Highest Common Factor - HCF

HCF: The HCF of two (or) more numbers is the greatest number that can divide each one of them exactly.

Method of finding HCF:

There are two methods to find HCF

    1. By factorisation

    2. By division

1. By factorisation: In this type express the given numbers in to the prime factors.

    "The product of least powers of common prime factors gives HCF"

Ex: i. Find the HCF of 108, 288 and 360.

Ans:

2	108
2	54
3	27
3	9
	3

2	288
2	144
2	72
2	36
2	18
3	9
	3

2	360
2	180
2	90
3	45
3	15
	5

$108 = 2^2 \times 3^3$ $288 = 2^5 \times 3^2$ $360 = 2^3 \times 3^2 \times 5$

HCF = Primes common with least power

    $= 2^2 \times 3^2 = 36$

ii. Find the HCF of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$.

    a) $2 \times 5 \times 7$ b) $2^2 \times 5 \times 7$ c) $2^2 \times 5 \times 7^2$ d) None

Ans: HCF = Product of common prime numbers with least power

HCF = $2 \times 5 \times 7$ (common primes)

= $2^2 \times 5 \times 7^2$ (least power)

2. By Division Method:

Ex: i. Find the HCF of 108, 288 and 360.

Ans: Take any two numbers 108 and 288

```
        108) 288 (2
              210
              ___
             72) 108 (1
                  72
                  ___
                 36) 72 (2
                      72
                      ___
                       0
```

HCF of 108 & 288 is " 36 "

Divide 360 by 36,

```
    36) 360 (10
         360
         ___
          0
```

" 36 " is the HCF of 108, 288 and 360.

Problems:

1. HCF of 1056, 1584 and 2178 is -

    a) 60   b) 61   c) 65   d) 66

```
  1056) 1584 (1
         1056
         ____
         528) 1056 (2
               1056
               ____
                 0
```

528) 21784 (4

    2112

      66) 528 (8

          528   HCF = " 66 "

            0

2. Find the largest number that can exactly divide by 513, 783 and 1107.

   a) 25   b) 26   c) 27   d) 28

Ans: 513) 783 (1

          513

         270) 513 (1

           270

           243) 270 (1

             243

              27) 243 (9

               243

                0

   HCF = " 27 "

Diffrent models on HCF

Type I : Find the greatest number that will exactly divide x, y, z .

     Required number = HCF of (x, y & z)

Type II : Find the greatest number that will exactly divide x, y & z leaving the remainder p, q & r respectively.

    Required number = HCF of [(x-p), (y-q), (z-r)]

Type III : Find the greatest number that will divide x, y & z leaving the same remainder in

    each case.

    Required number = HCF of [(x-y), (y-z), (z-x)]

Examples

1. Find the greatest number which can divide 284, 698 and 1618 leaving the same remainder ' 8 ' in each case.

    a) 20   b) 22   c) 23   d) 46

Ans: Required number = HCF [(284 - 8), (698 - 8), (1618 - 8)]

    Required. number = HCF of 276, 690 & 1610

```
276) 690 (2              138) 1610 (11
      552                      138
     ─────                     ─────
     138) 276 (2               230
          276                  138
         ─────                ─────
            0                 92) 138 (1
                                   92
                                  ─────
                                  46) 92 (2
                                       92
                                      ─────
                                       0
```

2. Find the greatest number which will divide 1050, 1250 and 1650 leaving the remainders 43, 31 and 7 respectively.

    a) 73   b) 63   c) 59   d) 53

Ans: Required numbers = HCF of [(1050 - 43) : (1250 - 31), (1650 - 7)]

= 1007, 1219 & 1643

```
1007) 1219 (1              53) 1643 (3
      1007                     159
     ─────                    ─────
     212) 1007 (4             53) 53 (1
          848                     53
         ─────                   ─────
         159) 212 (1              0
              159
             ─────
             53) 159 (3
```

$$\frac{159}{0}$$

Ans: " 53 "

3. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.

a) 12　　　　　　b) 18　　　　　　c) 24　　　　　　d) 32

Ans: Required number = HCF of [25 - 73), (73 - 97), (97 - 25)]

= HCF of 48, 24, 72

```
24) 48 (2          24) 72 (3
    48                 72
   ----               ----
     0                  0
```

Ans: " 24 "

4. Find the size of the largest square marble which can be paved on the floor of a room 5 meteres 44 cm long and 3 meters 74 cm broad.

a) 56　　　　　　b) 42　　　　　　c) 38　　　　　　d) 34

Ans: Size of square marble = HCF of 544 cm; 374 cm

```
374) 544 (1
     374
    ----
     170)374 (2
         340
        ----
          34) 170 (5
              170
             ----
                0
```

Ans: 34 cm

```
108) 288 (2
     216
    ----
     72) 108 (1
          72
         ----
          36) 72 (2
              72
             ----
               0
```

4. A room is 4 meteres 37 cm long and 3 metres 23 cm broad. If is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose.

a) 485   b) 481   c) 391   d) 381

Ans: Length of room (L) = 437 cm

Breadth (B) = 323 cm

Area of room (A) = l × b = 437 × 323 cm^2

Size of marble = HCF of 437 & 323

323) 437 (1

    323
    ̅ ̅ ̅ ̅
114) 323 (2

    228
    ̅ ̅ ̅ ̅
  95) 114 (1

    95
   ̅ ̅ ̅
  19) 95(5

    95
   ̅ ̅ ̅
    1

area of marble = 19 × 19cm^2 No. of marbles $= \dfrac{\text{Area of room}}{\text{Area of marble}}$

$$= \dfrac{\cancel{437}^{23} \times \cancel{323}^{17}}{\cancel{19} \times \cancel{19}} = 391$$

5. Three different containers contain 496 litres, 403 liters and 713 litres of mixture of milk and water respectively. What biggest measure can measure all the different quantities exactly ?

a) 1 litre   b) 7 litre   c) 31 litres   d) None

Ans: HCF of 496, 403 & 713 is the required measerement

HCF = 31 litres

6. A trader has two varieties of sugar 204 kg and 1190 kg by weights. Find the number

of minimum bags of equal size in which he can store the sugar without mixing.

a) 39 　　　　　 b) 40 　　　　　 c) 41 　　　　　 d) 2

Ans: Size of the bag = HCF of 204 kg & 1190 kg

```
204) 1190 (5
     1020
     ‾‾‾‾
      170) 204 (1
           170
           ‾‾‾
            34) 170 (5
                170
                ‾‾‾
                 0
```

HCF = " 34 "

∴ Size of the bag is 34 kg

Number of bags required = $\dfrac{204}{34} + \dfrac{1190}{34}$

= 6 + 35 = 41

7. A worker was engaged for a certain number of days and was promised to be paid Rs.

1189. He remained absent for some days and was paid Rs.1073 only. What were his

daily wages?

a) 29 　　　　　 b) 30 　　　　　 c) 40 　　　　　 d) 50

Ans: Daily wage is common factor of 1189 & 1073 = HCF of 1189 & 1073 = 29/-

8. Two numbers are in the ratio of 15:11 if their HCF is 13 then the numbers are -

a) 195,140 　　　 b) 195,143 　　　 c) 190, 80 　　　 d) 100,50

Ans: Let, the numbers be 15x & 11x

Their HCF (x) = 13

The numbers are = [(15×13): (11×13)]

= (195, 143)

9. Three numbers are in the ratio 1 : 2 : 3 and their HCF is 12, then the numbers are -

a) 10, 20, 30   b) 12, 20, 24   c) 12, 24, 36   d) None

Ans: The numbers be x, 2x & 3x

Their HCF (x) = 12

The numbers are 12, (2×12), (3×12)

= 12, 24 & 36

10. If the sum of two numbers is 216 and their HCF is 27, then the numbers are -

a) 27,189   b) 27,100   c) 20,200   d) 27,180

Ans: HCF of two numbers be (x) = 27

Let, the numbers are 279 & 276

sum (27a + 27b) = 216

27 (a + b) = 216

a + b = 216 / 27 $\Rightarrow$ a + b = 8

Now, co-primes with sum " 8 " are (1, 7) & (3, 5)

Required numbers are 27a & 27b

(27 × 1) : (27 × 7)

27 : 189