# VECTORS

## 2011

**1. If vectors $\hat{i}-3\hat{j}+5\hat{k}$ and $\hat{i}-3\hat{j}-a\hat{k}$ are equal vectors, then the value of a is**

a) 5 b) 2 c) -3 d) 4

## 2010

**2. If a + b = c and a + b = c, then the angle included between a and b is**

a) $90^0$ b) $180^0$ c) $120^0$ d) zero

**3. Three equal masses of 1kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}m$ from each of the vertices of the triangle. The force, in newton, acting on the mass of 2kg is**

a) 2 b) 1 c) 1 d) zero

**4. Find the torque of a force $F=3\hat{i}+2\hat{j}+\hat{k}$ acting at the point $r=8\hat{i}+2\hat{j}+3\hat{k}$**

a) $14\hat{i}-38\hat{j}+16\hat{k}$ b) $4\hat{i}+4\hat{j}+6\hat{k}$ c) $-14\hat{i}+38\hat{j}-16\hat{k}$ d) $-4\hat{i}-17\hat{j}+22\hat{k}$

**5. A variable force, given by the two dimensional vector $F=(3x^2\hat{i}+4\hat{j})$, acts on a particle. The force is in newton and x is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2, 3) to (3, 0)? (The coordinates are in metres)**

a) – 7 J b) zero c) +7J d) 19J

**6. the centre of mass of a system of three particles of masses 1 g, 2g and 3g is taken as the origin of a coordinates system. The position vector of a fourth particle of mass 4g such that the centre of mass of the four particle system lies at the point (1, 2, 3) is $\alpha(\hat{i}+2\hat{j}+3\hat{k})$, where is a constant. The value of $\alpha$ is**

a) 10/3 b) 5/2 c) 1/2 d) 2/5

## 2009

**7. If $a_1$ and $a_2$ are two non-collinear unit vectors and if $|a_1+a_2|=\sqrt{3}$, then the value of $(a_1-a).(2a_1+a_2)$ is**

a) 2 b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) 1

**8. There are N coplanar vectors each of magnitude V. Each vector is inclined to the preceding vector at angle $\frac{2\pi}{N}$. What is the magnitude of their resultant?**

a) $\frac{V}{N}$ b) V c) zero d) $\frac{N}{V}$

## 2008

**9. The value of P so that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+P\hat{j}+5\hat{k}$ are coplanar should be**

a) 16 b) -4 c) 4 d) -8

**10. Two forces of 12N and 8N act upon a body. The resultant force on the body has a maximum value of**

a) 4N b) zero c) 20N d) 8N

**11. The condition under which vectors (a + b) and (a – b) should be at right angles to each other is**

a) $a \neq b$ b) a.b = 0 c) $|a|=|b|$ d) a. b = 1

**12. A car travels 6km towards north at an angle of $45^0$ to the east and then travels distance of 4kg towards north at an angle $135^0$ to east. How far is the point from the starting point ? What angle does the straight line joining its initial and final positions makes with the east ?**

a) $\sqrt{50}km$ and $\tan^{-1}$ b) 10 km and $\tan^{-1}(\sqrt{5})$

c) $\sqrt{52}$ km and $\tan^{-1}(5)$ d) $\sqrt{52}$ km and $\tan^{-1}(\sqrt{5})$

**13. A train of 150 m length is going towards north direction at a speed of $10ms^{-1}$. A parrot flies at a speed of $5ms^{-1}$ towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to**

a) 12s b) 8s c) 15s d) 10s

**14. Rain is falling vertically downwards with a velocity of $4kmh^{-1}$. A man walks in the rain with a velocity of $3kmh^{-1}$. The raindrops will fall on the man with a velocity is**

a) $1kmh^{-1}$ b) $3kmh^{-1}$

c) $4kmh^{-1}$ d) $5kmh^{-1}$

**15. A proton in a cyclotron changes its velocity from $30kms^{-1}$ north to $40kms^{-1}$ east in 20s. What is the magnitude of average acceleration during this time?**

a) $2.5kms^{-2}$ b) $12.5kms^{-2}$

c) $22.5kms^{-2}$ d) $32.5kms^{-2}$

## 2006

**16. A police jeep is chasing with velocity of $45kmh^{-1}$, a thief in another jeep moving with velocity $153kmh^{-1}$. Police fires a bullet with muzzle velocity of $180ms^{-1}$. The velocity with which will strike the car of the thief, is**

a) $150ms^{-1}$ b) $27ms^{-1}$ c) $450ms^{-1}$ d) $250ms^{-1}$

## 2005

**17. Minimum of unequal vectors which can gives zero resultant are**

a) Two b) three c) four d) more than four

**18. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of $\alpha$ is**

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

**19. The vectors from origin to the points A and B are $A=3\hat{i}-6\hat{j}+2\hat{k}$ and $B=2\hat{i}+\hat{j}-2\hat{k}$ respectively. The area of the triangle OAB is**

a) $\frac{5}{2}\sqrt{17}$ b) $\frac{2}{5}\sqrt{17}$ c) $\frac{3}{5}\sqrt{17}$ d) $\frac{5}{3}\sqrt{17}$

## KEY

**1) c 2) d 3) d 4) d 5) c 6) b 7) c 8) c 9) b 10) c**

**11) c 12) c 13) d 14) d 15) a 16)a 17) b 18) c 19) a**

## HINTS

1. Given vectors

$A=\hat{i}-3\hat{j}+5\hat{k}$

$B=\hat{i}-3\hat{j}-a\hat{k}$

According to problem both vectors are equal then A = B, so that the value of a =-5

2. We have a + b = c and c = a + b

$\Rightarrow c=\sqrt{a^2+b^2+2ab\cos\theta}$

$\Rightarrow a+b=\sqrt{a^2+b^2+2ab\cos\theta}$

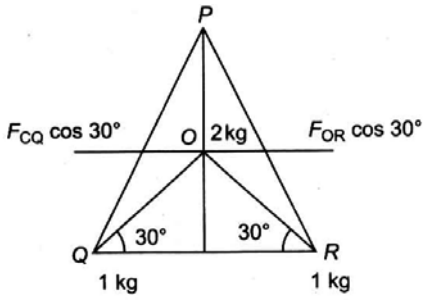
$\Rightarrow a^2+b^2+2ab=a^2+b^2+2ab\cos\theta$

$\Rightarrow \cos\theta=1\Rightarrow\theta=0^0$

3. Given, OP = OQ = OR = $\sqrt{2}m$

The gravitational force on the mass 2kg due to the 1kg mass at P is

$F_{OP}=G\frac{2\times1}{(\sqrt{2})^2}=G$ along OP




Similarly, $F_{OQ}=G_1\frac{2\times1}{(\sqrt{2})^2}=G_1$ along OQ and

$F_{OR}=G_1\frac{2\times1}{(\sqrt{2})^2}=G_1$ along OR

$F_{OQ}\cos30^0$ and $F_{OR}\cos30^0$ are equal and acting in opposite directions, thus they cancel out.

Hence, the resultant force on the 2kg mass at O, is

$F=F_{OP}=(F_{OQ}\sin30^0+F_{OR}\sin30^0)$

$=G_1-\left(\frac{G_1}{2}+\frac{G_1}{2}\right)$

= 0

4. Torque of the force, $\tau=r\times F$

So, $\tau=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\8&2&3\\-3&2&1\end{vmatrix}$

$=\hat{i}(2-6)-\hat{j}(8+9)+\hat{k}(16+6)$

$=-4\hat{i}-17\hat{j}+22\hat{k}$

5. Given two dimensional force

$F=3x^2\hat{i}+4\hat{j}$

$r=x\hat{i}+y\hat{j}$

$dr = dx\hat{i} + dy\,\hat{j}$

Kinetic energy = work done

$W = \int F.dr$

$= \int_{(2,3)}^{(3,0)} (3x^2\hat{i} + 4\hat{j}).(dx\hat{i} + dy\,\hat{j})$

$= \int_2^3 (3x^2 dx + 4dy)$

$= [x^3]_2^3 + 4[y]_3^0 = (27-8) + 4(-3)$

= 19 -12 = 7J

6. The coordinates (x, y, z) of masses 1g, 2g, 3g and 4g are $(x_1 = 0, y_1 = 0, z_1 = 0)$, $(x_2 = 0, y_2 = 0, z_2 = 0)$

$\Rightarrow x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$= \frac{4\alpha}{1+2+3+4}$

$= \frac{4\alpha}{10}$

Hence, $\frac{4\alpha}{10} = 1$

$\Rightarrow \alpha = \frac{5}{2}$

$\Rightarrow y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4} = \frac{8\alpha}{10} = 2$

$\Rightarrow \alpha = \frac{5}{2}$

$\Rightarrow z_{CM} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + m_4z_4}{m_1 + m_2 + m_3 + m_4}$

$= \frac{12\alpha}{10} = 3 \Rightarrow \alpha = 5/2$

7. Since, $a_1$ and $a_2$ are non=collinear

$\therefore a_1 = a_2 = 1$

And $|a_1 + a_2| = \sqrt{3}$

$\Rightarrow a_1^2 + a_2^2 + 2a_1a_2\cos\theta = (\sqrt{3})^2$

$\Rightarrow 1+1+2\cos\theta = 3 \Rightarrow \cos\theta = \frac{1}{2}$

Now $(a_1 + a_2).(2a_1 + a_2)$

$= 2a_1^2 - a_2^2 - a_1 a_2 \cos\theta = 2 - 1 - \frac{1}{2} = \frac{1}{2}$

8. Since each of N-coplanar vectors is inclined at $\frac{2\pi}{N}$ to the preceding hence, they will form a closed polygon. Therefore, their resultant must be zero

9. For coplanarity

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & P & 5 \end{vmatrix} = 0$$

Or 2(10 +3P) +1(5 + 9) + 1 (P – 6) =

Or 20 + 6P + 5 + 9 + P – 6 = 0

Or 7P + 34 – 6 = 0

Or 7P + 28 = 0

Or 7P = -28

$\Rightarrow P = -\frac{28}{7} = -4$

10. When the two forces of 12N 8N act upon a body, the resultant force on the body has maximum value when resultant force

= 12N + 8N = 20N

11. The dot product of two vectors should be equal to zero is (a + b). (a – b) =0

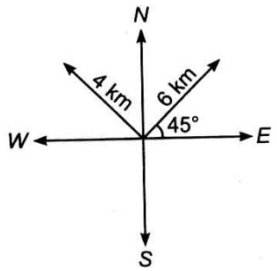
$\Rightarrow a^2 - b^2 = 0$

$\Rightarrow |a| = |b|$

12. Net movement along X-direction

$S_x = (6-4)\cos 45^0 \hat{i}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} km$




Net movement along Y-direction

$S_y = (6+4)\sin 45^0 \hat{j}$

$= 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2} km$

Net movement from starting point

$|S| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2}$

$= \sqrt{52} km$

Angle which resultant makes with the east direction

$\tan\theta = \frac{y - component}{x - component}$

$= \frac{5\sqrt{2}}{\sqrt{2}}$

$\theta = \tan^{-1}(5)$

13. Relative velocity of the parrot w.r.t the train

$= [10 - (-5)] ms^{-1} = 15 ms^{-1}$

Time taken by the parrot to cross the train

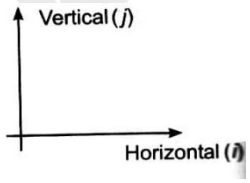
$\frac{150}{15} = 10s$

14. Relative velocity of man w.r.t rain

$v_{rm} = v_r - v_m$

$= 4\hat{j} - 3\hat{i}$

$= -3\hat{i} + 4\hat{j}$




Or $= 5 kmh^{-1}$

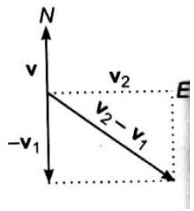
$|v_{rm}| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$

16. Change in velocity $= \sqrt{(40)^2 + (30)^2}$

$= 50 kms^{-1}$




Average acceleration $= \frac{50}{20}$

$= 2.5 kms^{-2}$

17. The relative velocity of thief's jeep with respect to police jeep = 153 – 45

$= 108 kmh^{-1}$

$= 108 \times \frac{5}{18} = 30 ms^{-1}$

Therefore, striking speed = relative speed of bullet with respect to theif's car

= 180 – 30 = $150 ms^{-1}$

18. By triangle law of vectors minimum three vectors are required to give zero resultant

19. Let $a = 2\hat{i} + 3\hat{j} + 8\hat{k}$

$b = 4\hat{i} - 4\hat{j} + \alpha\hat{k}$

$= -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Given $a \perp b$

$\Rightarrow a.b = 0$

$\Rightarrow (2\hat{i} + 3\hat{j} + 8\hat{k}).(-4\hat{i} + 4\hat{j} + \alpha\hat{k}) = 0$

$\Rightarrow -8 + 12 + 8\alpha = 0$

$\Rightarrow 8\alpha = -4$

$\therefore \alpha = -\frac{4}{8} = -\frac{1}{2}$