	1	2	1216 (longest wavelength)
	1	∞	912 (shortest wavelength)
Balmer	2	3	6563 (longest wavelength)
	2	∞	3646 (shortest wavelength)
Paschen	3	4	18751 (longest wavelength)
	3	∞	8220 (shortest wavelength)
Bracket	4	5	40533 (longest wavelength)
	4	∞	14592 (shortest wavelength)
Pfund	5	6	74618 (longest wavelength)
	5	∞	22800 (shortest wavelength)

13. The wavelengths of some of the spectral lines obtained in hydrogen spectrum are 1216 Å, 6463 Å and 9546 Å. Which one of these wavelengths belongs to the Paschen Series?

A. The Paschen series lies in the infrared region. So, the wavelength of any line in the Paschen Series has to be in between 8220 Å - 18751 Å. Clearly 9546 Å belongs to the Paschen Series.

14. Give two drawbacks of Rutherford's atomic model?

A. Drawbacks of Rutherford's atomic model:

(i) An accelerating charge loses energy continuously by radiation. Due to this continuous loss of energy, the electron should spiral towards the nucleus and fall into it. Thus an atom should be unstable.

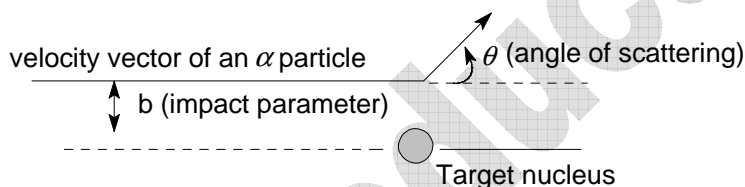
(ii) The electron moving around the nucleus continuously experience centripetal acceleration and therefore it must lose energy continuously. Hence the atom must be able to emit continuous energy spectrum. But the observed spectrum from the atom is line spectrum.

Short Answer Questions

1. **What is impact parameter and angle of scattering? How are they are related to each other?**

A. **Impact Parameter:** The impact parameter b is defined as the perpendicular distance of the velocity vector of the α -particle from the centre of the nucleus when it is far away from the atom.

Angle of Scattering: The angle between the initial and final directions of motion of a scattered α -particle is called angle of scattering.



A beam of α -particles have nearly same kinetic energy close to the nucleus. (Small impact parameter) suffers large scattering. In case of head - on collision, the impact parameter is minimum and the α -particles rebounds back ($\theta \cong \pi$). For a large impact parameter, the particle goes nearly undeviated and has a small deflection ($\theta \cong 0$).

The fact that only a small fraction of the number of incident particles rebound back indicated that the number of α -particles undergoing head on collision is small. This in turn, implies that the mass of the atom is concentrated in a small volume.

2. Derive an expression for potential and kinetic energy of an electron in any orbit of a hydrogen atom according to Bohr's atomic model. How does P.E. change with increasing 'n'?

A. Potential Energy:

An electron possesses some potential energy because it is found in the field of nucleus.

Potential energy of electron in n^{th} orbit is given by,

$$P.E. = \frac{-1}{4\pi\epsilon_0} \frac{(Ze)e}{r}$$

But, $r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}$ And for hydrogen atom $Z = 1$,

$$P.E. = - \frac{m e^4}{4 \epsilon_0^2 n^2 h^2}$$

Kinetic Energy:

The Coulomb's force of attraction between electron and the positively charged nucleus provides necessary centripetal force.

$$\frac{m v^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z e^2}{r^2}$$

Or
$$m v^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z e^2}{r}$$

Or
$$\frac{1}{2} m v^2 = \frac{1}{8\pi\epsilon_0} \cdot \frac{Z e^2}{r}$$

But, $r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}$ And for hydrogen atom $Z = 1$,

$$K.E. = \frac{m e^4}{8 \epsilon_0^2 n^2 h^2}$$

As the value of 'n' increases, the potential energy of the electron increases.

3. What are the limitations of Bohr's theory of hydrogen atom?

- A. Bohr's theory was able to explain successfully a number of experimental observed facts and has correctly predicted the spectral lines of neutral hydrogen atom and singly ionized helium atom, etc. in terms of only principal quantum number n. However, the theory fails to explain the following facts.

- (i) The theory could not account the spectra of atoms more complex than hydrogen.
- (ii) The theory does not give any information regarding the distribution and arrangement of electrons in atom.
- (iii) It does not explain the experimentally observed variations in intensity of the spectral lines of an element.
- (iv) This theory cannot be used to calculate about transitions from one level to another such as the rate at which they occur or the selection rules which apply to them.
- (v) This theory fails for accounting the fine structure of spectral line. Actually it was found that when spectral lines emitted by an atom are examined, each line is composed of several lines closely packed together. Bohr's theory does not throw any light on it.
- (vi) This theory cannot be used for the quantitative explanation of chemical bonding.
- (vii) The theory fails to give correct result when an electric or magnetic field is applied to the atom. It is found that when electric or magnetic field is applied to the atom, each spectral line splits into several lines. The former effect is called as Stark effect while the later as Zeeman Effect.

4. Explain the distance of closest approach and impact parameter?

A. Distance of Closest Approach (r_0):

The minimum distance from the nucleus up to which the α -particle approach, is called the **Distance of Closest Approach** (r_0).

As the α -particle approaches the nucleus, the electrostatic repulsive force due to the nucleus increases and kinetic energy of the alpha particle goes on converting into electrostatic potential energy. When whole of the kinetic energy is converted into electrostatic potential energy, the particle cannot further move towards the nucleus but returns back on its initial path i.e. α -particle is scattered through an angle of 180° . The distance of particle from the nucleus in this stage is called the distance of closest approach.

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{2} m_\alpha v_\alpha^2$$

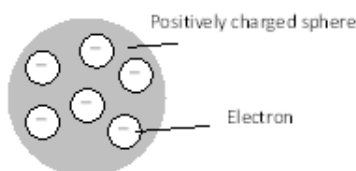
$$\therefore r_0 = \frac{1}{4\pi\epsilon_0} \frac{4Ze^2}{m_\alpha v_\alpha^2}$$

5. Give a brief account of Thomson model of atom. What are its limitations?

A. Thomson's Atomic Model:

J.J. Thomson gave the first idea regarding structure of atom. According to this model.

1) An atom is a solid sphere in which entire and positive charge and its mass is uniformly distributed and in which negative charge (i.e. electron) are embedded like seeds in watermelon.



2) This model explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization.

3) The model fail to explain the scattering of α - particles and it cannot explain the origin of spectral lines observed in the spectrum of hydrogen and other atoms.

6. Describe Rutherford atom model. What are the draw backs of this model?

A. Postulates: The experiments carried out by Geiger and Marsden gave rise to the Rutherford's nuclear model of atom. This model of the atom is also known as Rutherford's planetary model of the atom. This model is based on the following postulates.

i) The whole of the positive charge and nearly the entire mass of the atom is concentrated in a very small volume of the atom called nucleus. The nuclear radius is about $(1/10,000)$ of the atomic radius.

(ii) The electrons are distributed around the nucleus. So, there is lot of empty space in the atom.

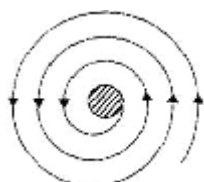
(iii) The amount of positive charge in the nucleus is equal to the amount of negative charge on the electrons. So, the atom, as a whole, is an electrically neutral entity.

(iv) Electrons were continuously revolving around the nucleus in circular orbits. The electrostatic force of attraction provides the necessary centripetal force.

Merits (i) Large angle scattering of alpha-particles through thin foils could be explained.

(ii) The classification of elements in the periodic table on the basis of their atomic number, instead of atomic weight, was justified.

Limitations: (i) According to the classical electromagnetic theory, a charged particle in accelerated motion should radiate energy in the form of electromagnetic radiation. As a result of continuous emission of radiation, the energy of the electron should gradually decrease. This should lead to a constant decrease in the radius of the electronic orbit. In other words, the electron should follow a spiral path (as shown in figure) and finally fall into the nucleus. This would mean a collapse of atomic structure. But the atom is a very stable structure. Thus, Rutherford model fails to account for the stability of the atom.



(ii) Rutherford model does not envisage any particular values of the radius of the electronic orbit. This should mean that an electron can emit radiations of all possible frequencies. In other words, an atom should have a continuous radiation spectrum. But this is contrary to the experimental result. The spectrum of an atom is a series of sharp lines. Thus, Rutherford model could not explain the line spectra of the atoms.

7. Distinguish between excitation potential and ionization potential?

A. Excitation is the process of absorption of energy by an electron so that the electron gets excited from a lower energy level to some higher energy level.

Excitation energy is the energy required to excite an electron from its ground state to the excited state.

The energy required to excite the electron from $n = 1$ to $n = 2$ orbit of hydrogen atom is called first excitation energy of the hydrogen atom. Its value is $[-3.4 - (-13.6)]$ eV or 10.2 eV.

The energy required to excite the electron from $n = 1$ to $n = 3$ orbit of hydrogen atom is called second excitation energy of the hydrogen atom. Its value is $[-1.51 - (-13.6)]$ eV or 12.09 eV.

Excitation Potential of an excited state is the potential difference through which electron in an atom has to be accelerated so as to excite it from its ground state to the given excited state.

The first excitation potential of hydrogen atom is 10.2 V. The second excitation potential of hydrogen atom is 12.09 V

Ionisation and Ionisation Potential

Ionisation is the process of knocking an electron out of the atom.

Ionisation energy is the energy required to knock an electron completely out of the atom. It is the energy required to excite an electron from $n = 1$ to $n = \infty$

Ionisation energy of hydrogen atom

$$= E_{\infty} - E_1 = 0 - (-13.6) \text{ eV} = 13.6 \text{ eV}$$

Numerically the ground state energy is equal to the ionisation energy.

8. Explain the different types of spectral series?

A.

Name of the series	Final state	Initial state n_2	Series limit ∞ to n_1	Maximum $\lambda(n_1+1)$ to n_1	Region
Lyman	$n_1 = 1$	2, 3, 4,	$\lambda = \frac{1}{R}$	$\lambda = \frac{4}{3R}$	UV
Balmer	$n_1 = 2$	3, 4, 5,	$\lambda = \frac{4}{R}$	$\lambda = \frac{36}{5R}$	Visible
Paschen	$n_1 = 3$	4, 5, 6,	$\lambda = \frac{9}{R}$	$\lambda = \frac{144}{7R}$	Near IR
Brackett	$n_1 = 4$	5, 6, 7,	$\lambda = \frac{16}{R}$	$\lambda = \frac{400}{9R}$	Middle IR
Pfund	$n_1 = 5$	6, 7, 8,	$\lambda = \frac{25}{R}$	$\lambda = \frac{900}{11R}$	Far IR

9. Write a short note on De Broglie's explanation of Bohr's second postulate of quantization?

A. The wave associated with a moving particle is called matter wave and the wavelength is called

The De Broglie Wavelength. For a photon, momentum $p = \frac{E}{c}$ (or) $p = \frac{h\nu}{c}$. If λ is the wavelength of the wave,

$$p = \frac{h}{\lambda} \left(\because \nu = \frac{c}{\lambda} \right) \text{ (Or) } \lambda = \frac{h}{p}$$

De Broglie tried to explain Bohr's criterion to select the allowed orbits in which angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. According to his hypothesis, an

electron revolving round nucleus is associated with certain wavelength ' λ ' which depends on

its momentum mv . It is given by $\lambda = \frac{h}{mv} = \frac{h}{p}$

In an allowed orbit, an electron can have an integral multiple of this wavelength. That is the n^{th} orbit consists of n complete de-Broglie wavelengths i.e. $2\pi r_n = n\lambda_n$, where r_n is the radius of n^{th} orbit and λ_n is the wavelength of n^{th} orbit $\lambda_n = \frac{2\pi r_n}{n}$ (or) $\lambda_n = \frac{2\pi}{n}(0.53 \times n^2) \text{ \AA}$ (or) $\lambda_n = 2\pi r_1 n \text{ \AA}$, where r_1 is radius of first orbit of H -atom.

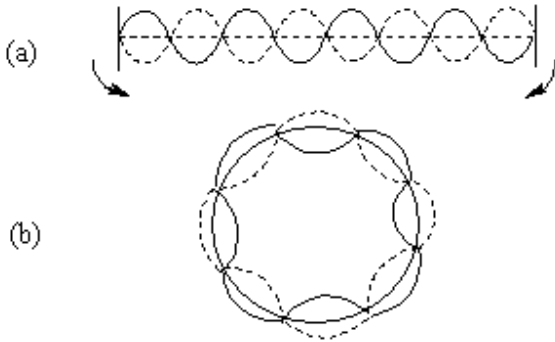


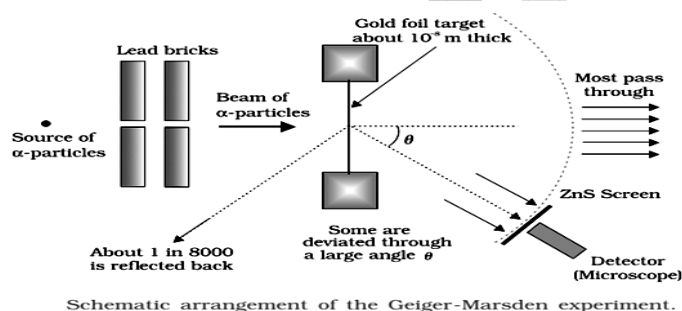
Figure (a) shows the waves on a string having a wavelength related to the length of the string allowing them to interfere constructively. If we imagine the string bent into a closed circle we get an idea of how electrons in circular orbits can interfere constructively as shown in figure (b). If the wavelength does not fit in to the circumference, the electron interferes destructively and it cannot exist in such an orbit.

Long Answer Questions

1. Describe Geiger - Marsden Experiment on scattering of α -particles. How is the size of the nucleus estimated in this experiment?

A. Geiger - Marsden Experiment:

Figure shows a schematic diagram of this experiment. Alpha-particles emitted by a radioactive source were collimated into a narrow beam by their passage through lead bricks. The beam was allowed to fall on a thin foil of gold of thickness 2.1×10^{-7} m. The scattered α -particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. The scattered alpha-particles on striking the screen produced brief light flashes or scintillations. These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.



Observations:

Experimental Observations:

- Most of the α -particles were found to pass through the gold- foil without being deviated from their paths.
- Some α -particles were found to be deflected through small angles $\theta < 90^\circ$.
- Few α -particles were found to be scattered at fairly large angles from their initial path $\theta > 90^\circ$
- A very small number of α - particles about in 8000 practically retraced their paths or suffered deflections of nearly 180° .

Conclusions:

- The entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away.

ii) The electrons would be moving in orbits about the nucleus just as the planets do around the sun.

iii) The distance of closest approach of the particles corresponding to that maximum value of kinetic energy for which the particle is not scattered back, will be a measure of the radius of the nucleus.

iv) Rutherford's experiments suggested the size of the nucleus to be about 10^{-15} m to 10^{-14} m.

Size of the nucleus: The fact that very small fraction of alpha particles undergoes large scattering indicates that the size of the nucleus is very small.

The fact that a large fraction of particles passes through the foil without deviation indicates that there is a large distance or empty space between the revolving e^- and the nucleus

Size of nucleus (Distance of Closest Approach): Consider an α -particle of mass m moving directly towards a nucleus with velocity v at any given time. As this α -particle approaches the nucleus, its velocity and hence kinetic energy continues to decrease. At a certain distance d from the nucleus, the α -particle will stop and then start retracing its path as depicted. This distance is called the distance of closest approach. At this distance, the kinetic energy of the α -particle is transformed into electrostatic potential energy.

If Z be the atomic number of the nucleus, then $\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(2e)}{d}$

$$\Rightarrow d = \frac{1}{4\pi\epsilon_0} \frac{4Ze^2}{mv^2}$$

$$\text{Here } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

The distance of closest approach is of the order of 10^{-14} m. So, the radius of the nucleus should be less than 10^{-14} m.

2. Discuss Bohr's theory of the spectrum of hydrogen atom?

A. According to the postulate of Bohr's model, when an atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number n_f ($n_f < n_i$), the difference of energy is carried away by a photon of frequency ν_{if} such that

$$h\nu_{if} = E_{n_i} - E_{n_f}$$

Hydrogen Spectrum (Origin of Spectral Lines)

When the hydrogen atom is subjected to external energy, the electron jumps from lower energy state to a higher energy state i.e., the hydrogen atom is excited. The excited state is not stable hence the electron returns to its ground state in about 10^{-8} seconds. The excess of energy is now radiated in the form of radiations of different wavelengths. The different wavelengths constitute spectral series which are characteristics of atom emitting them.

The wavelength of the different members of the series can be found from the following derivation

According to Bohr's frequency condition,

$$h\nu_{if} = E_{n_i} - E_{n_f}$$

Using the equation $E_n = \frac{me^4}{8n^2\epsilon_0^2h^2}$ for E_{n_f} and E_{n_i} , we get

$$h\nu_{if} = \frac{me^4}{8\epsilon_0^2h^2} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Or $\nu_{if} = \frac{me^4}{8\epsilon_0^2h^3} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$ this equation is called Rydberg formula, for the spectrum of the

hydrogen atom. The Rydberg constant R is given by $R = \frac{me^4}{8\epsilon_0^2h^3c}$

If we insert the values of various constants in above equation we get $R = 1.03 \times 10^7 m^{-1}$

If c is the velocity of light in vacuum and is the wavelength of radiation emitted, then

$$\frac{c}{\lambda} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2me^4}{h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad [\nu = c/\lambda]$$

$$\frac{1}{\lambda} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2me^4}{ch^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Here $1/\lambda$ is the wave number. It is defined as the number of waves in unit distance. It is denoted by $\bar{\nu}$

$$\bar{\nu} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2me^4}{ch^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\bar{\nu} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

The term is known as Rydberg constant it is denoted by R. Its value is $1.0974 \times 10^7 \text{ m}^{-1}$.

The above relation, known as Rydberg formula for the spectrum of the hydrogen atom, various spectral lines of different frequencies are produced for different values n_1 and n_2 .

- (i) **Lyman Series.** The spectral lines of this series correspond to the transition of an electron from some higher energy state to the innermost orbit ($n = 1$).

For Lyman series, $n_1 = 1$ and $n_2 = 2, 3, 4, \dots$

The wave numbers and the wavelengths of the spectral lines constituting the Lyman series are given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n_2^2} \right)$$

Lyman series was first predicted by Bohr. This series lies in the ultra-violet region of the spectrum.

- (ii) **Balmer series.** The spectral lines of this series correspond to the transition of an electron from some higher energy state to an orbit having $n = 2$.

For Balmer series, $n_1 = 2$, $n_2 = 3, 4, 5, \dots$

The wave numbers and the wavelengths of spectral lines constituting the Balmer series are given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right)$$

Balmer series is so named because it was discovered by Balmer in 1885. The first four lines of Balmer series lie in the visible region of the spectrum.

- (iii) **Paschen Series.** The spectral lines of this series correspond to the transition of an electron from some higher energy state to an orbit having $n = 3$.

For Paschen series, $n_1 = 3$, $n_2 = 4, 5, 6$,

The wave numbers and the wavelengths of the spectral lines constituting the Paschen series are given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n_2^2} \right)$$

Paschen series is so named because it was discovered by Paschen. Just like other series, this series was first predicted by Bohr. Paschen series lies in the infrared region of the spectrum and is invisible.

(iv) **Bracket Series:** The spectral lines of this series correspond to the transition of an electron from a higher energy state to the orbit having $n = 4$. For this series, $n_1 = 4$ and $n_2 = 5, 6, 7, \dots$

The wave numbers and the wavelengths of the spectral lines constituting the Brackett series are given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n_2^2} \right)$$

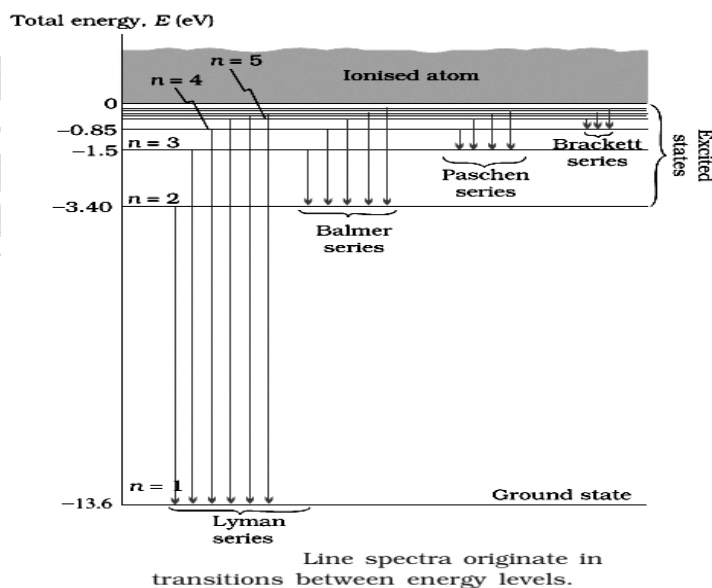
This series lies in the infrared region of the spectrum.

(v) **Pfund Series:** The spectral lines of this series correspond to the transition of electron from a higher energy state to the orbit having $n = 5$. For this series, $n_1 = 5$ and $n_2 = 6, 7, 8, \dots$

The wave numbers and the wavelengths of the spectral lines constituting the Pfund series are given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n_2^2} \right)$$

This series lies in the far infrared region of the spectrum.



3. State the basic postulates of Bohr's theory of atomic spectra. Hence obtain an expression for the radius of orbit and the energy of orbital electron in a hydrogen atom?

A. Postulates:

1) Electrons revolve only in certain allowed circular orbits called stationary orbits while in these orbits they do not radiate energy.

2) The angular momentum of an electron is an integer multiple of $\frac{h}{2\pi}$

$$\therefore mvr = \frac{nh}{2\pi}$$

When m = mass of electron, v = linear velocity of the electron, r = radius of the stationary orbit, h = Planck's constant and $n = 1, 2, 3, 4, \dots$ whole number integer. Angular momentum and energy both are quantized for an electron.

3) An orbiting electron emits energy in the form of electromagnetic waves when it jumps from outer stationary orbit to the inner orbit.

$$E_2 - E_1 = h\nu \text{ Where } \nu \text{ is the frequency of radiation.}$$

4) The coulomb force of attraction between the nucleus and the electron supplies the necessary centripetal force

$$\frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r^2} = \frac{mv^2}{r}$$

Radius of Orbit:

The coulomb force of attraction between the nucleus and the electron supplies the necessary centripetal force

$$\frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r_n^2} = \frac{mv_n^2}{r_n} \quad \text{Or} \quad \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} = mv_n^2$$

$$\text{But, } mvr_n = \frac{nh}{2\pi} \text{ (or) } v_n = \frac{nh}{2\pi nr_n}$$

$$\therefore \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} = m \times \frac{n^2 h^2}{4\pi m^2 r_n^2}$$

$$r_n = \left(\frac{\epsilon_0 h^2}{\pi m e^2} \right) \frac{n^2}{Z} = (0.53 \times 10^{-10}) \frac{n^2}{Z} m$$

$$\text{Or } r_n = 0.53 \times \frac{n^2}{Z} \text{ \AA} \quad \text{For } H_2 \text{ atom } z = 1$$

$$\therefore r_n = 0.53n^2 \quad \text{Or } r_n \propto n^2$$

For $n = 1$; $r_1 = 0.53$; For $n = 2$; $r_2 = 2.12 \text{ \AA}$

Energy of Orbital Electron:

Potential Energy:

An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit is given by,

$$P.E. = \frac{-1}{4\pi\epsilon_0} \frac{(Ze)e}{r}$$

$$\text{But, } r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z} \quad \text{and for hydrogen atom } Z = 1 ,$$

$$P.E. = - \frac{m e^4}{4 \epsilon_0^2 n^2 h^2}$$

Kinetic Energy:

The Coulomb's force of attraction between electron and the positively charged nucleus provides necessary centripetal force.

$$\frac{m v^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z e^2}{r^2}$$

$$\Rightarrow m v^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z e^2}{r}$$

$$\Rightarrow \frac{1}{2} m v^2 = \frac{1}{8\pi\epsilon_0} \cdot \frac{Z e^2}{r}$$

$$\text{But, } r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z} \quad \text{And for hydrogen atom } Z = 1 ,$$

$$K.E. = \frac{m e^4}{8 \epsilon_0^2 n^2 h^2}$$

Total Energy:

Total energy = P.E. + K.E.

$$\therefore T.E. = - \frac{m e^4}{4 \epsilon_0^2 n^2 h^2} + \frac{m e^4}{8 \epsilon_0^2 n^2 h^2} = - \frac{m e^4}{8 \epsilon_0^2 n^2 h^2}$$

The negative energy indicates that the electron is bound to be nucleus.

PROBLEMS

1. The radius of the first electron orbit of a hydrogen atom is $5.3 \times 10^{-11} m$. what is the radius of the second orbit?

Sol: $r_1 = 5.3 \times 10^{-11} m$, $n_1 = 1$

$$n_2 = 2, \quad r_2 = 2$$

$$r \propto n^2 \Rightarrow \frac{r_2}{r_1} = \frac{n_2^2}{n_1^2}$$

$$\Rightarrow \frac{r_2}{5.3 \times 10^{-11}} = \left(\frac{2}{1}\right)^2 = 4$$

$$\Rightarrow r_2 = 4 \times 5.3 \times 10^{-11} = 21.2 \times 10^{-11} m$$

2. Determine the radius of the first orbit of the hydrogen atom. What' would be the velocity and frequency of the electron in the first orbit?

Given: $h = 6.62 \times 10^{-34} J s$, $m = 9.1 \times 10^{-31} kg$, $e = 1.6 \times 10^{-19} C$, $k = 9 \times 10^9 m^2 C^{-2}$

A. The radius of the n^{th} orbit of hydrogen atom is given by $r_n = 0.53 n^2 A^\circ$

For first orbit, $n = 1$, $r_1 = 0.53 A^\circ$

Velocity of electron in n^{th} orbit, $v_n = \frac{nh}{2\pi m r_n}$

For first orbit, $n = 1 \therefore v_1 = \frac{h}{2\pi m \times 0.53 \times 10^{-10}} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-10}} = 2.19 \times 10^6 ms^{-1}$

$$f = \frac{m r^2 4\pi^2}{nh} \Rightarrow f = \frac{4\pi^2 m (0.53)^2 n^4}{nh} \Rightarrow f = \frac{4\pi^2 m (0.53)^2 n^3}{h} = 6.6 \times 10^{15} Hz (n = 1)$$

3. The total energy of an electron in the first excited state of the hydrogen atom is -3.4eV .
What is the potential energy of the electron in this state?

Sol: $TE_1 = -3.4\text{ eV}$

$$PE_1 = 2TE_1 = 2(-3.4)$$

$$= -6.8\text{ eV}$$

4. The total energy of an electron in the first excited state of the hydrogen, atom is -3.4eV .
What is the kinetic energy of the electron in this state?

A. $E_k = \frac{ke^2}{2r}, E = -\frac{ke^2}{2r}$ In the first excited state

$$E_k = -E = -(-3.4)\text{ eV} = 3.4\text{ eV}$$

5. Find the radius of the hydrogen atom in its ground state. Also calculate the velocity of the electron in $n=1$ orbit?

Given $h = 6.63 \times 10^{-34}\text{J s}$, $m = 9.1 \times 10^{-31}\text{kg}$, $e = 1.6 \times 10^{-19}\text{ C}$, $k = 9 \times 10^9\text{ N m}^2\text{C}^{-2}$.

A. The radius of the n^{th} orbit of hydrogen atom is given by $r_n = 0.53 n^2 \text{ \AA}$

For first orbit, $n = 1$, $r_1 = 0.53 \text{ \AA}$

Velocity of electron in n^{th} orbit, $v_n = \frac{nh}{2\pi m r_n}$

For first orbit, $n = 1$ $\therefore v_1 = \frac{h}{2\pi m \times 0.53 \times 10^{-10}}$

$$= \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-10}} = 2.19 \times 10^6 \text{ ms}^{-1}$$

6. Prove that the ionization energy of hydrogen atom is 13.6 eV ?

Sol: $n_1 = 1, n_2 = \infty, E_{n_1} = -13.6\text{ eV}$

$$E_{n_2} = E_{\infty} = 0$$

Ionization energy $E_{n_2} - E_{n_1} = E_{\infty} - E_{n_1} = 0 - (-13.6) = 13.6\text{ eV}$

7. Calculate the ionization energy for a lithium atom?

Sol: $n = 2$

$$E = E_{\infty} - E_2$$

$$E = 0 - \left(\frac{-13.6}{n^2} \right) = \frac{13.6}{4} = 3.4 \text{ ev.}$$

8. The wavelength of the first member of Lyman Series is 1216 A. Calculate the wavelength of second member of Balmer series?

Sol: wavelength of different members of Lyman series is given by

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n_2^2} \right]$$

$$(n_2 = 2, 3, 4, \dots)$$

For first member, $n_2 = 2$

$$\Rightarrow \frac{1}{\lambda_{\alpha}} = R \left[\frac{1}{1} - \frac{1}{4} \right] = \frac{3R}{4} \Rightarrow \lambda_{\alpha} = \frac{4}{3R} = 1216 \text{ A}^{\circ}$$

Wavelength of different members of Balmer series is given by

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right]$$

Where $n_2 = 3, 4, 5, \dots$

For 2nd member, $n_2 = 4$

$$\therefore \frac{1}{\lambda_{\beta}} = R \left[\frac{1}{4} - \frac{1}{16} \right] = \frac{3R}{16} \Rightarrow \lambda_{\beta} = \frac{16}{3R}$$

$$\therefore \frac{\lambda_{\beta}}{\lambda_{\alpha}} = \frac{\frac{16}{3R}}{\frac{4}{3R}} = 4 \Rightarrow \lambda_{\beta} = 4\lambda_{\alpha} = 4(1216) = 4864 \text{ A}^{\circ}$$

9. The wavelength of first member of Balmer Series is 6563 \AA . Calculate the wavelength of second member of Lyman series?

Sol: From $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right)$

For first member of Balmer series

$$n_2 = 3$$

$$\frac{1}{\lambda_\alpha} = R \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R}{36} \quad \text{OR} \quad \lambda_\alpha = \frac{36}{5R} = 6563 \text{ \AA}$$

From $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n_2^2} \right)$

For second member of Lyman series

$$n_2 = 3$$

$$\frac{1}{\lambda_\beta} = R \left(1 - \frac{1}{9} \right) = \frac{8R}{9}$$

$$\therefore \lambda_\beta = \frac{9}{8R} \Rightarrow \frac{\lambda_\beta}{\lambda_\alpha} = \frac{\frac{9}{8R}}{\frac{36}{5R}} = \frac{5}{32}$$

$$\lambda_\beta = \frac{5}{32} \lambda_\alpha = \frac{5}{32} \times 6563 = 1025.5 \text{ \AA}$$

10. The second member of Lyman Series in hydrogen spectrum has wavelength 5400 \AA . Find the wavelength of first member?

A. $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

For Lyman series $n_1 = 1$ and $n_2 = 2, 3, 4, \dots$

Let λ_1 and λ_2 be the wavelengths of the first and second lines respectively.

$$\frac{1}{\lambda_1} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = R \left(1 - \frac{1}{4} \right) = \frac{3}{4} R$$

$$\frac{1}{\lambda_2} = R \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = R \left(1 - \frac{1}{9} \right) = \frac{8}{9} R$$

$$\frac{1}{\lambda_2} \times \frac{\lambda_1}{1} = \frac{1}{\lambda_2} = \frac{8}{9} R \times \frac{4}{3R} = \frac{32}{27}$$

$$\lambda_1 = \frac{32}{27} \times \lambda_2 = \frac{32}{27} \times 5400 \text{ \AA} = 6400 \text{ \AA}$$

11. Calculate the shortest wavelength of Balmer Series. Or calculate the wavelength of the Balmer Series limit. Given: $R=10970000\text{m}^{-1}$

A. The last spectral line of Balmer series has the shortest wavelength or highest frequency. This line is obtained when the electron jumps from last orbit ($n_2 = \infty$) to second orbit ($n_1 = 2$)

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) \text{ Or } \frac{1}{\lambda} = \frac{R}{4} \text{ or } \lambda = \frac{R}{4}$$

$$\lambda = \frac{4}{10970000} \text{ m} = 3.646 \times 10^{-7} \text{ m}$$

$$= 3646 \times 10^{-10} \text{ m} = 3646 \text{ \AA}$$

12. Using the Rydberg formula, calculate the wavelength of the first four spectral lines in the Balmer series of the hydrogen spectrum.

A. The Rydberg formula is

$$\frac{hc}{\lambda_{if}} = \frac{1}{2} mc^2 \alpha^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The wavelength of the first four lines in the Balmer series correspond to transitions from $n_1 = 3, 4, 5, 6$, to $n_f = 2$. We know $\frac{1}{2} mc^2 \alpha^2 = 13.6\text{eV} = 21.76 \times 10^{-19} \text{ J}$

$$\lambda_{i2} = \frac{hc}{21.76 \times 10^{-19} \times \left(\frac{1}{4} - \frac{1}{n_i^2} \right)} \text{ m}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8 \times 4 n_i^2}{21.76 \times 10^{-19} \times (n_i^2 - 4)} \text{ m}$$

$$\frac{3.653 n_i^2}{(n_i^2 - 4)} \times 10^{-7} \text{ m} = \frac{3653 n_i^2}{(n_i^2 - 4)} \text{ \AA}$$

Substituting $n_i = 3, 4, 5$ and 6 , we get $\lambda_{32} = 6575 \text{ \AA}$, $\lambda_{42} = 4870 \text{ \AA}$, $\lambda_{52} = 4348 \text{ \AA}$ and $\lambda_{62} = 4109 \text{ \AA}$