MOTION IN A PLANE

Horizontal projection

1. Consider a body horizontally from the top of a tower with a velocity 'u'.

a) It reaches the ground along a parabolic path.

b) Its time of descent is $\sqrt{2h/g}$.

c) The horizontal displacement is $R = u\sqrt{2h/g}$

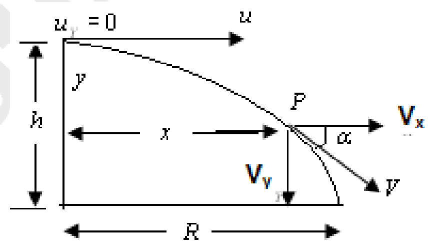
d) The angle α with which it strikes the ground is given by $\tan\alpha = \frac{\sqrt{2h/g}}{u} = \frac{gt}{u}$

e) The velocity with which it hits the ground is given by $v = \sqrt{u^2 + 2gh}$ or $v = \sqrt{u^2 + (gt)^2}$.

Position after time t:

Horizontal displacement, $x = u\,t$

Vertical displacement, $y = \frac{1}{2}gt^2$



2. Velocity after time t :

$v = \sqrt{u^2 + (gt)^2} = \sqrt{u^2 + 2gh}$

If angle made with the horizontal is α $\tan\alpha = \frac{gt}{u} = \frac{gt}{\sqrt{2gh}}$.

3. Equation of path:

$y = \frac{1}{2}g\frac{x^2}{u^2}$

4. Form a certain height. If two bodies are projected horizontally with velocities u_1 and u_2 in opposite directions.

a) Time after which velocity vectors are perpendicular is $t = \frac{\sqrt{u_1u_2}}{g}$

b) Time after which displacement vectors are perpendicular is $t = \frac{2\sqrt{u_1u_2}}{g}$

c) Distance between the two bodies when velocity vectors are perpendicular is $\frac{\sqrt{u_1u_2}}{g}(u_1 + u_2)$

d) Horizontal distance between the two bodies when displacement vectors' are perpendicular is

$2\frac{\sqrt{u_1u_2}}{g}(u_1 + u_2)$

5. Body is dropped from the window of the moving train. The path of the body appears as

a) Vertical straight line for an observer in the train

b) Parabolic for an observer outside the train

6. From the top of a tower a stone is dropped and simultaneously another stone is projected horizontally with a uniform velocity. Both of them reach the ground simultaneously.

7. Motion of a body along an inclined plane :

a) A body is projected up with a speed u from an inclined plane which makes an angle α with the horizontal and velocity of projection makes an angle θ with the inclined plane.

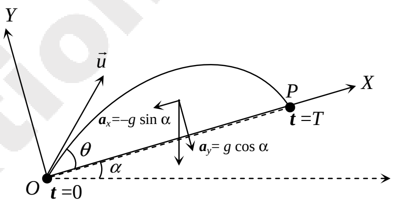
b) The component of initial velocity parallel and perpendicular to the plane are equal to $u \cos \theta$ and $u \sin \theta$ respectively *i.e.* $u_{\parallel} = u\cos\theta$ and $u_{\perp} = u\sin\theta$.

c) The component of g along the plane is $g\sin\alpha$ and perpendicular to the plane is $g\cos\alpha$ as shown in the figure *i.e.* $a_{\parallel} = -g\sin\alpha$ and $a_{\perp} = g\cos\alpha$.

d) **Time of flight :**

Time of flight on an inclined plane $T = \dfrac{2u_{\perp}}{a_{\perp}}$

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$



c) **Maximum height :**

Maximum height on an inclined plane $H = \dfrac{u_{\perp}^2}{2a_{\perp}}$

$$H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$$

d) **Horizontal range :**

$$R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$$

(i) Maximum range occurs when $\theta = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R_{\max} = \frac{u^2}{g(1+\sin\alpha)}$$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R_{\max} = \frac{u^2}{g(1-\sin\alpha)} \quad \text{and} \quad T^2 g = 2R_{max}$$