Problems Based on Quotient, Remainders

Quotient:

A quotient is the result of division. When 'N' is divided by "x" we find a quotient..

Ex: If 6 is divided by 3, the quotient is "2"

This quotient gives number of numbers less than a given number "N"

Problems:

1) Find the number of numbers upto 500 which are divisible by 13.

    a) 36   b) 37   c) 38   d) 39

Ans: Divide 500 by 13

    13) 500 (38

        39

      ————

      110

      104

      ————

       6

The quotient is "38". It means, 38 numbers less than 500 which are divisible by 13.

2) How many numbers upto 100 are divisible by 7?

    a) 14   b) 67   c) 93   d) 100

Ans:

    7) 100 (14

       7

      ——

      30

      28

      ——

       2   The quotient is "14"

3) How many numbers upto 500 are divisible by 23?

    a) 23   b) 27   c) 21   d) 19

Ans: 23) 500 (21

        46

      ——

      40

      23

      ——

17

The quotient is "21"

4) How many numbers upto 200 are divisible by 2 and 3 both?

    a) 35   b) 33   c) 29   d) 27

Ans: L.C.M. of 2 & 3 is "6"

  6) 200 (33

      $\underline{18}$

      20

      $\underline{18}$

      2

The quotient is 33

5) How many numbers between 100 and 300 are divisible by "11" ?

    a) 22   b) 21   c) 20   d) 18

Ans:

11) 100 (9   11) 300 (27

   $\underline{99}$ $\underline{22}$

   1   80

                      $\underline{77}$

                      3

∴ Between 300 and 100, there are 18 numbers (27 - 9 = 18)

6) How many numbers between 300 and 700 are divisible by 2, 3 and 7 together ?

    a) 9   b) 8   c) 10   d) 11

Ans: L.C.M of 2, 3 & 7 is "42"

42) 300 (7   42) 700 (16

  $\underline{294}$ $\underline{42}$

   6   280

                    $\underline{252}$

                    28

9 numbers divisible by 42 (16 - 7 = 9)

7. What is the sum of all the numbers between 3,000 and 5,000 which are divisible by 563?

    a) 11,723   b) 11,833   c) 11,823   d) 11,923

Ans: The numbers just more than 3000 and divisible by 563 is

$$563 \times 6 = 3,378$$
$$563 \times 7 = 3,941$$
$$563 \times 8 = 4,504$$
$$\overline{11,823}$$

8) Find the sum of all the numbers upto 10,000 which are divisible by 563.

    a) 86,139   b) 78,479   c) 73,671   d) 98,711

Ans: Number of numbers below 10,000 divisible by 563 is "17"

```
563) 10000 (17
       563
     ─────
     4370
     3941
     ─────
      329
```

$\therefore$ Sum of such numbers

$$= 563 \ (1 + 2 + 3 + \dots 17\ )$$

$$= 563 \times \frac{17 \times (17 + 1)}{2}$$

$$= 86,139$$

9) What is the sum of all the numbers between 300 and 1,000 which are divisible by 179?

    a) 2,517   b) 2,527   c) 2,607   d) 2,506

Ans:

$$\Rightarrow 179 \times 2 + 179 \times 3 + 179 \times 4 + 179 \times 5$$

$$\Rightarrow 179 \ (2 + 3 + 4 + 5)$$

$$\Rightarrow 179 \times 14 = 2506$$

Remainders

The remainder or residue is the amount "left over" after performing the division of two integers.

Problems:

1. A certain number 'X' when divided by 51 leaves a remainder 26 what is the remainder if the number X is divided by 17?

    a) 6   b) 7   c) 8   d) 9

Ans: X = 51Q + 26

             X = 3 × 17Q + 17 + 9

             X = 17 (3Q + 1) + 9

             Remainder = 9

2. A number when divided by 119 leaves 19 as remainder if the same number is divided by 17 the remainder obtained.

    a) 2   b) 3   c) 1   d) 5

Ans: Let, the Number = x

    Quotient = q

    Divisor = (d) = 119

    Remainder = (r) = 19

    x = (d × q) + r

    x = (119 × q) +19

    Same number when divided by "17"

    x = (17 × 7 × q) + 17 + 2

    = 17 (7q + 1) + 2

    Remainder is "2"

3. A certain umber when divided by 39 leaves a remainder 20, what is the remainder when the same number be divided by 13?

    a) 7   b) 11   c) 0   d) 5

Ans: Let the number = x

    Divisor (d) = 39

    Remainder (r) = 20

$x = (d \times q) + r$

$x = (39 \times q) + 20$

When divided by "13"

$x = 13 \times 3 \times q + 13 + 7$

$x = 13 (3q + 1) + 7$

Remainder = 7

4. When "N" is divided by "4" the remainder is '3' what is the remainder when "2N" is divided by 4?

    a) 1   b) 2   c) 3   d) 4

Ans: When 'N' is divided by 4 the remainder is '3'

    Q = Quotient

    $N = 4Q + r \implies N = 4Q + 3$

    When '2N' is divided by 4..

    $2N = 2 (4q + 3) \implies 2N = 8q + 6$

    $2N = 4 (2q + 1) + 2$

    Remainder = 2

5. What least number must be subtracted from 6,500 to get number exactly divisible by 135?

    a) 10   b) 15   c) 20   d) 25

Ans: On dividing 6,500 by 135

$$135) \overline{6500} (98$$
$$\underline{540}$$
$$1100$$
$$\underline{1050}$$
$$20 \longrightarrow \text{Remainder}$$

If "20" is substracter from 6,500 it is divisible exactly by 135.

6. Find the number which is nearest to 3,105 and exactly divisible by 21.

    a) 3,106   b) 3,108   c) 3,110   d) 3,111

Ans: On dividing 3,105 by 21, remainder obtained '18'

    Number to be added (21-18) = 3

    Required number = (3105 + 3) = 3108

7. What least number must be added to 3,000 to a number exactly divisible by 19?

    a) 1   b) 2   c) 3   d) 4

Ans: $3{,}000 \div 19 \Rightarrow$ remainder = 17

    Number to be added = (19 - 17) = 2

8. A number 'X' when divided by 73 gives a quotient 50 and a remainder one tenth of the quotient the number 'X' is -

    a) 3,500   b) 6,000   c) 3,655   d) 4,050

Ans:

    Divisor (d) = 73

    Quotiant (Q) = 50

    Remainder (r) = $\dfrac{Q}{10} = \dfrac{\cancel{50}}{\cancel{10}} = 5$

    X = [(d × q) + r]

    X = [73 × 50) + 5]

    X = 3655

Complete Remainder:

    "A remainder obtained by dividing a given number by the method of successive division is called complete remainder"

Ex: Divide 132 by 35

    35) 132 (3

          105

           27

$d_1 \leftarrow 5 \,|\, 132$

$d_2 \leftarrow 7 \,|\, 26 \longrightarrow 2 \longrightarrow r_1$

            $3 \longrightarrow 5 \longrightarrow r_2$

Complete remainder = $d_1 r_2 + r_1$

    = (5 × 5) + 2

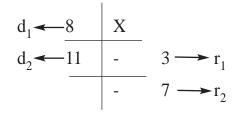   = 27

Problems:

1. A certain number when successively dividided by "8" and "11" leave remainder 3 and 7 respectivly. Find the complete remainder.

    a) 57   b) 58   c) 59   d) 60

Ans:

Let, the number 'X'

$$
\begin{array}{r@{\,}l|l}
d_1 \leftarrow 8 & & X \\
\hline
d_2 \leftarrow 11 & & -\quad 3 \longrightarrow r_1 \\
\hline
& & -\quad 7 \longrightarrow r_2
\end{array}
$$

Complete remainder $= d_1 r_2 + r_1$

$$= (8 \times 7) + 3$$

$$= 59$$

2. A certain number when sucessively divided by 3 and 7 it leaves remainders 1 and 3 respectively. Find the complete remainder.

    a) 7   b) 8   c) 9   d) 10

Ans:

$d_1 = 3 \qquad d_2 = 7$

$r_1 = 1 \qquad r_2 = 3$

Complete remainder $= d_1 r_2 + r_1$

$$= 3 \times 3 + 1 \qquad = 10$$

3. A certain number when sucessively divided by 2, 3 and 5 leave remaindery 1, 2 and 3 respectively. Then what is the complete remainder?

    a) 20   b) 21   c) 22   d) 23

Ans:

$d_1 = 2 \qquad d_2 = 3 \qquad d_3 = 5$

$r_1 = 1 \qquad r_2 = 2 \qquad r_3 = 3$

Complete remainder $= d_1 d_2 r_3 + d_1 r_2 + r_1$

$$= 2 \times 3 \times 3 + 2 \times 2 + 1$$

$$= 18 + 4 + 1 \qquad = 23$$

4. A number when sucessively divided by 3, 4 and 5 leaves remainders 1, 2 and 3 respectively. if the last quotient is 3, then the number is..

a) 221   b) 222   c) 223   d) 230

Ans:

Let the number be "2"

```
3 | Z
  |_____
4 | Y-1 ——→1
  |_____
5 | X-2 ——→2
  |_____
  | 3-3 ——→3
```

$X = (5 \times 3) + 3 = 18$

$Y = 4x + 2 = 4 (18) + 2 = 74$

$Z = 3Y + 1 = 3 (74) + 1 = 223$

5. Find that number which when successively divided by 7, 5 and 4 leaves remainder 1, 2, 3 respectively the lest quotiant being the sum of the remainders

a) 900   b) 950   c) 960   d) 970

Ans: Let the number be 'Z'

```
7 | Z
  |_____
5 | Y ——→1
  |_____
4 | X ——→2
  |_____
  | 6 ——→3
```

$X = 4 \times 6 + 3 = 27$

$Y = 5x + 2 = 5 (27) + 2 = 137$

$Z = 7Y + 1 = 7 (137) + 1 = 960$