VERTICAL CIRCULAR MOTION

1. **When a body is whirled in a vertical circle such that its total mechanical energy is constant with critical velocities then tension in the string changes with time. Arrange the following in increasing order of tension when**

   i) String is horizontal

   ii) String makes an angle 0^0 with downward vertical

   iii) String makes an angle of 180^0 with downward vertical

   iv) String makes an angle 120^0 with downward vertical

   1) iii, iv, i, ii 2) iii, i, ii, iv 3) i, ii, iii, iv 4) iv, iii, ii, i

2. **The bob (mass m) of a simple pendulum is imparted a horizontal velocity $u=\sqrt{5gl}$ where is the length of the pendulum. The tension in the string will be**

   (a) 6 mg wt at the bottom

   (b) Zero at the top

   (c) 4 mg wt in the horizontal direction

   (d) 5 mg wt at 60^0 with the downward vertical

   (1) a & d (2) b & d (3) c & c (4) a & b

3. **A small ball of mass m, initially at rest rolls down the loop track from height h. If h = 2.5R, where R is the radius of the truck, then**

   (a) the normal reaction of the bottom of the loop will be equal to 6 mg.

   (b) the normal reaction at the bottom of the loop will be less than 6 mg.

   (c) the ball will be able to just complete the circular track

   (d) the ball will not be able to complete the circular path.

   (1) a, c (2) a, d (3) b, c (4) b, d

4. A car of mass m is moving on a horizontal circular path of radius r. At certain instant its speed is v and has tangential acceleration 'a'.

(a) The acceleration of the car points towards the centre of the path.

(b) The coefficient of friction between the ground and the car is $\tan^{-1}\frac{v^2}{rg}$

(c) The coefficient of friction between the ground and the car is larger than a/g

(d) The frictional force on the car is larger than mv^2/r.

(1) a & b (2) b & c (3) c & d (4) b & d

5. A stone is tied to the end of a string and whirled in a vertical circle. The tension in the string is T_1 when the stone is at the highest point and T_2 when it is at the lowest point. Then

1) $T_1=T_2$ 2) $T_1=-T_2$ 3) $T_1>T_2$ 4) $T_1<T_2$

6. The minimum speed for a particle at the lowest point of a vertical circle of radius R to describe the circle is V. If the radius of the circle is reduced to one fourth its value, the corresponding minimum speed will be

1) V/4 2) V/2 3) 2V 4) 4V

7. A stone tied to a string is rotated in a vertical circle. The minimum speed with which the string has to be rotated

1) decreases with increasing mass of the stone

2) is independent of mass of the stone

3) decreases with increasing length of the string

4) is independent of length of the string

8. A 5 kg body is rotated in a vertical circle with a constant speed of 4 ms^{-1} using a string of length 1 m, when the tension in the string is 31N, then the body will be

1) at the lowest point 2) making an angle 30^0 with vertical

3) at the highest point 4) at horizontal position

9. A simple pendulum of length ‘’ carries a bob of mass m. If the breaking strength of the string is 2 mg, the maximum angular amplitude from the vertical can be

1) 0^0 2) 30^0 3) 60^0 4) 90^0

10. Two identical cars A and B are moving at 36 kmph. A goes on a bridge, convex upward and ‘B’ on concave upward. If the radius of curvature of bridge is 20 m, the ratio of normal forces exerted on the cars when they are at the middle of bridges (g = 10 ms^{-2})

1) 1 : 3 2) 1 : 2 3) 2 : 3 4) 1 : 5

11. The velocity of a body revolving in a vertical circle of radius ‘r’ at the lowest point $\sqrt{7gr}$. The ratio of maximum to minimum tensions in the string is

1) 8 : 1 2) 4 : 1 3) $\sqrt{7}:1$ 4) $1:\sqrt{7}$

12. An inclined track ends in a circular loop of diameter ‘D’. From what height on the track a particle should be released so that it completes that loop in the vertical plane ?

1) $\frac{5D}{2}$ 2) $\frac{2D}{5}$ 3) $\frac{5D}{4}$ 4) $\frac{4D}{5}$

13. A particle suspended by a thread of length ‘l’ is projected horizontally with a velocity $\sqrt{3g\ell}$ at the lowest point. The height from the bottom at which the tension in the string becomes zero is

1) $\frac{4\ell}{3}$ 2) $\frac{2\ell}{3}$ 3) $\frac{5\ell}{3}$ 4) $\frac{\ell}{3}$

14. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum of length 1m. The pendulum is released from the horizontal position. If it rotates in a vertical circle with nail as centre, the distance of nail from point of suspension is

1) 0.5 m 2) 0.6 m 3) 0.4 m 4) 0.8m

15. A body is moving in a vertical circle of radius ‘r’ by a string. If the ratio of maximum to minimum speed is $\sqrt{3}:1$, the ratio of maximum to minimum tensions in the string is

1) 3 : 1 2) 5 : 1 3) 7 : 1 4) 9 : 1

16. A test tube of mass 10g closed with a cork of mass 1g having some ether is suspended by a thread of length 4cm. On heating the tube the cork flies out. The minimum velocity of cork, so that the test tube just describes the vertical circle is

1) 14 ms^{-1} 2)7 ms^{-1} 3)3.5 ms^{-1} 4)12.4 ms^{-1}

17. A mass 'm' is released, with a horizontal speed v from the top of a smooth and fixed, hemispherical bowl, of radius r. The angle θ w.r.t. the vertical where it leaves contact with the bowl is

1) $\sin^{-1}\left(\frac{v^2}{3rg}+\frac{2}{3}\right)$ 2) $\cos^{-1}\left(\frac{v^2}{3rg}+\frac{2}{3}\right)$ 3) $Tan^{-1}\left(\frac{v^2}{3rg}+\frac{2}{3}\right)$ 4) $\cos^{-1}\left(\frac{2}{3}\right)$

18. A simple pendulum of length '*l*' carries a bob of mass 'm'. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a vertical circle about the point of suspension. When the string is horizontal, the net force on the bob is

1) mg 2) 3mg 3) $\sqrt{10}\, mg$ 4) 4 mg

19. A simple pendulum is vibrating with angular amplitude of 90^0. For what values of θ with vertical, the acceleration is directed horizontally

1) $\cos^{-1} \frac{1}{3}$ 2) $\cos^{-1} \frac{1}{\sqrt{3}}$ 3) $\cos^{-1} \frac{1}{\sqrt{2}}$ 4) $\cos^{-1} \frac{2}{\sqrt{3}}$

20. **A body of mass m kg is rotating in a vertical circle at the end of a string of length r metre. The difference in the kinetic energy at the top and the bottom of the circle is**

1) $\frac{mg}{r}$ 2) $\frac{2mg}{r}$ 3) $2mgr$ 4) mgr

KEY

1) 1 2) 1 3) 4 4) 4 5) 4 6) 2 7) 2 8) 3 9)3

10) 1 11) 2 12) 3 13) 1 14) 2 15) 3 16) 1 17) 2 18)3

19)2 20) 3

HINTS

8. $T_\theta = \frac{mv^2}{r} + mg\cos\theta$

$31 = \frac{5x16}{1} + 5x9.8\cos\theta$

$\therefore 31 = 80 + 49\cos\theta \quad 49\cos\theta = -49$

$\cos\theta = -1$

$\therefore \theta = 180°$ [Highest Point]

9. $T = mg(3 - 2\cos\theta)$

$\therefore 2mg = mg(3 - 2\cos\theta)$

$\therefore 2\cos\theta = 1$

$\therefore \cos\theta = \frac{1}{2}$

$\therefore \theta = 60°$

10. $\frac{N_1}{N_2} = \frac{mg - \frac{mv^2}{r}}{mg + \frac{mv^2}{r}} \quad \frac{N_1}{N_2} = \frac{1}{3} \quad = 1:3$

11. $\frac{T_1}{T_3} = \frac{mv_1^2}{r} + mg = \frac{\frac{m}{r}(7gr) + mg}{\frac{m}{r}(3gr) - mg}$

$\frac{T_1}{T_3} = \frac{8mg}{2mg} = 4:1$

12. $h = \frac{5}{2}r \quad = \frac{5}{2}\left(\frac{D}{2}\right) = \frac{5D}{4}$

13. $h = \frac{u^2 + lg}{3g} = 4l/3$

14. $mgl(1-\cos\theta)=\frac{1}{2}mv^2 \qquad v^2=2gl(1-\cos\theta)$

$5g(l-r)=2gl(1-\cos\theta)$

$5(l-r)=2(1-\cos 90)$

$r=0.6m$

15. $\frac{v_1}{v_2}=\frac{\sqrt{3}}{1} \quad v_1=\sqrt{3}v_2$

$\frac{T_1}{T_2}=\frac{mv_1^2/r+mg}{mv_2^2/r-mg} \quad \frac{1}{2}mv_1^2=mg.2r+\frac{1}{2}mv_2^2$

$\frac{3v_2^2}{2}=2gr+\frac{v_2^2}{2} \quad v_2^2=2gr$

$\frac{T_1}{T_2}=\frac{7}{1}$

16. $mv=M\sqrt{5gl} \qquad v=\frac{10}{1}\text{x}\sqrt{5\text{x}9.8\text{x}4\text{x}10^{-2}}$

$=10 \text{ x } 7 \text{ x } 2 \text{ x } 10^{-1} \quad =14ms^{-1}$

17. $\frac{1}{2}m\left(v_1^2-v^2\right)=mg(R-h)$ ------- (1)

$mg\cos\theta-N=\frac{mv^2}{R}$

If - N = 0. $mg\cos\theta=\frac{mv^2}{R}$ ------- (2)

From $\theta=\cos^{-1}\left(\frac{v^2}{3rg}+\frac{2}{3}\right)$

18. $R^2 = \sqrt{(3mg)^2 + (mg)^2}$

$R = \sqrt{10}\, mg$

19. $T\cos\theta = mg$

$T - mg\cos\theta = \frac{mv^2}{r}$

$\frac{mg}{\cos\theta} - mg\cos\theta = \frac{m}{L}(2gl\cos\alpha)$

$\theta = \cos^{-1}\frac{1}{\sqrt{3}}$

20. Difference in K.E. = Difference in P.E. = $2mgr$