ALTERNATING CURRENT

Important Points:

1. The alternating current (AC) is generally expressed as

$I = I_0 \sin(\omega t + \phi)$ Where i_0 = peak value of alternating current.

2. emf of an alternating current source is generally given by $E = E_0 \sin(\omega t + \phi)$

Where $e_0 = BAN\omega$ is known as peak value of alternating emf

3. Average or Mean Value:

a) It is the steady current (DC) which when passes through a circuit for half time period of AC sends the same charge as done by the AC in the same time.

b) The average value of AC for the complete cycle is zero.

c) The average value of AC for the half cycle of AC is given by.

$$I_{avg} = \frac{2I_0}{\pi} 0.636 I_0$$

Similarly, E_{avg} is also $\frac{2E_0}{\pi}$ for half cycle of AC

4. RMS Value OR Virtual Value OR Effective Value:

The RMS value of AC is the steady current (DC) which when flowing through a given resistance for a given time, produces the same amount of heat as produced by the AC when flowing through the same resistance for the same time.

$I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$ And $E_v = \frac{E_0}{\sqrt{2}} = 0.707 E_0$

RMS value of AC for half cycle is also same as above

5. AC through A Pure Resistor

$E = E_0 \sin \omega t$ And $I = I_0 \sin \omega t$ Where E_0 and I_0 are the peak values of

Voltage and current of AC respectively

Since the emf and current raise or fall simultaneously they are in phase with each other.

6. AC through A Pure Inductor:

$$E = E_0 \sin \omega t \quad \text{And} \quad I = \frac{E_0}{L\omega}\sin\left(\omega t - \frac{\pi}{2}\right)$$

The term L_ω has the units of resistance and it is called as resistance of the inductor or inductive reactance (X_L)

$$I = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \Rightarrow I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where I_0 is the peak value of current. Hence the current is lagging behind emf **or** emf is leading current by a phase difference of $\frac{\pi}{2}$.

7. AC through A Pure Capacitor:

$$E = E_0 \sin \omega t \quad \text{And} \quad I = \frac{E_0}{1/c\omega}\cos \omega t$$

The term $\frac{1}{c\omega}$ is called capacitive reactance where it has the dimensions of resistance. It is also called resistance of capacitor.

$$I = \frac{E_0}{X_c}\sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$ Where I_0 is peak value of AC

Hence current leads emf or emf is lagging behind current by $\frac{\pi}{2}$

8. AC through LR Circuit:

Let V_L and V_R be the instantaneous voltages across the inductor and resistor respectively.

$$V_L = IX_L = IL\omega \quad \text{And} \quad V_R = IR$$

Where X_L is the inductive reactance when AC flow through a pure resistor,

$$\therefore E = I\sqrt{R^2 + (L\omega)^2}$$

$$Z = \sqrt{R^2 + (L\omega)^2} \quad \text{And} \quad \tan\phi = \frac{L\omega}{R}$$

This is the phase angle by which the emf leads the current in L – R circuit. z is called impedance of LR circuit.

9. AC through C – R Circuit:

$V_c = I \times X_c = \frac{I}{C_\omega}$ And $V_R = IR$ Where X_C = capacitive reactance

$$E = \sqrt{V_R^2 + V_C^2} \Rightarrow E = \sqrt{(IR)^2 + \left(\frac{I}{C_\omega}\right)^2}$$

$$E = I\left(\sqrt{R^2 + \frac{1}{(C_\omega)^2}}\right) \quad \text{And} \quad Tan\phi = \frac{V_C}{V_R} = \frac{1}{C_\omega R}$$

This is the phase angle by which the emf lags behind the current in C – R circuit

Here $Z = \sqrt{R^2 + \left(\frac{1}{C_\omega}\right)^2}$ is called the impedance of C – R circuit.

10. AC through L – C – R Circuit:

a) $V_L = LX_L = IL\omega; V_C = IX_C = \frac{1}{C\omega}$ and $V_R = IR$

b) $E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

c) $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ is called the impedance of L – C – R circuit.

d) $Tan\phi = \frac{V_L - V_C}{V_R} = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$ This is the phase angle by which the emf leads the current.

e) If $L\omega = \frac{1}{C\omega}$, then $Tan\phi = 0$ or $\phi = 0$ and hence emf and current are in phase. The circuit in this condition behaves like a pure resistor circuit. This condition is called resonance condition.

f) At resonance, $L\omega = \frac{1}{C\omega}$ or $\omega^2 = \frac{1}{LC}$

$$\omega = 2\pi n = \frac{1}{\sqrt{LC}} \Rightarrow n = \frac{1}{2\pi\sqrt{LC}}$$

11. Advantages of AC Over DC:

a) The generation of AC is more economic than DC

b) AC voltages can be easily stepped up or stepped down using transformers.

c) AC can be transmitted to longer distances with less loss of energy.

d) AC can be easily converted into DC by using rectifiers.

12. Disadvantages:

a) AC is more fatal and dangerous than DC.

b) AC always flows on the outer layer of the conductor (skin effect) and hence AC requires stranded wires.

c) AC cannot be used in electrolysis like electroplating etc.

13. Transformer:

This works on the principle of mutual inductance between two circuits linked by common magnetic flux.

Step up transformers converts low voltage high current into high voltage low current.

Step down transformer: Converts high voltage low current into low voltage high current.

E.g.: In bed lamps, we will use this type of transformer.

14. Transformer works only with ac (Alternating Current)

15. Transformation ratio $\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$

Very Short Answer Questions

1. A transformer converts 200 V ac into 2000 V ac. Calculate the number of turns in the secondary if the primary has 10 turns?

A. $V_P = 200V$ $V_S = 2000V$

$n_P = 10$ $n_S = ?$

$\frac{V_P}{V_S} = \frac{n_P}{n_S}$

$n_S = \frac{V_S}{V_P}.n_P = \frac{2000}{200} X 10 = 100$

2. What type of transformer is used in a 6V bed lamp?

A. Step down transformer.

3. What is the phenomenon involved in the working of a transformer?

A. A transformer works on the principle of mutual induction.

4. What is transformer ratio?

A. $\frac{v_S}{v_P} = \frac{\text{Number of turns in secondary}(N_S)}{\text{Number of turns in primary}(N_P)}$

5. Write the expression for the reactance of i) an inductor and ii) a capacitor?

A. Inductive reactance $X_L = \omega L$

Capacitive reactance $X_C = \frac{1}{\omega C}$

6. What is the phase difference between AC emf and current in the following Pure resistor, pure inductor and pure capacitor?

A. Phase difference between ac emf and current

i) In pure resistor: zero

ii) In pure Inductor: Voltage leads current by 90^0

iii) In pure capacitor: Current leads voltage by 90^0

7. Define power factor. On which factors does power factor depend?

A. The average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called power factor.

$p = VI\cos\phi$

Power factor depends on nature of elements (Resistor, Inductor, Capacitor) in the circuit.

8. What is meant by wattless component of current?

A. If the voltage and current differ in phase by $\pi/2$, then Power factor, $\cos\phi = \cos 90^0 = 0$.

In this case, the current has no power. Such a current is, therefore, called **wattless current**. Since this current does not perform any work, this current may also be called idle current. Such a current flows only in purely inductive or in purely capacitive circuits.

9. When does a LCR circuit have minimum impedance?

A. When $X_L = X_C$ *or* $L\omega = \frac{1}{C\omega}$. Then $\tan\phi = 0$ *or* $\phi = 0°$ $\tan\phi = 0$ *or* $\phi = 0°$. Thus, there is no phase difference between current and potential difference. Therefore, the given LCR circuit is equivalent to a pure resistive circuit. The impedance of such LCR circuit is given by Z = R. which is minimum.

10. What is the phase difference between voltage and current when the power factor in LCR series circuit is unity?

A. The phase difference between voltage and current is zero**.**

Power factor $= \cos\phi = 1$

$\phi = 0$

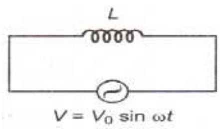
ϕ is phase difference between Voltage and current

Short Answer Questions

1. Obtain an expression for the current through an inductor when an AC emf is applied.

A. A.C. through a Pure Inductance:

Consider a pure inductor of inductance L (no resistance) connected to a source of emf ε. The instantaneous emf is given by $v = v_m \sin \omega t$



Let I be the current through the circuit and $\frac{di}{dt}$ be the rate of change of current in the circuit at any instant. The net emf in the circuit is given by

$$v - L\frac{di}{dt} = 0$$

Since there is no resistance in the circuit, there is no P.D. in the circuit.

$$\frac{di}{dt} = \frac{v}{L} = \frac{v_m}{L}\sin \omega t$$

Integrating the above equation,

$$\int \frac{di}{dt}dt = \frac{v_m}{L}\int \sin \omega t\, dt$$

$$\therefore i = \frac{v_m}{L\omega}(-\cos \omega t) + \text{Constant}$$

Since the current is oscillatory, time independent constant does not exist.

$$\therefore i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

The term $L\omega$ has the units of resistance and it is called as inductive reactance (X_L)

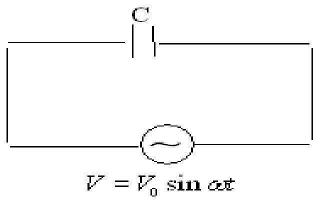
$i = \frac{v_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \Rightarrow i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$ Here is the peak value of current. Hence the current is lagging behind emf or emf is leading current by a phase difference of $\frac{\pi}{2}$

2. Obtain an expression for the current in a capacitor when an Ac emf is applied?

A. A.C. through a pure capacitor:

Consider a capacitor of capacity C connected to a source of alternating emf ε. The instantaneous emf is given by.

$$v = v_m \sin \omega t$$



The net emf in the circuit is given by,

$$v - \frac{q}{C} = 0 \quad \text{(Or)} \quad v = \frac{q}{C}$$

Or $\frac{q}{C} = v_m \sin \omega t$

$$\therefore i = \frac{dq}{dt} = \frac{d\left(v_m C \sin \omega t\right)}{dt} = C\omega v_m \cos \omega t$$

(or) $i = \frac{v_m}{1/c\omega} \cos \omega t$ Or $i = \frac{v_m}{X_L} \sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$

The term $\frac{1}{C\omega}$ is called capacitive reactance and it has the dimensions of resistance. Here i_m is peak value of AC. Hence current leads emf or emf is lagging behind current by $\frac{\pi}{2}$.

3. State the principle on which a transformer works. Describe the working of a transformer with necessary theory?

A. Transformer:

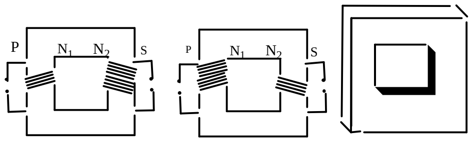
A transformer converts high voltage low currents into low voltage high currents and vice-versa. Transformer works only for AC.

Principle:

A transformer works on the principle of mutual inductance between two coils linked by a common magnetic flux.

Construction:

A transformer consists of two mutually coupled insulated coils of wire wound on a continuous iron core. One of the coils is called primary coil and the other is called secondary coil. The primary is connected to an AC emf. And secondary to a load. Due to this alternating flux linkage, an emf. Is induced in the secondary due to mutual induction.



(a) Soft iron core (b) (c) Laminated core

Working:

Let N_P and N_S be the number of turns in the primary and secondary coils respectively. The induced emf's produced in primary and secondary coils are given by

$$\varepsilon_p = -N_p\left(\frac{d\phi}{dt}\right) \text{ and } \varepsilon_s = -N_s\left(\frac{d\phi}{dt}\right),$$

Hence $\dfrac{\varepsilon_s}{\varepsilon_p} = \dfrac{N_s}{N_p}$ Or $\dfrac{v_s}{v_p} = \dfrac{N_s}{N_p}$

Where v_P and v_S are the primary and secondary voltages.

If the efficiency of the transformer is 100 % , then $v_s i_s = v_p i_p$ or $\dfrac{i_p}{i_s} = \dfrac{v_s}{v_p} = \dfrac{N_s}{N_p}$

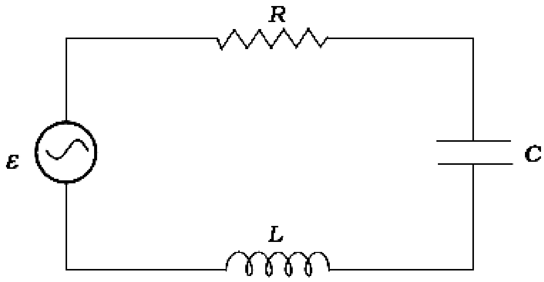
$\dfrac{N_s}{N_p}$ is called transformer ratio. If $N_s > N_p$, then it is called a step-up transformer. If $N_s < N_p$, then it is called a step-down transformer.

Long Answer Questions

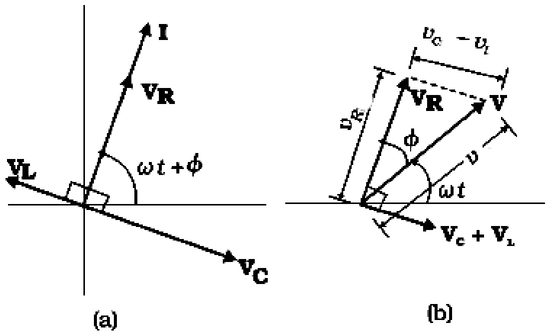
1. Obtain an expression for impedance and current in series LCR circuit. Deduce an expression for the frequency of an LCR series resonating circuit?

A. Phasor Diagram Solution:

Let an alternating emf $V=V_0 \sin \omega t$ be applied to a circuit containing a resistor of resistance R, a capacitor of capacitance C and an inductor of inductance L connected in series as shown in the fig.



A series *LCR* circuit connected to an ac source.



(a) Relation between the phasors V_L, V_R, V_C, and *I*, (b) Relation between the phasors V_L, V_R, and (V_L + V_C) for the circuit in Fig.

Let $i = i_0 \sin(\omega t + \phi)$ be ac current in each element at any time where ϕ is the phase difference between voltage of source and the current in the circuit. Let V_L, V_R, V_C and V represent the voltage across the inductor, resistor, capacitor and the source respectively which are shown in the phasor.

The voltage equation for the circuit is $V_L + V_R + V_C = V$ and the amplitudes of $V_L = i_0 X_L, V_R = i_0 R, V_C = i_0 X_e$ and $V = V_0$

From the diagram (B)

$$V_0^2 = V_R^2 + (V_C - V_L)^2$$

$$V_0^2 = (i_0 R)^2 + (i_0 X_C - i_0 X_L)^2 \quad = i_0 \left[R^2 + (X_C - X_L)^2\right]$$

Then $i_0 = \dfrac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}}$

Or $i_0 = \dfrac{V_0}{Z}$ where $Z = \sqrt{R^2 + (X_C - X_L)^2}$ impedance of the circuit

And also $\tan\phi = \dfrac{V_C - V_L}{V_R} = \dfrac{i_0 X_C - i_0 X_L}{i_0 R}$

i.e. $\tan\phi = \dfrac{X_C - X_L}{R}$ where ϕ is the phase angle between V_R and V

Resonance: At a resonant frequency, the total reactance of the circuit is zero and the impedance will be minimum.

From the expression $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$

The impedance of the circuit $Z=R$, $(\because\ X_L=X_C)$

$$\therefore \omega L=\frac{1}{\omega C}\Rightarrow \omega^2=\frac{1}{LC}\Rightarrow \omega=\frac{1}{\sqrt{LC}}$$

But $\omega=2\pi f$ where f is the resonant frequency then $2\pi f=\frac{1}{\sqrt{LC}}$ $\quad f=\frac{1}{2\pi\sqrt{LC}}$

At resonant frequency of LCR series circuit, the impedance is minimum equal to R and the current in the circuit is maximum.

PROBLEMS

1. **An ideal inductor (no internal resistance for the coil) of 20 mH is connected in series with an AC ammeter to an AC source whose emf is given by $e = 20\sqrt{2}\,\sin(200t + \pi/3)V$, where t is in seconds. Find the reading of the ammeter?**

Sol: $e = 20\sqrt{2}\ \sin\left(200t + \frac{\pi}{3}\right)$

Comparing with $e = e_o \sin(\omega t + \phi)$

$\omega = 200\ rads^{-1};\quad e_o = 20\sqrt{2}\ v.$

$L = 20\ mH = 20\times10^{-3}H$

$$i_o = \frac{e_o}{x_L} = \frac{e_o}{\omega L} = \frac{20\sqrt{2}}{200\times20\times10^{-3}}$$

$i_o = 5\sqrt{2}\ A$

$$i_{rms} = \frac{i_o}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5A$$

2. **The instantaneous current and instantaneous voltage across a series circuit containing resistance and inductance are given by $i = \sqrt{2}\sin(100t - \pi/4)A$ and v = 40 sin (100t) V. Calculate the resistance?**

Sol: $i = \sqrt{2}\ \sin\left(100t - \frac{\pi}{4}\right)A$

Comparing with $i = i_o\ \sin(\omega t - \phi)$

$i_o = \sqrt{2},\ \omega = 100\ rads^{-1}$

Comparing v = 40 sin (100t) with

$V = V_O\ \sin\ \omega t$

$v_o = 40v,\ \omega = 100\ rads^{-1}$

$$Z = \frac{v_o}{i_o} = \frac{40}{\sqrt{2}} = 20\sqrt{2}\ \Omega$$

$Z = \sqrt{R^2 + (L\omega)^2} = 20\sqrt{2}$

$R^2 + (L\omega)^2 = 800$

But $Tan\phi = \frac{L\omega}{R}$

Tan $\pi/4 = \frac{L\omega}{R} \Rightarrow L\omega = R$

$\therefore R^2 + R^2 = 800 \Rightarrow R^2 = 400$

$R = 20\Omega$

3. In an AC circuit a condenser, a resistor and a pure inductor are connected in series across an alternator (AC generator). If the voltage across them is 20 V, 35 V and 20V respectively, find the voltage supplied by the alternator?

Sol: $V_C = 20v,\ V_R = 35v,\ V_L = 20v$

$$V = \sqrt{V_2^2 + (V_L - V_C)^2} = \sqrt{(35)^2 + (20-20)^2} = 35V$$

4. An Ac circuit contains resistance R, an inductance L and a capacitance C connected in series across an alternator of constant voltage and variable frequency. At resonant frequency, it is found that the inductive reactance, the capacitive reactance and the resistance are equal and the current in the circuit is i_0. Find the current in the circuit at a frequency twice that of the resonant frequency?

A. $R = X_L = X_C$

$i_0 = \frac{V_0}{Z} = \frac{V_0}{R}$ (at resonance $X_L = X_C$)

When frequency is doubled $X_L^{'} = 2X_L \Rightarrow X_C^{'} = \frac{X_C}{2}$

$$Z' = \sqrt{R^2 + (X_L^{'} - X_C^{'})^2} = \sqrt{R^2 + \left(2R - \frac{R}{2}\right)^2} = \frac{\sqrt{13}R}{2}$$

$$i' = \frac{V_0}{Z'} = \frac{V_0}{\frac{\sqrt{13}R}{2}} = \frac{2V_0}{\sqrt{13}R} = \frac{2i_0}{\sqrt{13}}$$

5. A series resonant circuit L_1, R_1 and C_1. The resonant frequency is *F*. Another series resonant circuit contains L_2, R_2 and C_2. The resonant frequency is also F. If these two circuits are connected in series, calculate the resonant frequency?

A. $f = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{\omega}{2\pi}$ and

$f = \frac{1}{2\pi\sqrt{L_2C_2}} = \frac{\omega}{2\pi}$

$R_1 = L_1\omega = \frac{1}{C_1\omega}$ and $R_2 = L_2\omega = \frac{1}{C_2\omega}$

$L_1 = \frac{R_1}{\omega}$, $L_2 = \frac{R_2}{\omega}$

In series combination effective inductance (L) $= L_1 + L_2 = \frac{R_1 + R_2}{\omega}$

And $\frac{1}{C_1} = R_1\omega,\ \frac{1}{C_2} = R_2\omega$

In series combination effective capacitance (C) = $\frac{C_1C_2}{C_1+C_2} = \frac{1}{(R_1+R_2)\omega}$

$\therefore LC = \frac{1}{\omega^2}$

Resonating frequency $f' = \frac{1}{2\pi\sqrt{Lc}} = \frac{\omega}{2\pi} = f$

6. In a series LCR circuit R=200 W and the voltage and the frequency of the mains supply is 200V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 45^0. On taking out the inductor from the circuit the current leads the voltage by 45^0. Calculate the power dissipated in the LCR circuit?

A. $R = 200\Omega$

$V_{rms} = 200V$

$\tan\phi = \frac{Z}{R} = \frac{\sqrt{R^2+X_L^2}}{R} = \frac{\sqrt{R^2+X_C^2}}{R}$

$\therefore X_L = X_R \quad = P = \frac{V_{rms}^2}{R} = \frac{200\times200}{200} = 200W$

7. The primary of transformer with primary to secondary turns ratio of 1: 2, is connected to an alternator of voltage 200 V. A current of 4 A is flowing though the primary coil. Assuming that the transformer has no losses, find the secondary voltage and current are respectively?

Sol: $\frac{N_P}{N_S} = \frac{1}{2}, V_P = 200V.$

$\frac{N_S}{N_P} = \frac{V_S}{V_P} \Rightarrow \frac{2}{1} = \frac{V_S}{200} \Rightarrow V_S = 400V$