Electrostatics

I) COULOMBS LAW AND ELECTRIC FIELD

Synopsis:

1. Study of stationary electric charges at rest is known as **electrostatics**.
2. **Electric Charge** :
   i) It is a fundamental property of matter and never found free.
   ii) There are two kinds of charges namely positive and negative. If a body has excess of electrons, it is said to be **negatively charged** and if it is deficient in electrons, it is said to be **positively charged**.
   iii) Like charges repel and un-like charges attract.
   iv) Charge is conserved. It can neither be created nor destroyed. It can only be transferred from one object to other.
   v) Charge is quantised. The smallest charge is associated with electron (–) and proton (+) is 1.6×10^{-19} coulomb.
   vi) All charges in nature exist as integral multiples of electron charge. q = n.e. $n \to$ Integer
3. **Coulomb's law:**
   i) The force of attraction or repulsion between two charged bodies is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.
   ii) It acts along the line joining the two charges considered to be point charges.
   iii) $F \alpha \frac{q_1 q_2}{d^2}$
   iv) $F = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \cdot \frac{q_1 q_2}{d^2}$ (Or) $F = \frac{1}{4\pi\varepsilon_o K} \cdot \frac{q_1 q_2}{d^2}$ (or) $F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{d^2}$

   a) Where ε is **absolute permittivity?**
   K or ε_r is the **relative permittivity** or **specific inductive capacity** and ε_o is the **permittivity of free space**.

   b) K or ε_r is also called as **dielectric constant** of the medium in which the two charges are placed.

   v) a) **Relative permitivity of a material =**

   $$\varepsilon_r = K = \frac{\text{Force between two charges in air}}{\text{Force between the same charges in the medium at the same distance}}$$

   $$\varepsilon_r = \frac{F_a}{F_m}$$

   b) For air K = 1
   c) For metals K = infinity
   d) Force between 2 charges depends upon the nature of the intervening medium, where as gravitational force is independent of intervening medium.

vi) For air or vacuum, $F=\frac{1}{4\pi\varepsilon_o}.\frac{q_1q_2}{d^2}$

since for air or vacuum, $\varepsilon_r = K = 1$

vii) The value of $\frac{1}{4\pi\varepsilon_o}$ is equal to 9×10^9 Nm^2/C^2.

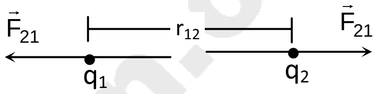
viii) A coulomb is that charge which repels an equal charge of the same sign with a force of 9×10^9 N when the charges are one metre apart in vacuum.

ix) The value of ε_o is 8.86×10^{-12} C^2/Nm^2 (or) 8.86×10^{-12} Fm^{-1}

x) Coulomb force is conservative mutual and internal force.

xi) Coulomb force is true only for static charges.

3. Coulomb's law in vector form:



1) $\vec{F}_{12}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$; $\vec{F}_{12}=-\vec{F}_{21}$

Here F_{12} is force exerted by q_1 on q_2 and F_{21} is force exerted by q_2 on q_1.

2) Coulomb's law holds for stationary charges only which are point sized.

3) This law obeys Newton's third law $\left(\text{ie } \vec{F}_{12}=-\vec{F}_{21}\right)$.

5. Force on a charged particle due to a number of point charges is the resultant of forces due to individual point charges i.e. $\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3+......$

6. i) If the force between two charges in two different media is the same for different separations, $F=\frac{1}{K}\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$ = constant .

ii) Kr^2 = constant or $K_1r_1^2 = K_2r_2^2$

iii) If the force between two charges separated by a distance 'r_0' in vacuum is same as the force between the same charges separated by a distance 'r' in a medium, $Kr^2 = r_0^2 \Rightarrow r=\frac{r_0}{\sqrt{k}}$

7. i) Two identical conductors having charges q_1 and q_2 are put to contact and then separated, then each have a charge equal to $\frac{q_1+q_2}{2}$. If the charges are q_1 and $-q_2$, then each have a charge equal to $\frac{q_1-q_2}{2}$.

ii) Two spherical conductors having charges q_1 and q_2 and radii r_1 and r_2 are put to contact and then separated then the charges of the conductors after contact are $q_1=\left(\frac{r_1}{r_1+r_2}\right)(q_1+q_2)$ & $q_2=\left(\frac{r_2}{r_1+r_2}\right)(q_1+q_2)$.

iii) The force of attraction or repulsion between two identical conductors having charges q_1 and q_2 when separated by a distance d is F. If they are put to contact and then separated by the same distance the new force between them is $F'=\frac{F(q_1+q_2)^2}{4q_1q_2}$

If charges are q_1 and $-q_2$ then $F'=\frac{F(q_1-q_2)^2}{4q_1q_2}$.

8. A charge Q is divided into q and (Q – q). Then electrostatic force between them is maximum when $\frac{q}{Q} = \frac{1}{2}$ (or) $\frac{q}{(Q-q)} = 1$

9. **Electric field and electric intensity:**
   i) The space around an electric charge in which its influence can be felt is known as **electric field**.
   ii) The **intensity of electric field (E)** at a point is the force experienced by a unit positive charge placed at that point.
   iii) It is a vector quantity.
   iv) E = F/ q, unit of E is NC^{-1} or Vm^{-1}
   v) Due to a point charge q, the intensity at a point d units away from it is given by the expression $E = \frac{q}{4\pi\varepsilon d^2}$ NC^{-1}. Another unit is volt/metre.
   vi) The electric field due to a positive charge is always directed away from the charge.
   vii) The electric field due to a negative charge is always directed towards the charge.
   viii) The intensity of electric field at any point due to a number of charges is equal to the vector sum of the intensities produced by the separate charges.

10. **Force experienced by a charge Q in an electric field.**
   $\vec{F} = Q\vec{E}$ Where E is the electric intensity
   i) If Q is positive charge, the force $\vec{F}$ acts in the direction of $\vec{E}$. Acceleration $a = \frac{F}{m} = \frac{QE}{m}$
   ii) If Q is negative charge, the force $\vec{F}$ acts in a direction opposite to $\vec{E}$ Acceleration $a = \frac{F}{m} = \frac{QE}{m}$
   iii) A charge in an electric field experiences a force whether it is at rest or moving.
   iv) The electric force is independent of the mass and velocity of the charged paerticle, it depends upon the charge.
   v) A proton and an electron in the same electric field experience forces of samemagnitude but in opposite directions.
   vi) Force on proton is accelerating force where as force on electron is retarding force. If the proton and electron are initially moving in the direction of electric field $\frac{\text{Acceleration of Proton}}{\text{Retardation of electron}} = \frac{\text{mass of electron}}{\text{mass of proton}}$

11. **Dielectric Strength:** It is the minimum field intensity that should be applied to break down the insulating property of insulator.
   i) Dielectric strength of air = 3×10^6 V/m
   Dielectric strength of Teflon = 60×10^6 Vm^{-1}
   ii) The maximum charge a sphere can hold depends on size and dielectric strength of medium in which sphere is placed.
   ii) The maximum charge a sphere of radius 'r' can hold in air = $4 \in_0 r^2 \times$ dielectric strength of air.

12. When the electric field in air exceeds its dielectric strength air molecules become ionised and are accelerated by fields and the air becomes conducting.

13. Motion of a charged particle in an electric field.

i) If a charged particle of charge Q is placed in an electric field of strength E, the force experienced by the charged particle = EQ.

ii) The acceleration of the charged particle in the electric field, $a = \frac{EQ}{m}$

iii)The velocity of charged particle after time "t" is $V = at = \left(\frac{EQ}{m}\right)t$ if the initial velocity is zero.

iv) The distance travelled by the charged particle is $S = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{EQ}{m}\right)t^2$ if the initial velocity is zero

v) When a charged particle is projected into a uniform electric field with some velocity perpendicular to the field, the path traced by it is **parabola**.

VI) The trajectory of a charged particle projected in a different direction from the direction of a uniform electric field is a **parabola.**

vii) When a charged particle of mass m and charge Q remains suspended in an vertical electric field then mg=EQ.

viii) When a charged particle of mass m and charge Q remains suspended in an electric field, the number of fundamental charges on the charged particle is n then

$$mg = E(ne)$$

$$n = \frac{mg}{Ee}$$

xi)The bob of a simple pendulum is given +ve charge and it is made to oscillate in vertically upward electric field, then the time period of oscillation is $T = 2\pi\sqrt{\frac{\ell}{g - \frac{EQ}{m}}}$

x) In the above case, if the bob is given a –ve charge then the time period is given by $T = 2\pi\sqrt{\frac{\ell}{g + \frac{EQ}{m}}}$

xi)A charged particle of charge ±Q is projected with an initial velocity u making an angle θ to the horizontal in an electric field directed vertically upward. Then

a) Time of flight = $\frac{2u\sin\theta}{g \mp \frac{EQ}{m}}$

b) Maximum height = $\frac{u^2\sin^2\theta}{2\left(g \mp \frac{EQ}{m}\right)}$

c) Range = $\frac{u^2\sin 2\theta}{\left(g \mp \frac{EQ}{m}\right)}$

xii) Density of electric field inside a charged hollow conducting sphere is zero

xiii) A sphere is given a charge of 'Q' and is suspended in a horizontal electric field. The angle made by the string with the vertical is, $\theta = \tan^{-1}\left(\frac{EQ}{mg}\right)$

xiv) The tension in the string is $\sqrt{(EQ)^2 + (mg)^2}$

xv) A bob carrying a +ve charge is suspended by a silk thread in a vertically upward electric field, then the tension in the string is, T = mg - EQ.

xvi) If the bob carries –ve charge, tension in the string is T = mg + EQ

14. **Surface charge density (σ):**

i) The charge per unit area of a conductor is defined as surface charge density.

ii) $\sigma = \frac{q}{A} = \frac{\text{total charge}}{\text{area}}$, when A=1 m^2 then σ = q

iii)Its unit is coulomb/ meter and its dimensions are ATL^{-2}.

iv) It is used in the formulae for charged disc, charged conductor and infinite sheet of charge etc.

v) $\sigma \propto \frac{1}{r^2}$ i.e. $\frac{\sigma_1}{\sigma_2} = \frac{r_2^2}{r_1^2}$

vi) σ is maximum at pointed surfaces and for plane surfaces it is minimum.

vii) σ depends on the shape of the conductor and presence of other conductors and insulators in the vicinity of the conductor.

viii) σ is maximum at the corners of rectangular laminas and at the vertex of conical conductor.