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# Curves

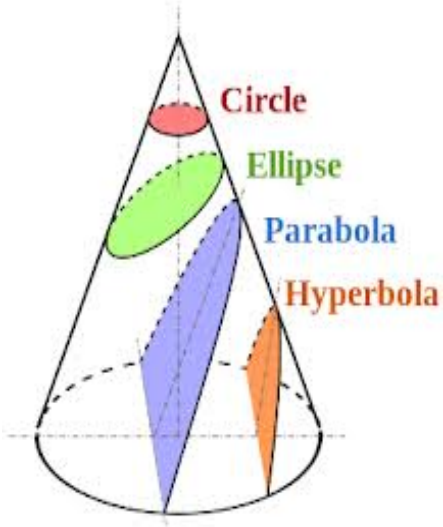
## CONICS

**Definition**: It is defined as the locus of point 'P' moving in a plane in such a way that the ratio of its distance from a fixed point 'F' to a fixed straight line is always a constant. This ratio is called eccentricity.

Eccentricity is denoted with 'e'.
F- Fixed point or Focus
Straight line is also called Directrix.

Based on eccentricity conics are:

1. Ellipse (e<1)
2. Parabola (e=1)
3. Hyperbola (e>1)

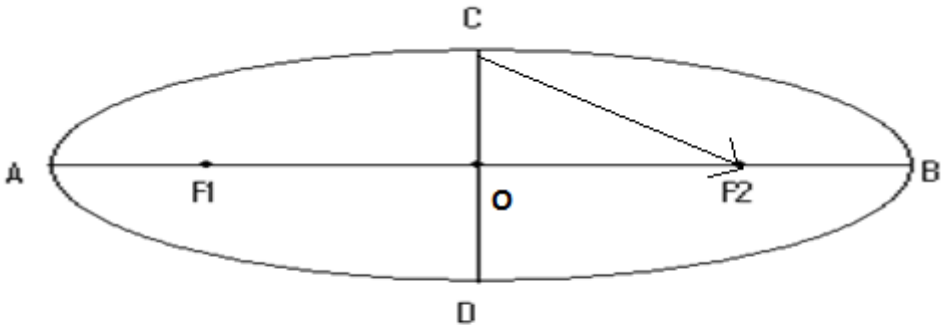


**ELLIPSE:**

The locus of a point 'P' which satisfies the equation $PF_1+PF_2=AB$ is called ellipse.

[OR]

The conic whose eccentricity is less than one is called ellipse. $(e < 1)$




**CF2=CF1=OA=OB =1/2 AB**

AB - major axis

CD - minor axis

$F_1$&$F_2$ - foci or fixed points

**Methods to draw Ellipse:**

1. General method (or) eccentricity method(or) Focus –Directrix method
2. Concentric circles method (or) Auxiliary circles method.
3. Rectangle method (or) Oblong method.
4. Parallelogram method.
5. Arcs of circles method (or) Intersecting arcs method (or) Foci method.

## Points to remember to solve conics problem

1. **First read the problem and decide which conic.**

    a. Sometimes conic will be mentioned.
        E.g.: draw the ellipse, draw parabola etc.
    b. Sometimes conic will not be mentioned.

**In this case decide the conic based on -**

i. Eccentricity
ii. On application
    A fountain jet discharges water (i.e. parabola)
    A foot ball is kicked (i.e. parabola)
    A Stone is thrown (i.e. parabola)
    A ball is thrown (i.e. parabola) etc.
iii. On Equation
    Equation $\mathbf{PF_1+PF_2=AB}$ is called Ellipse.
    Where AB is Major axis
    Equation $\mathbf{PF_1 \sim PF_2=AB}$ is called hyperbola.
    Where AB is Transverse axis

**Note 1:** The distance between the foci ($F_1$ $F_2$) is less than the Major axis (AB).

$\mathbf{F_1 F_2 < AB}$ **(i.e., Ellipse)**

**Note 2:** The distance between the foci ($F_1$ $F_2$) is greater than the Transverse axis (AB).

$\mathbf{F_1 F_2 > AB}$ **(i.e., Hyperbola)**

**2. After deciding the conic, next step is Choosing method.**

Decide the method based on the inputs given in the problem.

For this the student know the

i. Inputs required for each Method
ii. Other names of each Method

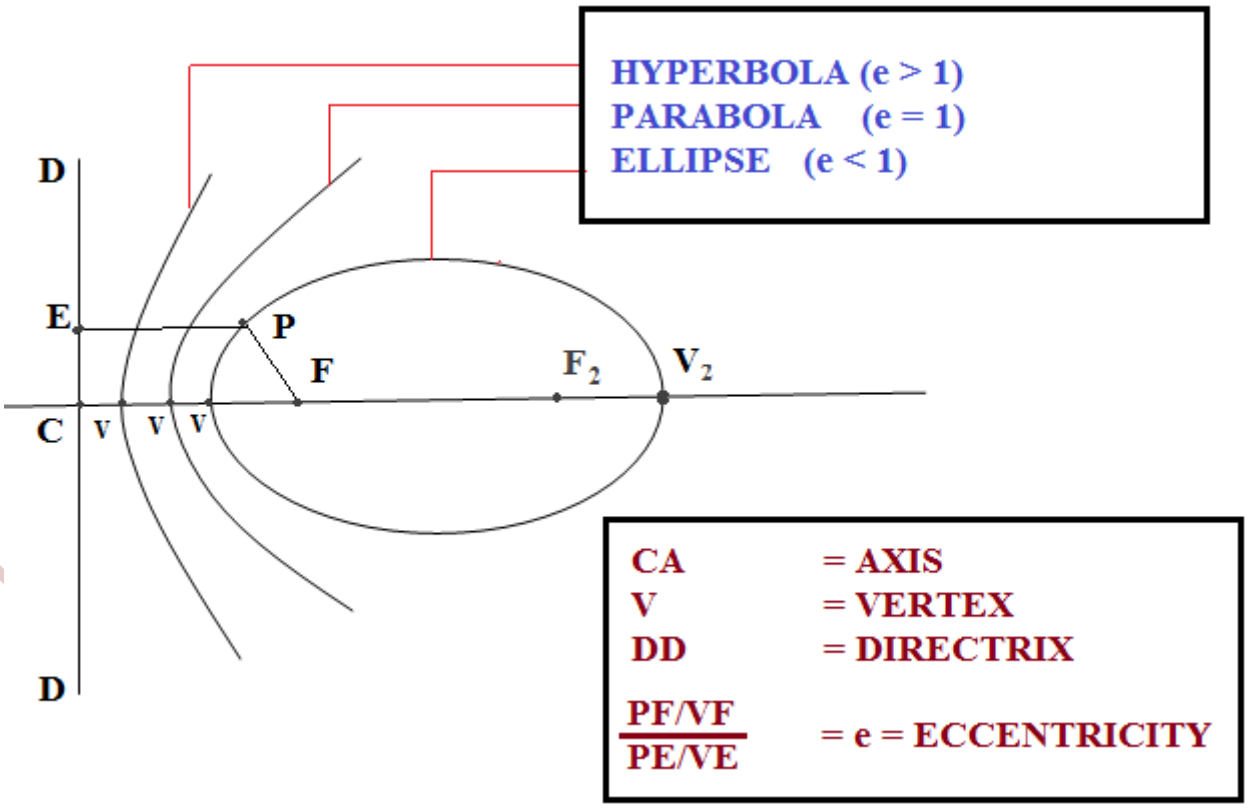
## Methods of Ellipse:

| Sl. No. | Method | Inputs Required | No. of inputs |
|---|---|---|---|
| 1. | General method<br>(or) eccentricity method<br>(or) Focus–Directrix method | Eccentricity,<br>distance from Directrix to focus/<br>distance from Directrix to vertex/<br>distance from vertex to focus | 2 |
| 2. | Concentric circles method<br>(or) Auxiliary circles method. | Major axis<br>Minor axis<br>(Know the relation between the major axis, minor axis and foci) | 2 |
| 3. | Rectangle method<br>(or) Oblong method | Major axis<br>Minor axis<br>(Know the relation between the major axis, minor axis and foci) | 2 |
| 4. | Parallelogram method | Major axis | 3 |

| | | | |
|---|---|---|---|
| | | Minor axis<br>Angle<br>(Know the relation between the major axis, minor axis and foci) | |
| 5. | Arcs of circles method<br>(or) Intersecting arcs method<br>(or) Foci method | Major axis<br>Minor axis<br>(Know the relation between the major axis, minor axis and foci) | 2 |

Table1: Methods of ellipse

## i. Eccentricity




$$\text{Eccentricity}(e) = \frac{\text{Distance of locus of point from focus}(VF)}{\text{Distance of locus of point from Directrix}(VC)}$$

$$e = \frac{VF}{VC}$$

Where V is the vertex (point on a conic)

And also vertex is the point of intersection of axis and conic.

## How to find the eccentricity for different cases:

**Case 1: By Value**

1. e = 0.67 (or) $e = \frac{2}{3}$
   Conic is ellipse since 0.67 is less than one (e<1)
2. e =1.5 (or) $e = \frac{3}{2}$
   Conic is hyperbola since 1.5 is greater than one (e>1)
3. $e = \frac{4}{3}$
   Conic is hyperbola since $\frac{4}{3}$ is greater than one (e>1)

**Case 2: By using definition of eccentricity**

1. The locus of a point P moving in a plane such a way that its distance from the straight line is $\frac{3}{2}$ times its distance from F.

**Solution:**

Distance from straight line = VC

Distance from focus = VF

$\Rightarrow$ $VC = \frac{3}{2} VF$

$\Rightarrow$ $\frac{VF}{VC} = \frac{2}{3}$

$\Rightarrow$ $e = \frac{2}{3}$

**Eccentricity is $\frac{2}{3}$ , it is less than 1 is so the conic is Ellipse.**

2. The locus of a point P moving in a plane such a way that its distance from the fixed point is always equal to its distance from Directrix.

**Solution:**
Distance from straight line/Directrix = VC
Distance from focus/fixed point = VF

$\Rightarrow$ VF = VC

$\Rightarrow$ $\frac{VF}{VC} = 1$

$\Rightarrow$ e = 1

**Eccentricity is 1. So the conic is Parabola.**

3. The locus of a point P moving in a plane such a way that its distance from the fixed point is twice its distance from Fixed straight line.

**Solution:**
Distance from straight line = VC
Distance from focus = VF

$\Rightarrow$ VF = 2 **VC**

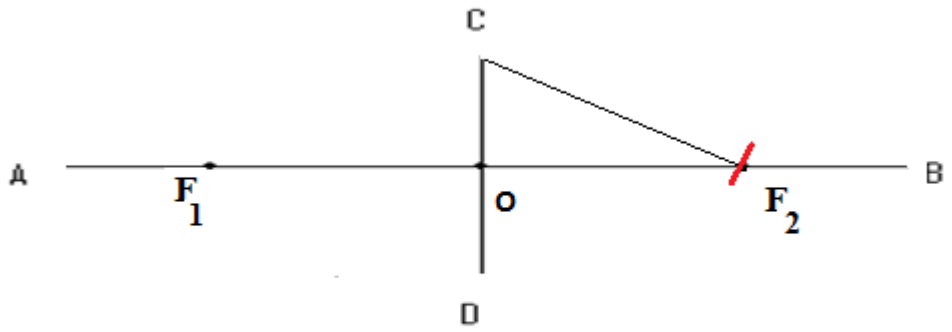
$\Rightarrow$ $\frac{VF}{VC} = 2$

$\Rightarrow$ e = 2

**Eccentricity is 2; it is greater than 1 is so the conic is Hyperbola.**

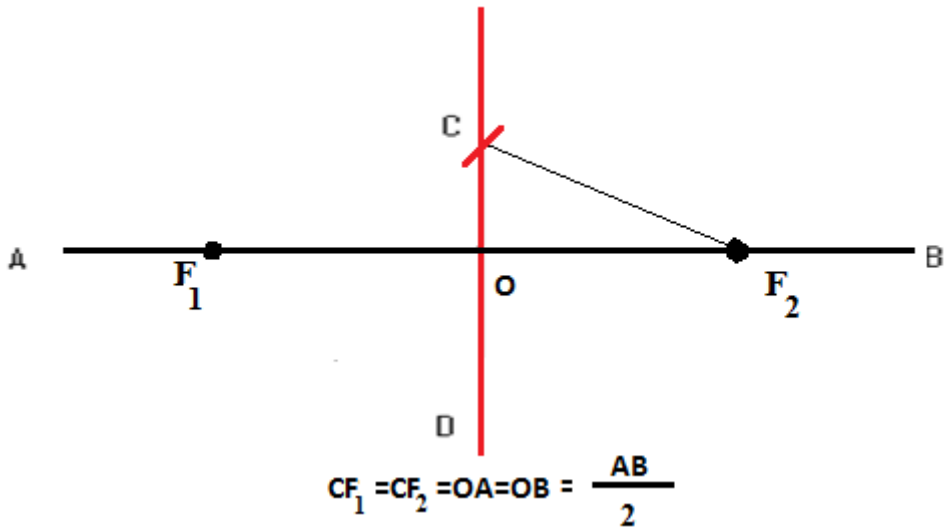
## Relation between Major axis (AB), Minor axis (CD) and Foci ($F_1F_2$):

**CASE 1:** How to locate $F_1$ & $F_2$ when AB & CD is given.




$$CF_1 = CF_2 = OA = OB = \frac{AB}{2}$$

**CASE 2:** How to locate CD when $F_1$ $F_2$ & CD is given.




$$CF_1 = CF_2 = OA = OB = \frac{AB}{2}$$

# Problems

## General method (or) eccentricity method (or) Focus–Directrix method

1. Draw a conic whose distance from Directrix to focus is 6cm and e=2/3.

   **Solution:**

   **Step1: Which conic: It is decided based on "e"**

   e = 2/3, which is less than 1, so the conic is Ellipse.

   In this case eccentricity is given directly as a value.

   **Step2: Which Method: It is decided based on inputs given in the problem. When method is not mention in the problem.**

   The inputs are 'e' and distance from Directrix to focus. (Refer Table1: Methods of ellipse)

   So the method is General method (or) eccentricity method (or) Focus–Directrix method

   **NOTE: The same problem may ask as below, Of course both are same.**

2. A fixed point 'F' is 6 cm from a fixed straight line. Draw the locus of a point 'p' moving in such a way that its distance from the straight line is 3/2 times its distance from 'F'. Name the curve.

   **Solution:**

   **Step1: Which conic: It is decided based on "e"**

   **In this case eccentricity is not given directly as a value, So use the definition of eccentricity.**

   Distance from straight line = VC

   Distance from F (focus) = VF

   $\Rightarrow \quad VC = \frac{3}{2}\,VF$

   $\Rightarrow \quad \frac{VF}{VC} = \frac{2}{3}$

   $\Rightarrow \quad e = \frac{2}{3}$

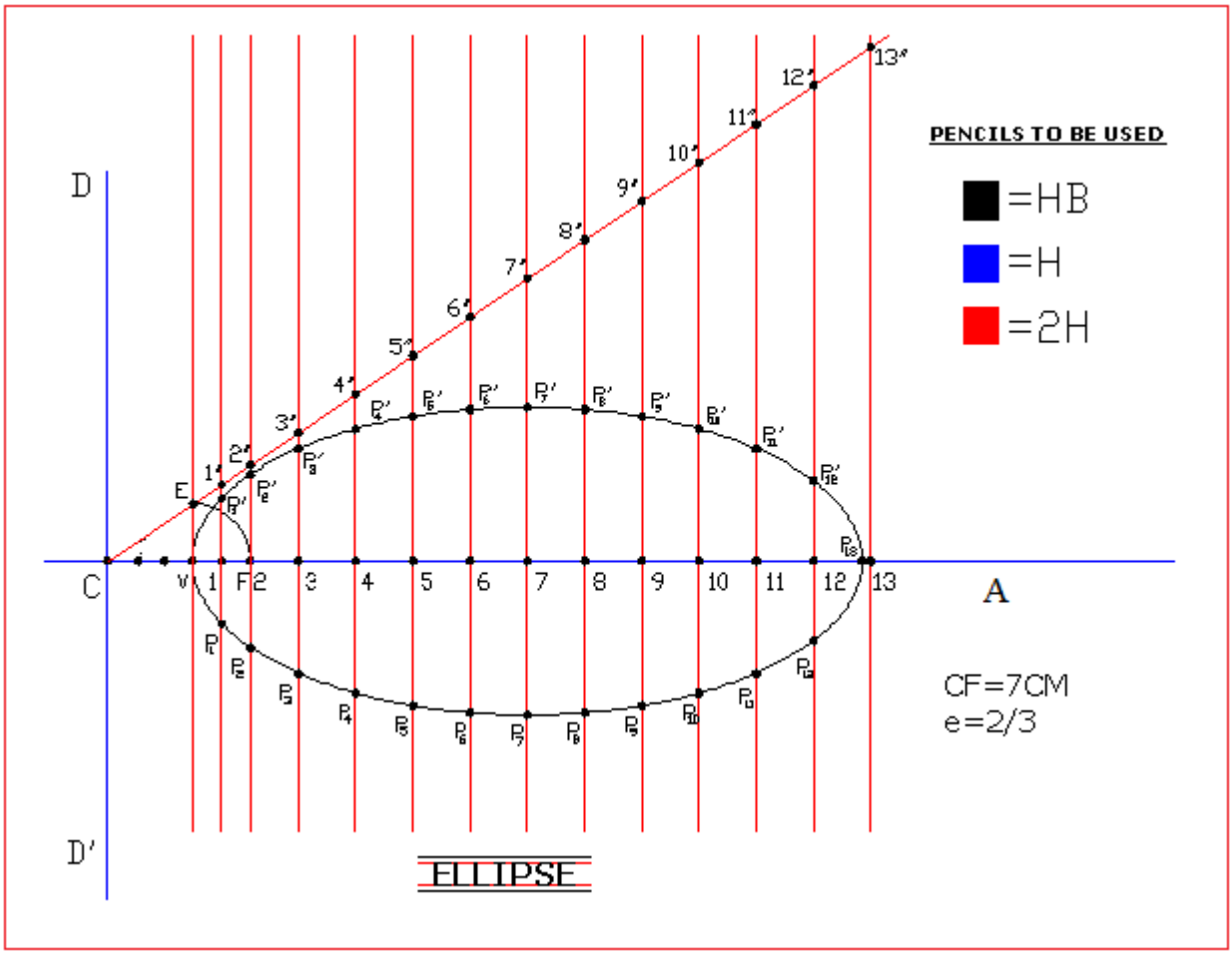
   e = 2/3, which is less than 1, so the conic is Ellipse.

   **Step2: Which Method: It is decided based on inputs given in the problem. When method is not mention in the problem.**

   The inputs are 'e' and distance from Directrix to focus. (Refer Table1: Methods of ellipse)

   So the method is General method (or) eccentricity method (or) Focus–Directrix method

1&2




**Drawing Procedure:**

1. Draw DD' line vertically and draw a horizontal line CA.
2. Locate F at a distance of 7cm and divide the CF into 5 equal parts since e is 2/3 (i.e., 2+3= 5 parts).by using a divide a line concept.
3. Locate V after two boxes (since 'e' is 2/3, numerator (VF) is 2 therefore 2 boxes) from F.
4. Draw a vertical line through V and locate E such that VE=VF.
5. Join CE and extend.
6. Now divide the CA line after V into some no. of equal parts. And name them as 1, 2. . etc.,
7. Draw vertical lines through 1, 2, and 3. . etc., these vertical lines intersect the CE extend line at 1', 2', 3' . . . so on.
8. Now take F as centre 11' as radius cut the vertical passing though 1 at p1 and p'1.
9. Similarly take F as centre 22' as radius cut the vertical passing though 2 at p2 and p'2.

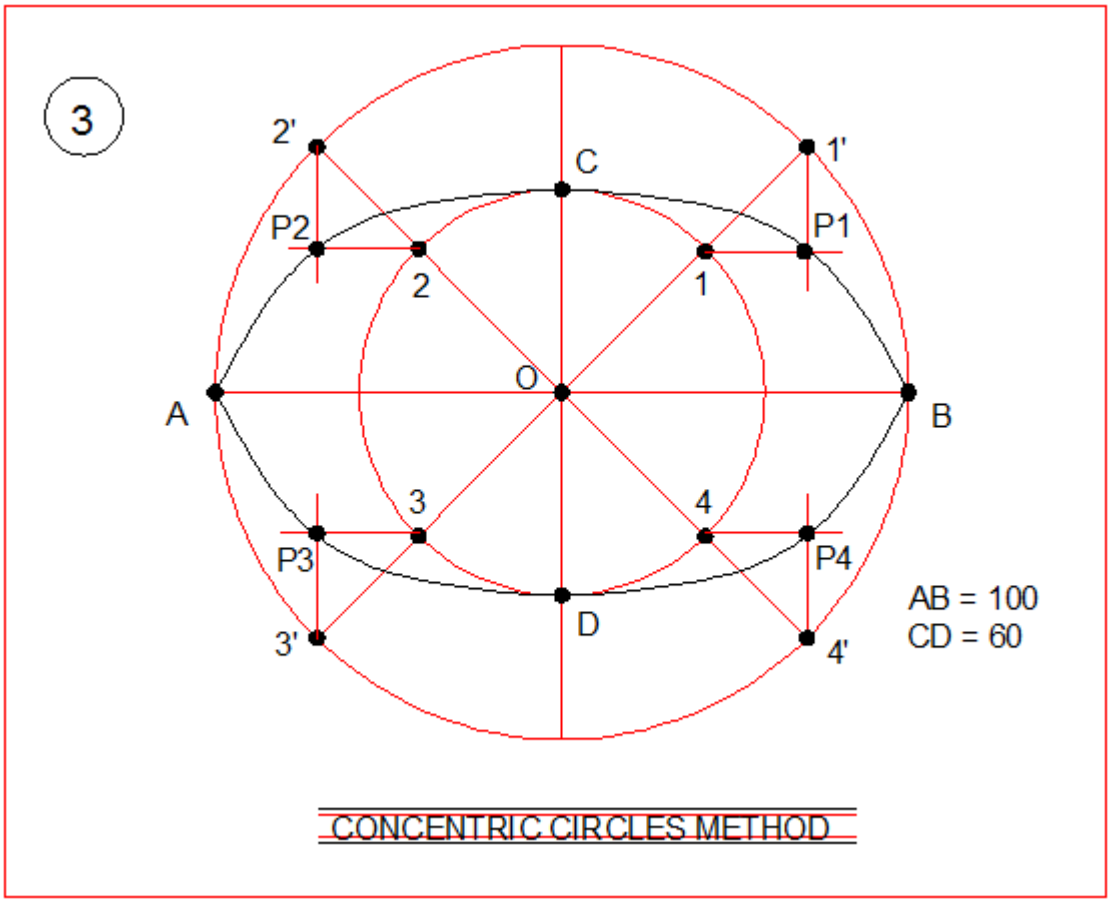
10. Continue this procedure till; the vertical will intersect at one point, instead of two points.
11. Now join all the points' p1, p2. .. P'1, p'2 . . . by maintaining a smooth curve with free hand.

**Note:**

1. Locate V properly.
2. Divide the axis line CA after V only.
3. The ellipse should pass through the vertex V.

## Concentric Circles Method (Or) Auxiliary Circles Method

3. Draw an ellipse with major axis 100mm and minor axis 60mm.




**Note:** Divide the circle into 12 parts to get the accurate ellipse.

**Drawing Procedure:**

1. Draw two concentric circles with diameter 100mm and 60mm.
2. Divide the circle into 8 or 12 parts.

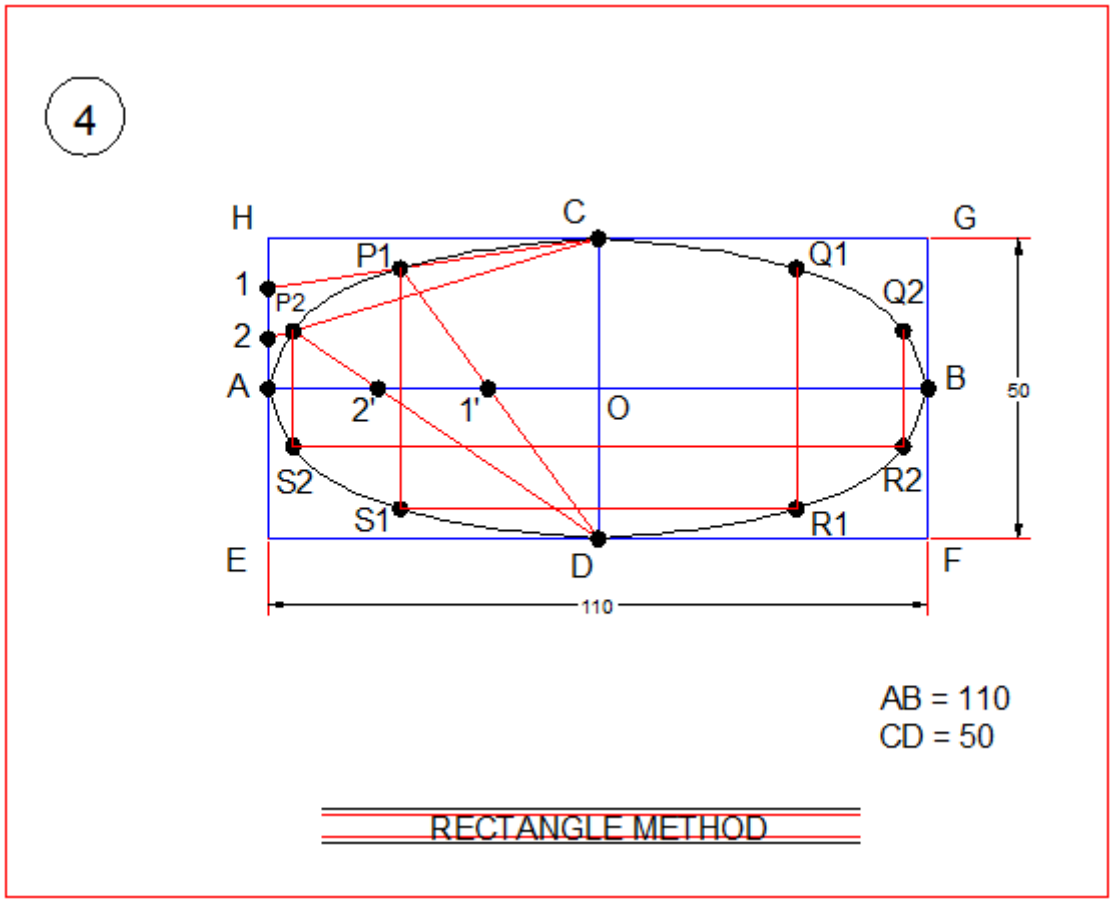
3. For major circle name the ends as A, B, 1', 2', 3', and 4'.
4. For minor circle name the ends as C, D, 1, 2, 3, 4.
5. Draw horizontal lines from 1, 2, 3 and 4 by using drafter only.
6. Draw vertical lines from 1', 2', 3' and 4' by using drafter only.
7. The intersection points of these horizontal lines and vertical lines are P1, P2, P3 and P4.
8. Draw the smooth curve by free hand through these points.

**Note:**

1. The ellipse should pass through the ends of major axis and minor axis i.e., A, B, C and D.
2. For ellipse major axis is AB and minor axis is CD.

## Rectangle Method (Or) Oblong Method

4. A lot of ground is in the shape of a rectangle 110m x 50m inscribed an elliptical lawn in it. Take a scale of 1:1000.

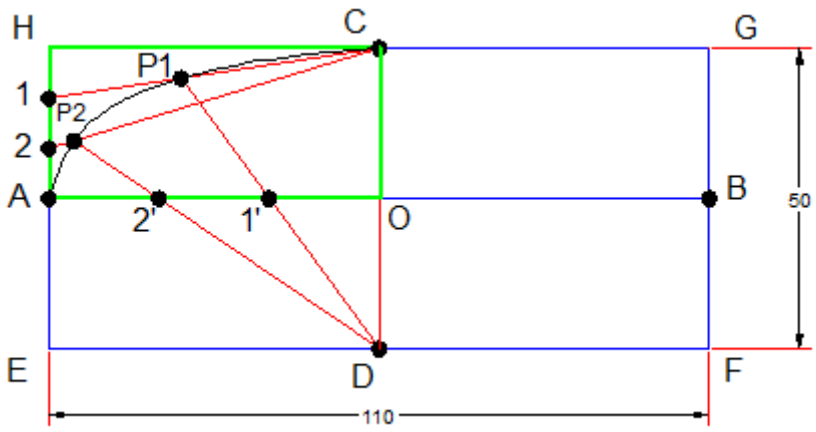


**Drawing Procedure:**

1. Draw a rectangle of 110X50.
2. Divide the rectangle into 4 parts. By drawing a lines AB and CD.
3. Divide HA length and AO length in to equal no. of parts say 3parts.(use divide a line concept if necessary).
4. Join C and 1, C and 2.
5. Join D and 1', D and 2'; extend the line D1' on to C1 and locate P1.
6. Extend the line D2' on to C2 and locate P2.
7. Since ellipse is symmetry about AB and CD locate the points Q1, Q2, R1, R2, S1 and S2.
8. Join these points Q1, Q2, R1, R2, S1 and S2 with free hand by maintaining a smooth curve.

**Note:**

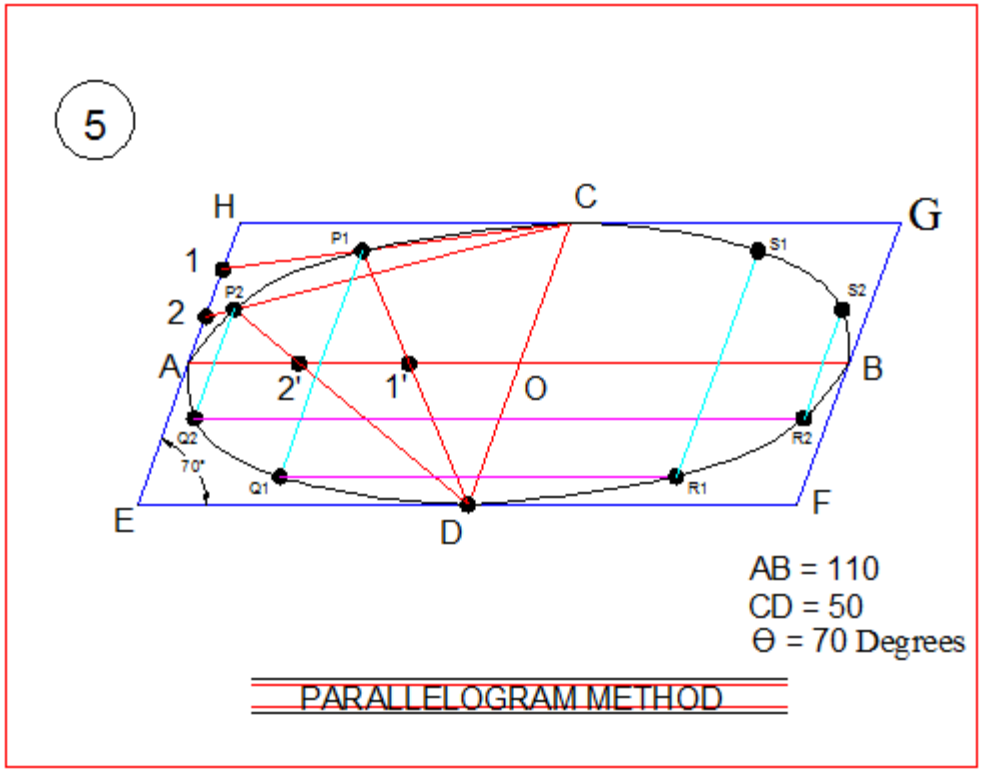
1. Ellipse is symmetry about two axes, so divide the rectangle into four parts.




2. Think about how to draw ellipse in one part i.e., in the rectangle HAOC. That is the reason we divide the lines AH and AO in to equal no. of parts.
3. Many students do the mistake by dividing the HE and AO in to equal parts. That is wrong practice.
4. The ellipse should pass through the ends of major and minor axis i.e., A,B,C and D.

## Parallelogram Method

5. Construct an ellipse when sides of parallelogram are AB and CD is equal to 110mm and 50mm respectively. The angle between the sides is $70^0$. (70 degrees).




**Note:**

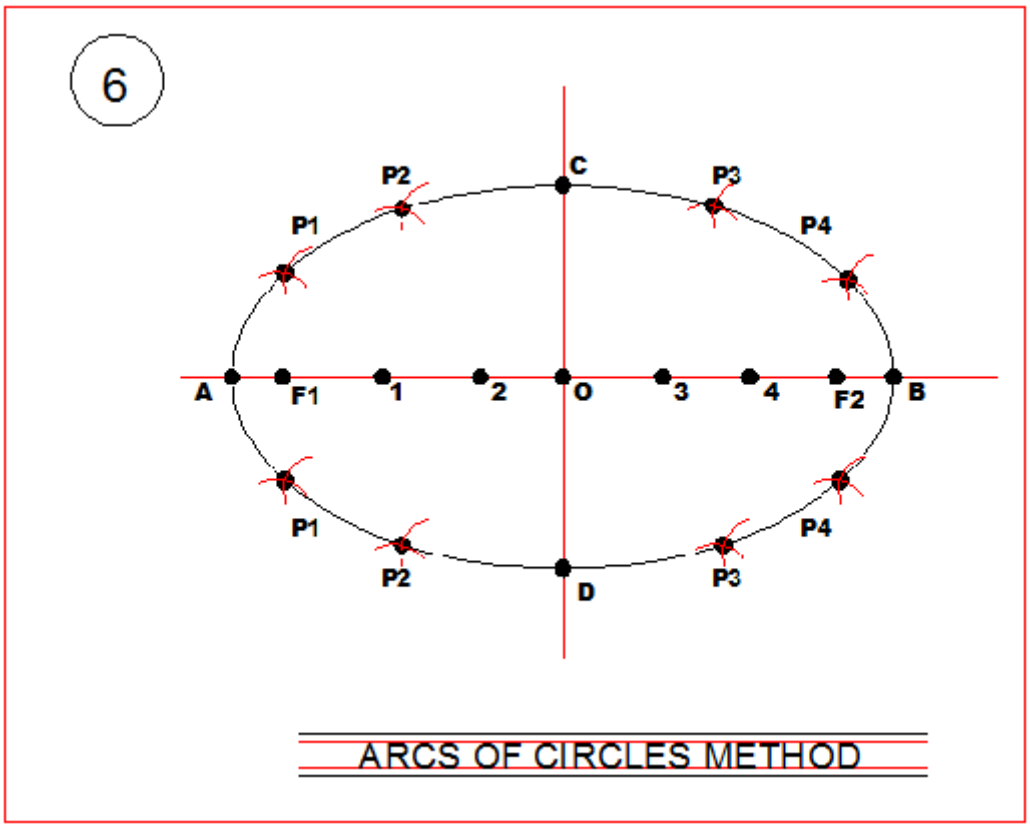
1. All the vertical lines in the rectangle method will become 70 degrees lines in this method.
2. P1 q1 and p2 q2 should parallel to CD or EH/HF. Many students do mistake here only.

## Arcs of circles method (or) Intersecting arc method (or) Foci method

6. Construct an ellipse where major axis is 125mm and distance between foci is 100mm. Determine the length of the minor axis.

**[OR]**

7. Two fixed points 'F1' and 'F2' are 100mm a part. Trace the curve in such a way that the sum of the distances from 'F1' and 'F2' is always equal to 125mm. Name the curves.




**Drawing Procedure:**

1. Draw Major axis and minor axis.(refer relation between major, minor and foci)
2. Locate f1 and f2. .(refer relation between major, minor and foci)
3. Divide f1 f2 in to some no. of equal parts and mark 1,2,3,4.
4. With A1 as radius f1 as centre, draw an arc above and below. Intersect this arc with B1 as radius F2 as centre and locate P1 above and below.
5. With A2 as radius f1 as centre, draw an arc above and below. Intersect this arc with B2 as radius F2 as centre and locate P2 above and below.
6. Similarly locate p3, p4.
7. Join p1, p2, p3, p4 with a smooth curve.

**Note:**

1. See that the ellipse should pass the ends of major and minor axis i.e. ., A, B, C and D.

# Tangent and Normal for the Conic:

## 1. When Directrix is there:

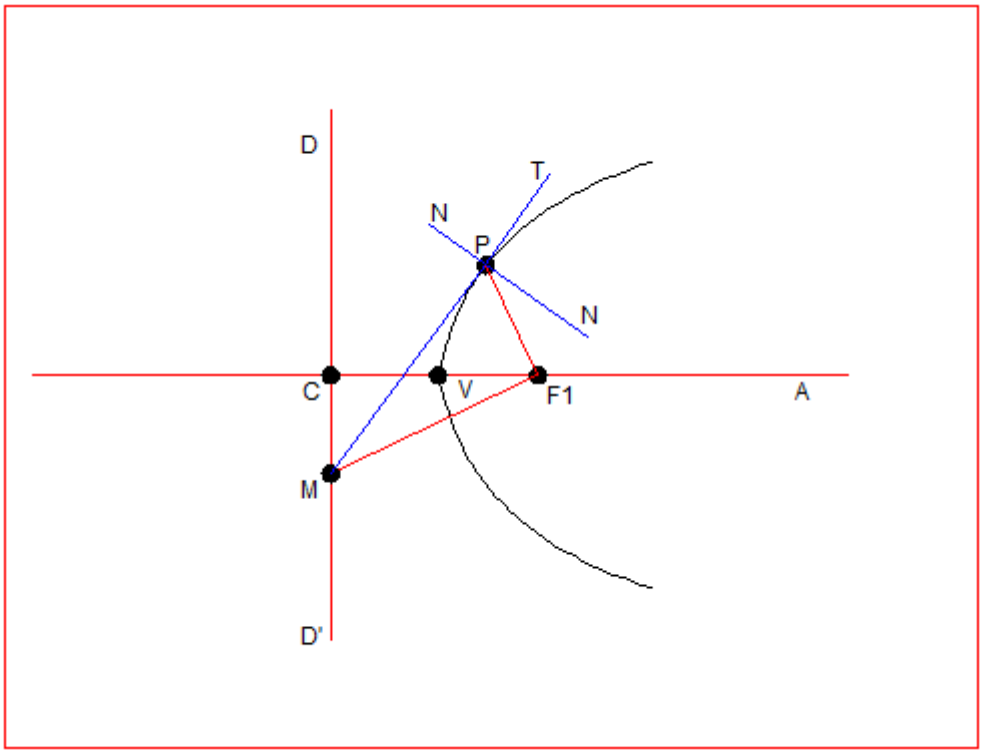
There are 3 ways of giving tangent and normal data.

Eg1: Draw the tangent & normal at any point on the curve.

Eg2: Draw the tangent & normal at a ____ distance from Directrix.

Eg3: Draw the tangent & normal at a ____ distance from focus.

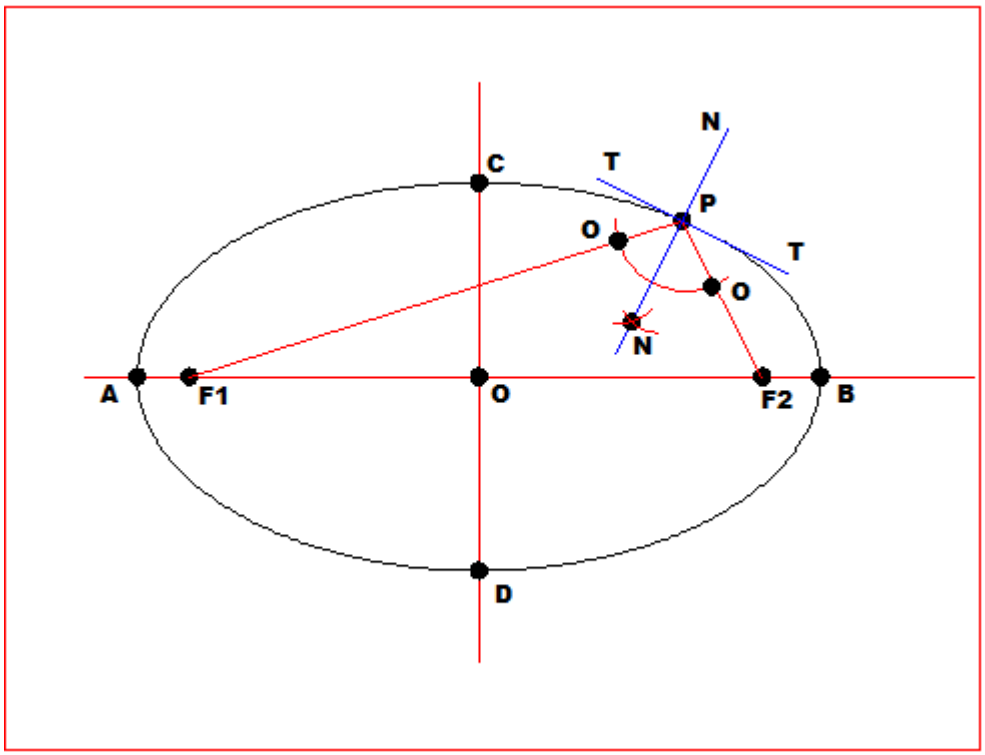
Based on the given input data locate P.




**Drawing procedure:**

1. Locate P, join p and f1.
2. Draw perpendicular to pf1 at f1 to DD'. locate M
3. Join m and p .i.e., tangent.
4. Perpendicular to tangent is normal.

**2. When directrix is not there for ellipse:**

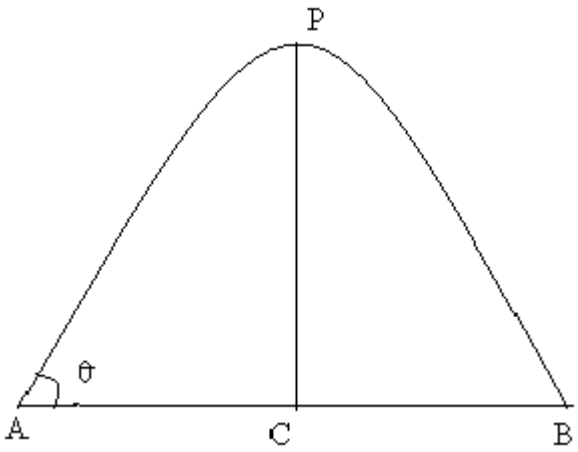



**Drawing procedure:**

1. Let the tangent and normal required at P. locate P as per the given data in the problem.
2. Join P and f1, p and f2.
3. The angular bisector (refer geometrical constructions) of angle pf1f2 is normal.
4. Perpendicular to the normal is the tangent.
5. Draw TT and NN only with HB. Remaining lines for procedure use 2H pencil.

## PARABOLA:

The conic whose eccentricity is equal to one is called parabola.




AB - Base/Double ordinate/ Horizontal Distance

PC - Axis/Abscissa/ Perpendicular distance/ Max height

AC & BC - Ordinate

θ - Angle of projection

**Methods to draw Parabola:**

1. General method/ Eccentricity/Focus- Directrix method.
2. Rectangle/Oblong method.
3. Parallelogram method.
4. Tangent method.

   a) With angle

   b) Without angle.

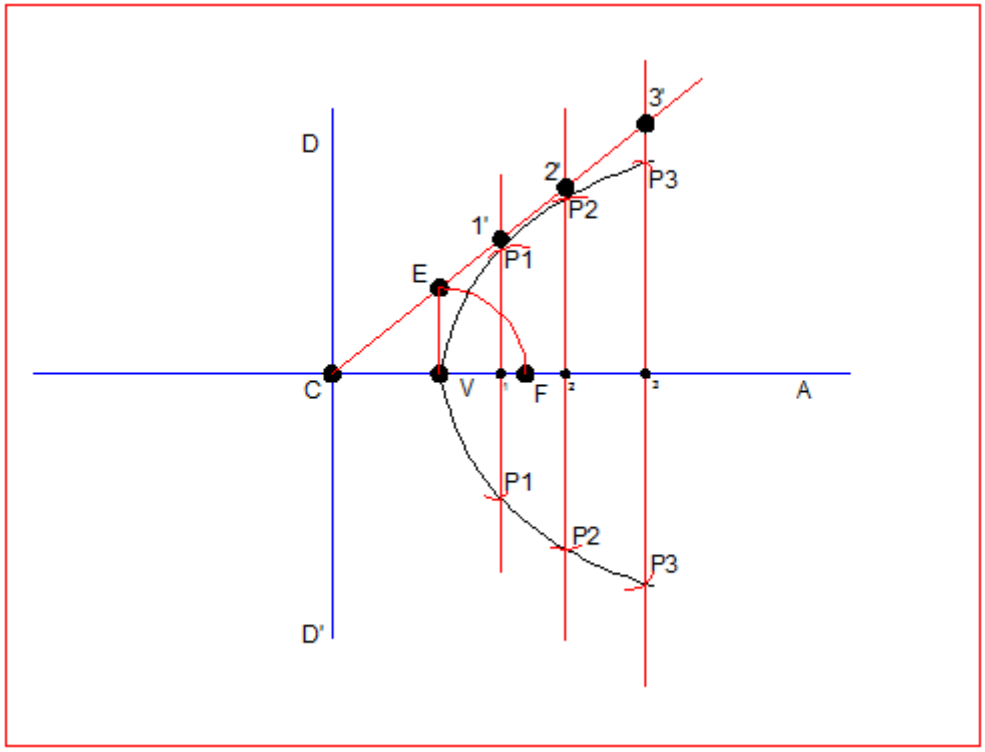
# QUESTIONS

## General Method/ Eccentricity Method /Focus- Directrix Method

9. The distance between the directrix and focus of a parabola is 50mm.Draw the curve. Draw a tangent and normal at a point of the curve 100mm from directrix.

[OR]

A fixed point ‘F’ is 50mm from a fixed straight line. Draw the locus of point ‘P’ moving in such a way that its distance from fixed point is equal to its distance from fixed line. Draw a tangent and normal at a point of the curve 100mm from directrix.




**Drawing procedure:**

1. Similar to general method of ellipse.
2. V is the midpoint of CF for parabola.

## Rectangle/Oblong Method

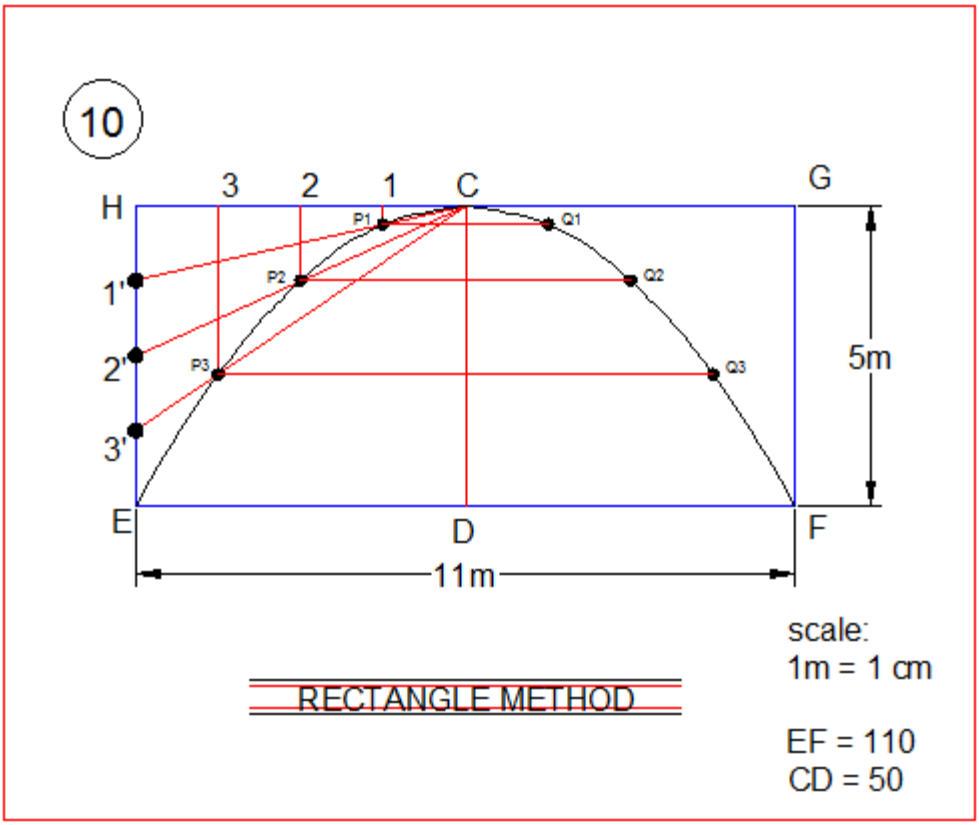
10. A ball thrown from ground level reaches a maximum height of 5m and travels horizontal distance of 11m from point of projection. Trace the path of the ball.

[OR]

Abscissa is 5m and double-ordinate is 11m.Draw a parabola.

[OR]

Base is 11m and axis is 5m.Draw a parabola.




**Drawing procedure:**

1. Draw a rectangle of 11X5.
2. Draw a CD line at midpoints.
3. Divide HC and HE into some no. of equal parts and give numbering in opposite direction to each line.
4. Join C and 1', 2', 3'...
5. And draw a perpendicular from 1 on to C1' and get the point P1.
6. Similarly get the point's p2, p3 and so on.

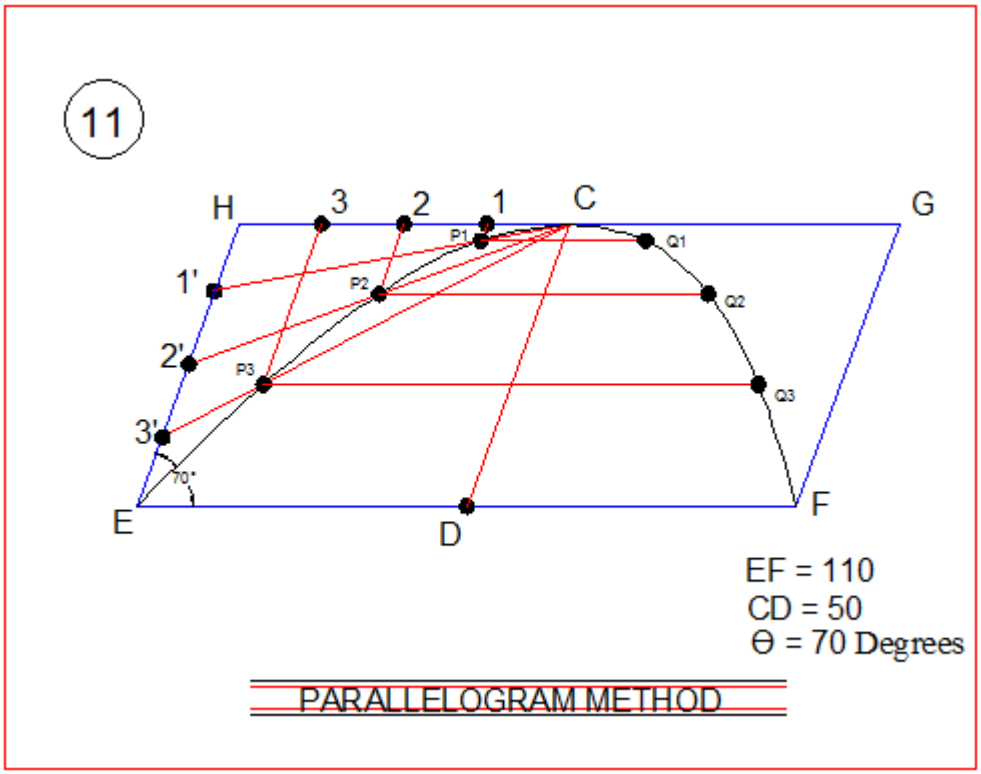
7. Join the points with smooth curve.

**Note:**

1. The curve should pass the points E, C and F.
2. See that the curve should be there between E and p3.
3. Parabola is symmetry is about one axis, so dividing into 2 parts, think about to complete the curve in one box. (I.e. HECD).

## Parallelogram Method

11. Construct a parabola with parallelogram of sides 110mm x 50mm.One of the included angle is $70^0$.



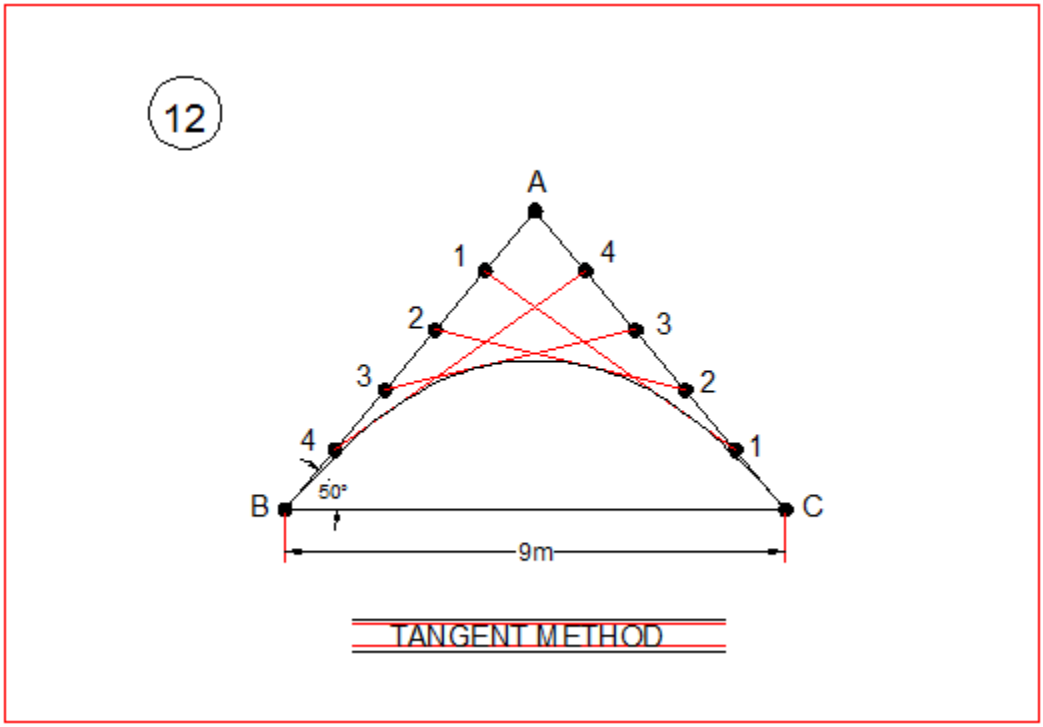

**Note:**

1. All the vertical lines in the above problem will become now inclined lines of 70 degrees.
2. See that the lines 1 p1, 2 p2 & 3 p3 should not be vertical. Those lines should be parallel to EH/CD.

## Tangent Method (With angle)

12. A fountain jet discharges water from the ground level at an inclination of $50^0$ to the ground. The jet travels a horizontal distance of 9m from point of discharge and falls on the ground. Trace the path of the jet. [OR]

Draw a parabola whose ordinate is 4.5m and angle of projection is $50^0$.




**Drawing procedure:**

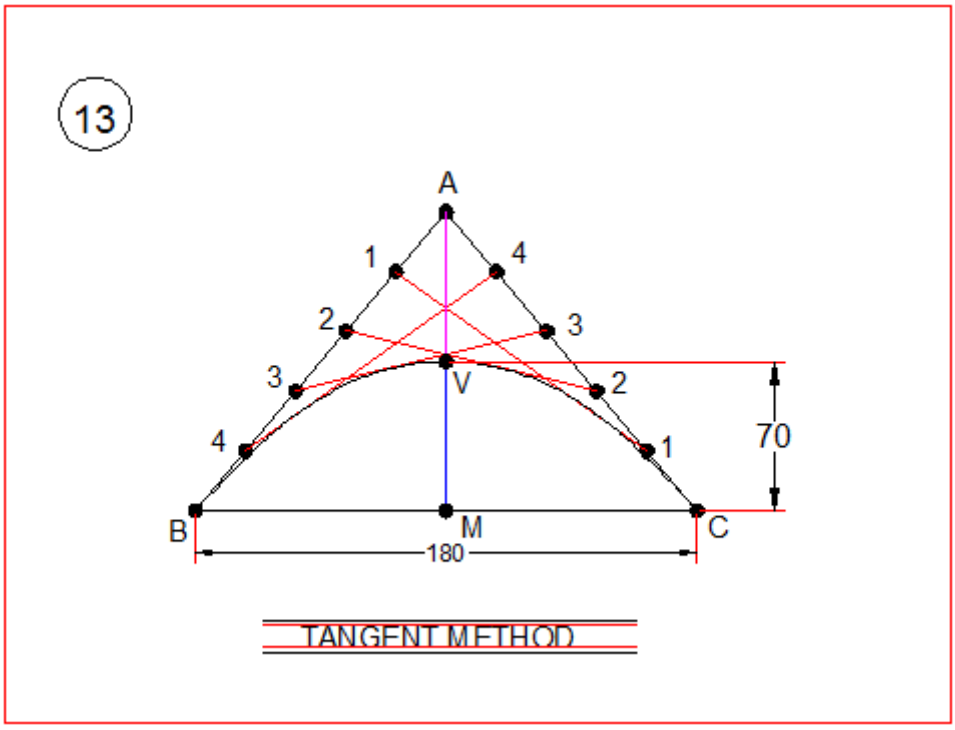
1. Draw a triangle ABC, with AB = 9 cm (follow scale) and 50 degrees inclination.
2. Divide AB and AC into some no. of equal parts and give numbering in opposite direction to each line.
3. Join 11 ,22 and so on
4. Draw a curve smooth curve by maintaining tangents to 11, 22….

## Tangent Method (With Out angle)

13. Draw a parabola using tangent method with its base equal to 180mm and axis 70mm.

[OR]

Draw a parabola; locus of a point 'P', its perpendicular distance from the double ordinate 180mm is 70mm.




**Drawing procedure:**

5. Draw BC line, locate midpoint as M.
6. MV is 70mm.
7. Locate A at a distance of 140 i.e. MV = VA
8. Divide AB and AC into some no. of equal parts and give numbering in opposite direction to each line.
9. Join 11 ,22 and so on
10. Draw a curve smooth curve by maintaining tangents to 11, 22….

**Note:**

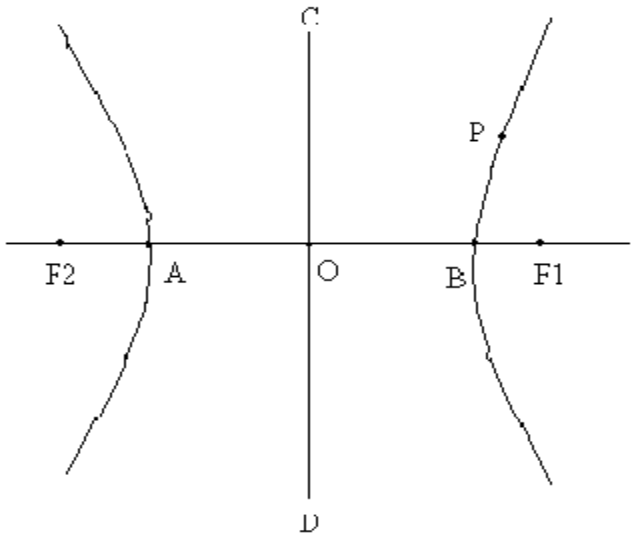
1. The curve should pass through the points B, V and C.
2. Many students take axis height as MA that is wrong.

# HYPERBOLA

The locus of a point 'P' which satisfies the equation $PF_1 \sim PF_2 = AB$ is called hyperbola.

[OR]

The conic whose eccentricity is greater than one is called hyperbola.




A, B-Vertices

AB-Transverse axis

CD-Conjugate axis

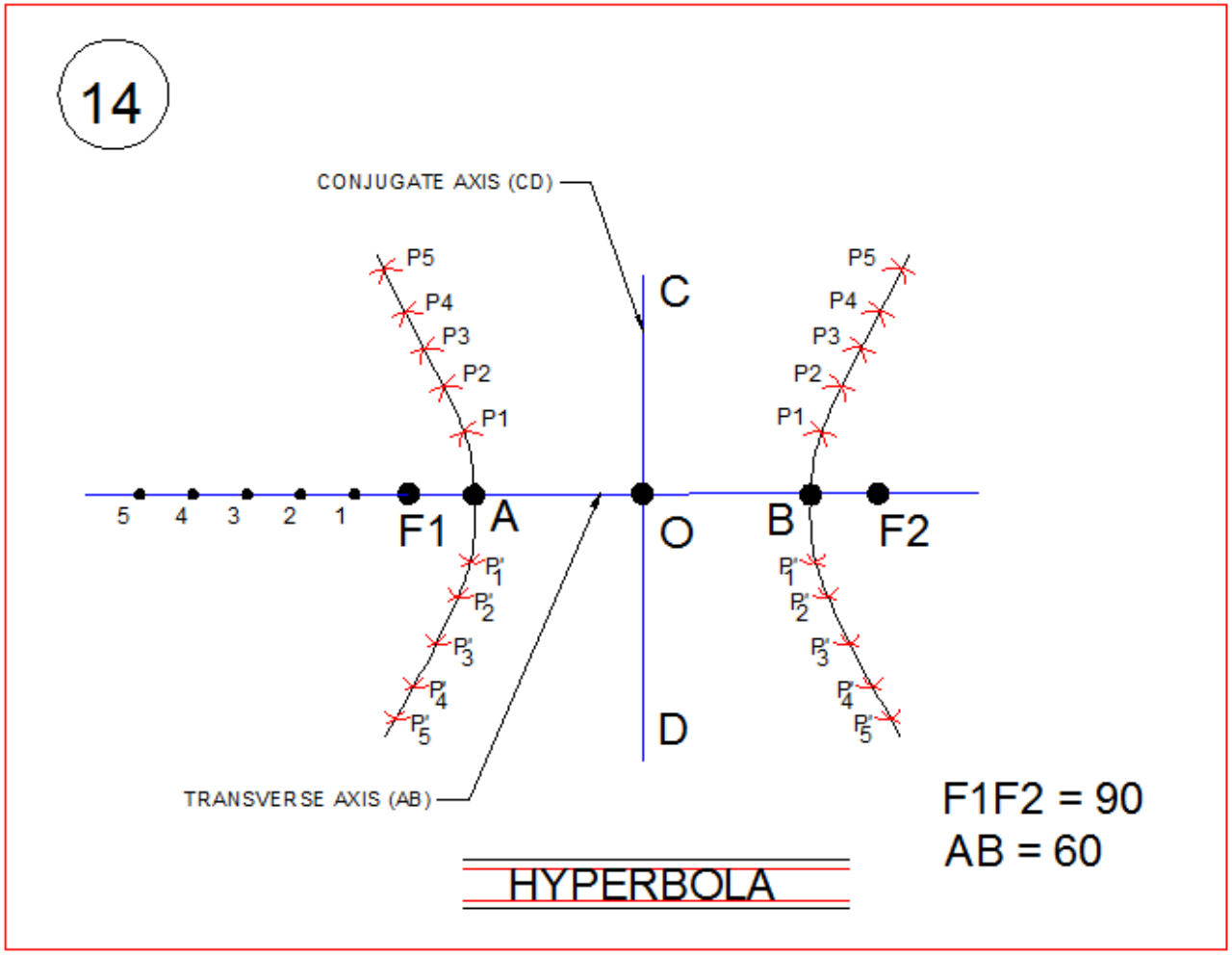
$F_1$ & $F_2$-Foci

**Methods to draw a Hyperbola:**

1. Arcs of arcs method.
2. General method.
3. Rectangle method.

## QUESTIONS

14. The distance between two fixed points is 90mm.Apoint ‘P’ moves such a way that the difference of its distance from the two fixed points is always equal to 60mm.Draw the locus of ‘P’.

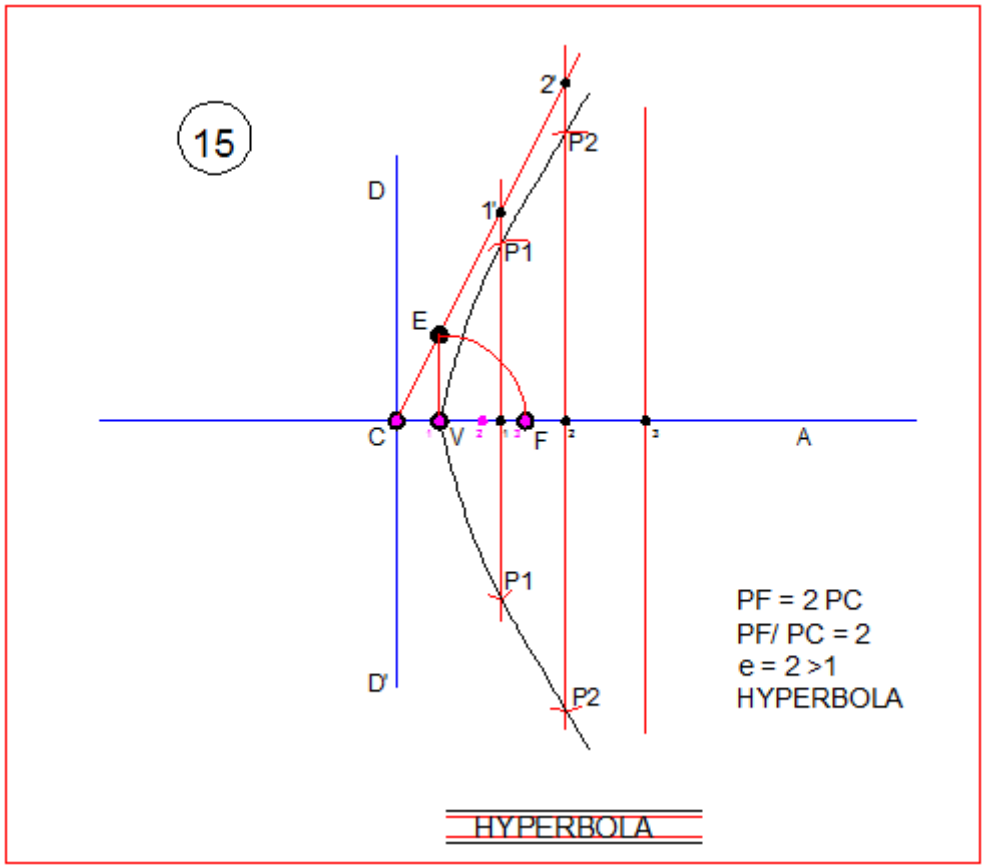



**Drawing procedure:**

1. Locate f1, f2, A, B and O.
2. Divide transverse axis before f1 into some no. of equal parts.
3. First complete the left side curve, and follow the same procedure for right side curve also.
4. Take A1 as radius and f1 as centre, draw an arc above and below.
5. Intersect this arc with the B1 as radius and f2 as centre.
6. Follow the same procedure to get all the points.
7. Join all the points with a smooth curve.

**Note:** PF2~ PF1 = AB, it is hyperbola.

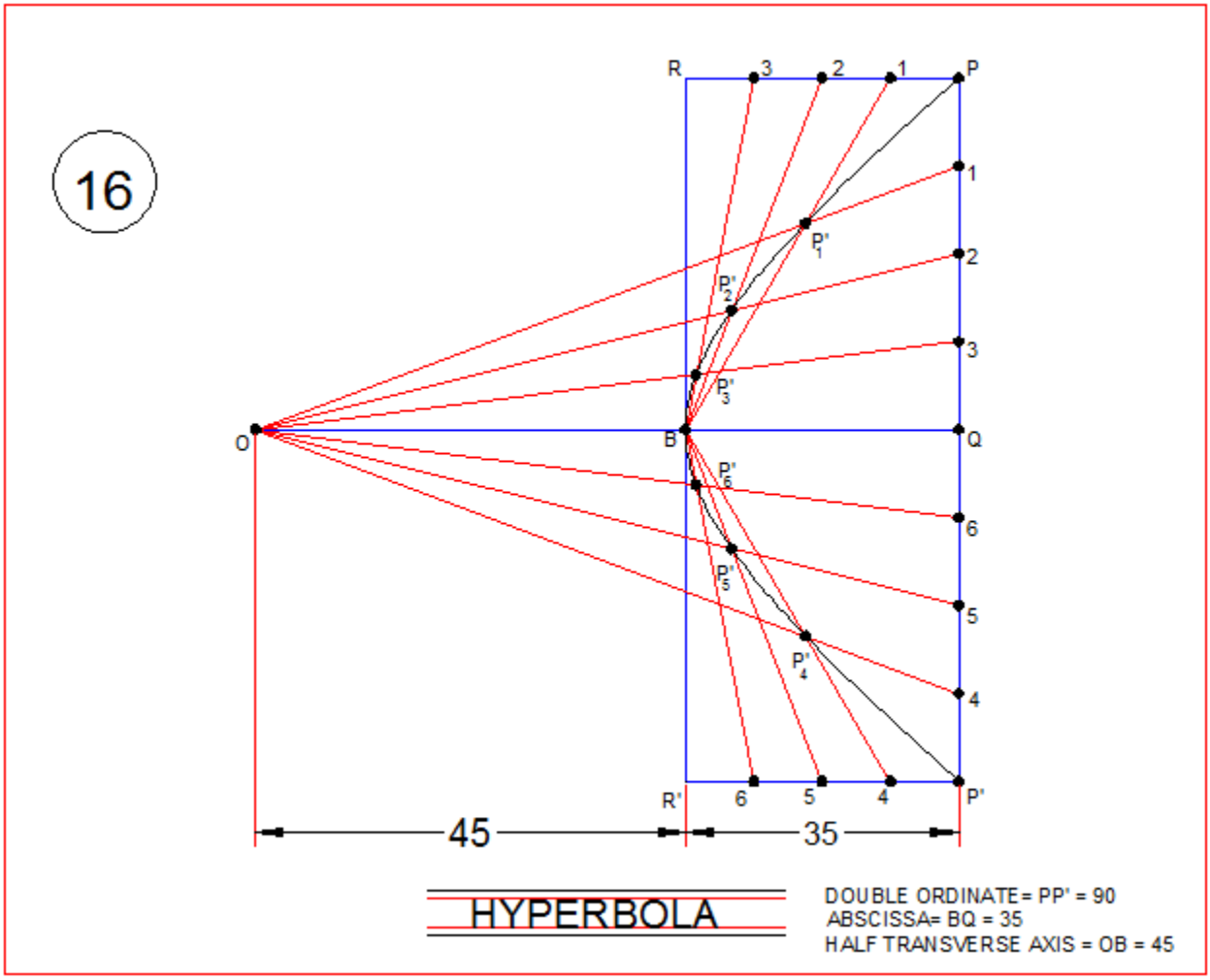
15. A fixed point is 90mm from fixed straight line. Draw locus of point ‘P’ in such a way that its distance from fixed point is twice the distance from fixed straight line.




**Note:**

1. Locate V based on eccentricity. (see magenta color points)
2. Follow the same procedure as general method of parabola.

16. Draw a hyperbola when its double ordinate is 90mm and abscissa is 35mm and half the transverse axis is 45mm.




**Drawing procedure:**

1. Draw a rectangle 35X90.
2. Locate O at a distance of 45mm.
3. Divide RP and PQ into some no. of equal parts. Say 1, 2, 3..
4. Join B and 1, 2, 3... i.e. divisions of RP
5. Join O and 1, 2, 3... i.e. divisions of PQ.
6. The intersection of these lines will give the points p1, p2, p3..
7. Joint the points with a smooth curve.
8. Since hyperbola is a symmetry curve get the remaining portion similarly.

# Rectangular Hyperbola (or) Equilateral Hyperbola

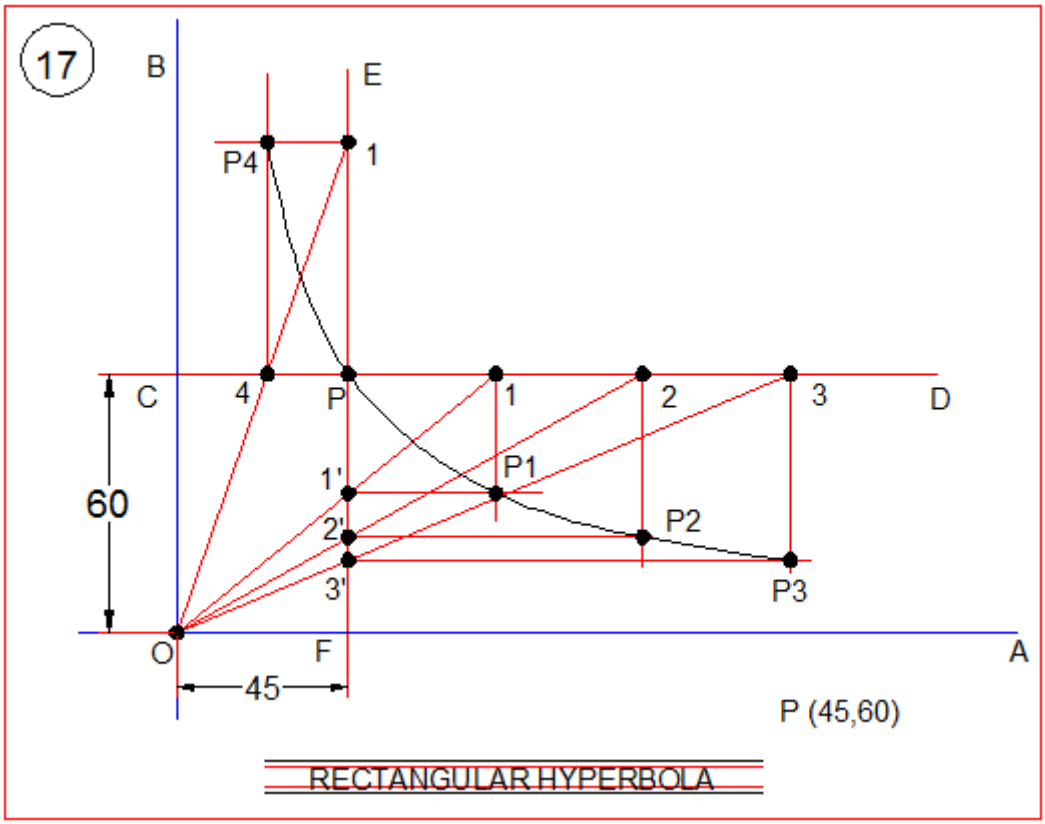
17. Construct a rectangular hyperbola when a point 'P' on it is at a distance of 45mm and 60mm respectively from two asymptotes.

[OR]

Draw a rectangular hyperbola for the point (45, 60).

[OR]

Draw a curve which satisfies the equation PV= const or Boyles law.

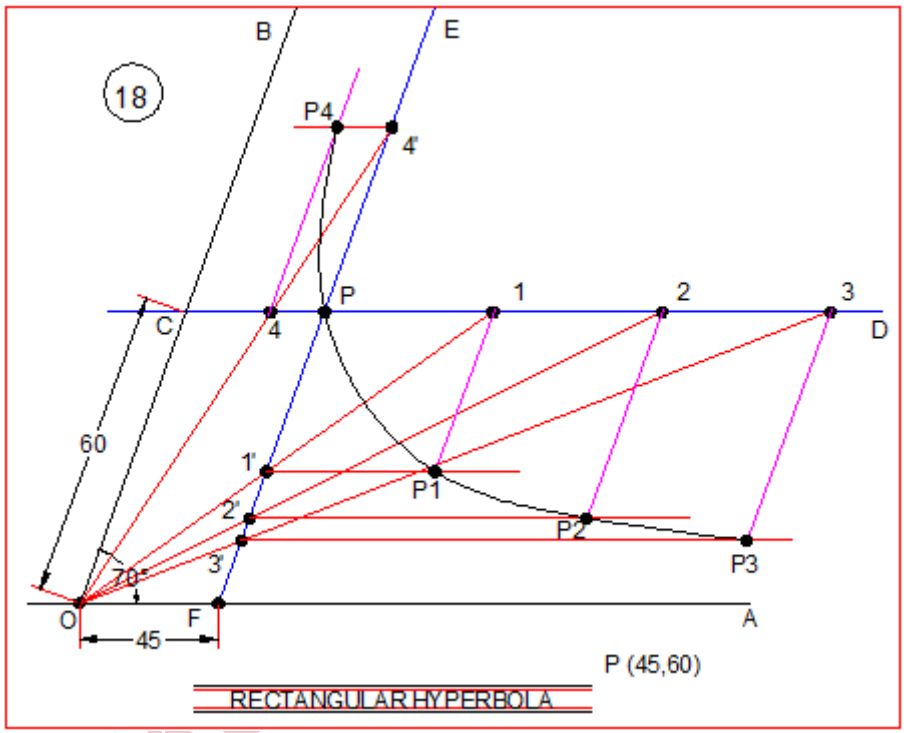



**Drawing procedure:**

1. Draw OA and OB lines perpendicular to each other.
2. Locate p(45 ,60) , for that draw EF and CD lines.
3. Divide CD line after P into some no. of equal parts or divide in increasing order. Mark 1,2,3..
4. Join O and 1, the line O1 intersect EF line at 1', similarly get 2', 3'..
5. Draw horizontal line from 1' and vertical line from 1, intersection of these two lines is P1, similarly get p2, p3..
6. Do the same procedures in the left side of P also, get p4, p5, p6…..
7. Join the point's p1, p2 … with a smooth curve.

**Note:** The curve should compulsory pass through the point P.

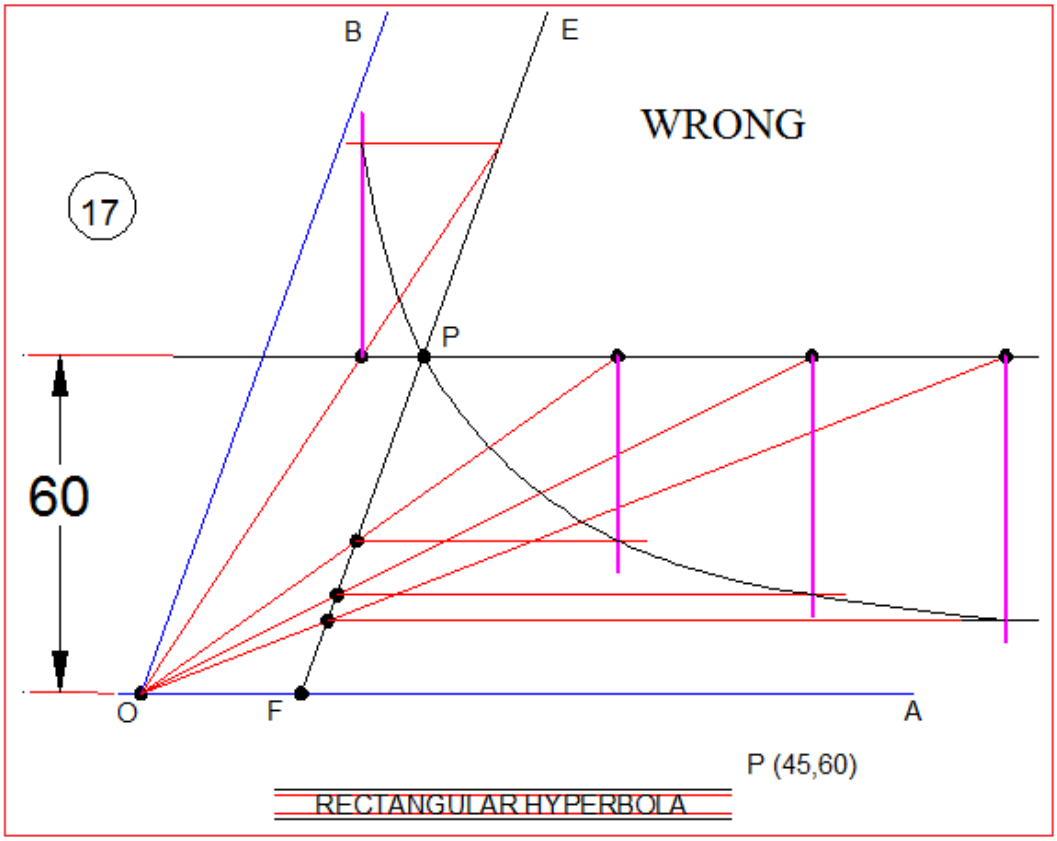
18. Construct a rectangular hyperbola when a point 'P' on it is at a distance of 45mm and 60mm from two asymptotes at an inclination of $70^0$.




**Drawing procedure:**

1. Same as above problem, but all the vertical lines in the above problem are now at inclination of 70 degrees to OA.

**Common mistake done by students:**




**Note:**

1. In this above drawing the pink color lines should not be vertical; these lines should be parallel to either OB or EF.
2. Taking the 60mm dimension vertically also wrong. The dimension 60 mm takes along OB.

## Model problems under conics:

| Sl. No | Methods | Ellipse | Parabola | Hyperbola |
|---|---|---|---|---|
| 1 | General method<br>(or) eccentricity method<br>(or) Focus–Directrix method | ✓ | ✓ | ✓ |
| 2 | Concentric circles method<br>(or) Auxiliary circles method. | ✓ | × | × |
| 3 | Rectangle method<br>(or) Oblong method | ✓ | ✓ | ✓ **(with extra input)** |
| 4 | Parallelogram method | ✓ | ✓ | × |
| 5 | Arcs of circles method<br>(or) Intersecting arc method<br>(or) Foci method | ✓ | × | × |