Rules on Natural Numbers

Formulae:

1) Sum of First 'n' natural numbers : $1 + 2 + 3 + \dots n$,

$$\sum n = \frac{n(n+1)}{2}$$

2) Sum of squares of 'n' natural numbers: $1^2 + 2^2 + 3^2 + \dots n^2$,

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

3) Sum of Cubes of 'n' natural numbers: $1^3 + 2^3 + 3^3 + \dots n^n$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

4) Sum of first 'n' even natural numbers: $2 + 4 + 6 + \dots n$,

$$\Rightarrow n(n+1)$$

5) Sum of First 'n' odd natural numbers: $1 + 3 + 5 + 7 + \dots n$.

$$\Rightarrow n^2$$

Problems:

1) Find sum of first 100 natural numbers.

    a) 5,000   b) 5,050   c) 5,500   d) 6,500

Ans: sum of first 'n' natural numbers

$$1 + 2 + 3 + \dots n \Rightarrow \sum n = \frac{n(n+1)}{2}$$

$$1 + 2 + 3 + \dots 100 \Rightarrow \sum n = \frac{100(100+1)}{2} = 5,050$$

2) The sum of squares of first 10 natural numbers is -

    a) 300   b) 385   c) 365   d) 400

Ans: $1^2 + 2^2 + 3^2 + \dots 10^2$, n = 10

$$\sum n2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2\times10+1)}{6}$$

$$= 385$$

3) Find the sum of first twenty multiples of 12.

   a) 2,000   b) 2,500   c) 2,520   d) 2,600

Ans: $12 \times 1 + 12 \times 2 + 12 \times 3 + ... 12 \times 20$

$= 12 [1 + 2 + 3 + ...20]$ $= 12 \times \left[\frac{20(20+1)}{2}\right] = \frac{12 \times 20 \times 21)}{2} = 2,520$

4) Sum of all odd numbers upto 100 is-

   a) 2,000   b) 2,500   c) 2,600   d) 3,000

Ans: From, 1 to 100 there are

   $50 \rightarrow$ even numbers

   $50 \rightarrow$ odd numbers

   There fore, $1 + 3 + 5 + ... 50$

   Sum of 50 odd numbers $= n^2$

   $= (50)^2 = 2,500$

5) Given that $(1^2 + 2^2 + 3^2 +..... 10^2) = 385$, **then the value of** $(2^2 + 4^2 + 6^2 +.... 20^2)$ **is -**

   a) 770   b) 1,540   c) 1,155   d) $(385)^2$

Ans: $2^2 + 4^2 + 6^2 + ... 20^2$

   It can be written as follows

   $(2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + ... (2 \times 10)^2$

   Take 2^2 common

   $= 2^2 [1^2 + 2^2 + 3^2 + ... 10^2]$

   $= 4[385] = 1,540$

6) The sum of [51 +52 + 53 +.... 100] is -

    a) 3,700   b) 3,775   c) 3,600   d) 3,800

Ans: [Sum of first 100 natural numbers] − [Sum of first 50 natural numbers]

    = [1 + 2 + 3 + ... 100] − [1 + 2 + ... 50]

$$= \frac{100\,(100+1)}{2} - \frac{50\,(50+1)}{2}$$

    = [50 × 101] − 25 (51)

    = 25 [2 × 101 − 51]

    = 25 [2 × 101 − 51] ⇒ 25 [202 − 51]

    = 25 × 151

    = 3,775

7) Sum of cubes of first ten natural numbers is -

    a) $(50)^2$ b) $(55)^2$ c) $(28)^2$ d) $(52)^2$

ans: $1^3 + 2^3 + 3^3 + ...10^3$

$$\sum n^3 = \left[\frac{n\,(n+1)}{2}\right]^2 = \left[\frac{10 \times (10+1)}{2}\right]^2 = \left[\frac{10 \times 11}{2}\right]^2 = (55)^2$$