Finding Unit's Digit

Unit's Digit:

A number is made up from digits in the numeral system is called **Unit's Digit**. We often use the decimal system in which we use 10 digits, In writing the any number, many digits are used, even repitation of digits. when we write any number using the digits, the last digit (from right side) in that number is called **unit digit**.

For example in the number 48750, here " **0** " is called unit digit.

To find the number in the unit place of N^n, where

N & n = positive numbers

"N" is called base

'n' is called index of power

* Base number "N" can has any of the following digit in the unit's place

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

Case-I:

When 0, 1, 5, 6 are the digits in the unit's place of the base number N, then the number in the unit's place of N^n will also be 0, 1, 5 & 6 respectively, whatever be the value of "n"

Ex: 1. Number in the unit's place of $(370)^{93}$ is " **0** "

2. Number in the unit's place $(391)^{775}$ is " **1** "

3. Number in the unit's place of $(75)^{73}$ is " **5** "

4. Number in the unit's place $(676)^{99}$ is " **6** "

Case-II

When 4 (or) 9 are in the unit's place of N, then

$(4)^1 = 4$	$(4)^2 = 16$
$(4)^3 = 64$	$(4)^4 = 256$
$(4)^5 = 1024$	$(4)^6 = 4096$

From the above figures we observe that..

$\Rightarrow 4^n \Rightarrow$ when " n " is odd number [i.e. 1, 3, 5, 7 etc.], it contains " 4 " in the unit's place

$\Rightarrow 4^n \Rightarrow$ when " n " even number [i.e. 2, 4, 6, 8 etc.], it contains " 6 " in the unit's place

In the same way..

$\Rightarrow 9^n \Rightarrow$ n = odd number $\Rightarrow$ " 9 " in the unit's place

$\Rightarrow 9^n \Rightarrow$ n = even number $\Rightarrow$ " 1 " in the unit's place

Example:

1) $(74)^{99} \Rightarrow$ n = 99 (odd) $\Rightarrow$ 4 in the unit's place

2) $(84)^{78} \Rightarrow$ n = 78 (even) $\Rightarrow$ 6 in the unit's place

3) $(79)^{33} \Rightarrow$ 9 in the unit's place

What is the unit's digit of the product $2^4 \times 5^2 \times 4^3$

$2^4 = 16$

$5^2 = 25$

$4^3 = 64$

$6 \times 5 \times 4 = 120$

unit's digit of the product is 0

Case-III

When ' 3 ' is the digit in the unit's place of ' N '

$(3)^1 = 3$ $(3)^2 = 9$

$(3)^3 = 27$ $(3)^4 = 81$

for any value of 3 is always odd

Problems

1. What is the number in the unit's place of $(743)^{74}$?

Ans: n = 74, it can be written as $74 = (4 \times 18) + 2$

'2' is remainder

The number in the unit's place (3) 2 = " **9** "

2. What is the number in the unit's place of $(72)^{75}$?

Ans: n = 75,

$75 = (4 \times 18) + 3$

Remainder = 3

$(72)^{75}$ (or) $(72)^3 \Rightarrow 2^3 \Rightarrow$ " **8** "

3. What is the number in the unit's place of $(788)^{94}$?

a) 2 b) 3 c) 4 d) 5

Ans: Divide 94 by 4 , $\frac{94}{4} = 23$

Remainder = 2

$(8)^n = 64$

$\therefore$ the number is "4" in the unit's place

4. The unit's digit in the product $(3127)^{173}$ is -

a) 5 b) 6 c) 7 d) 8

Ans: Divide 173 by 4,

Remainder is " 1 "

$(7)^1$ = " 7 " in the unit place

5. The unit's digit in the product of $(256 \times 27 \times 159 \times 182)$ is -

a) 4 b) 5 c) 6 d) 7

Ans: Product of unit's digits in all the number

$(6 \times 7 \times 9 \times 2) = 756$

Unit's digit in the given product is " 6 "

6) The unit's digit in the product of $(274 \times 318 \times 577 \times 313)$ is -

a)2 b) 3 c) 4 d)5

Ans: $4 \times 8 \times 7 \times 3 = 672$

unit's digit is " 2 "

7. If the unit's digit in the product of $(459 \times 46 \times 28x \times 484)$ is "2" then the value of "x" is -

a) 1 b) 2 c) 3 d) 4

Ans: $\Rightarrow 9 \times 6 \times x \times 4$

$\Rightarrow 216 \times x$

In order to obtain ' 2 ' in unit's place need to multiply by '7'

$\therefore x = 7$

8) The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is -

a) 3 b) 4 c) 5 d) 6

Ans: $7^4 = 1$, $((7^4)^4)^4 = 7^{64} = 1 \Rightarrow 7^{64} \times 7^4 \times 7^3 \Rightarrow 1 \times 1 \times 7^3 =$

Unit's digit in 7^{71} is " 3 "

6^{59} = every power of " 6 " will give the unit's digit " 6 "

$3^{65} = [(3^4)^4]^4 \times 3^1$

$= 1 \times 3 = 3$ ($\therefore$unit's digit of 3^4 is 1)

Units digit of 3^{65} is 3

$\therefore$ unit's digit in $(7^{71} \times 6^{59} \times 3^{65})$ is

$(3 \times 6 \times 3) = 54$

Unit's digit is " 4 "

9) The unit's digit in the product of $(3^{67} \times 6^{39} \times 7^{53})$ is -

a) 3 b) 4 c) 5 d) 6

Ans: $3^{67} = [[3^4]^4]^4 \times 3^3 \Rightarrow 1 \times 3^3 = 27 \Rightarrow 7$

$6^{39} = 6$

$7^{53} = 7$

$\Rightarrow (7 \times 6 \times 7) = 294$

the unit's digit is " 4 "