

Motion in One Dimension

2011

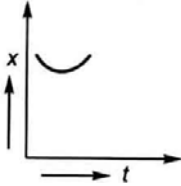
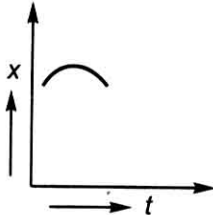
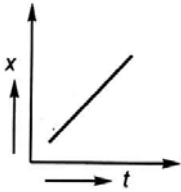
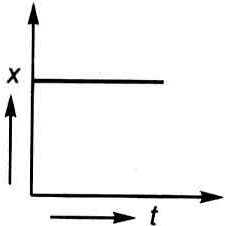
1. A body is moving with velocity 30ms^{-1} towards east. After 10s its velocity becomes 40ms^{-1} towards north. The average acceleration of the body is

- a) 7ms^{-2} b) $\sqrt{7}\text{ms}^{-2}$ c) 5ms^{-2} d) 1ms^{-2}

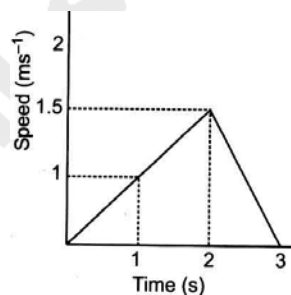
2. A boy standing at the top of a tower of 20m height drops a stone. Assuming $g = 10\text{ms}^{-2}$, the velocity with which it hits the ground is

- a) 20ms^{-1} b) 40ms^{-1} c) 5ms^{-1} d) 10ms^{-1}

3. Position-time graph for motion with zero acceleration is

- a)  b)  c)  d) 

4. The speed-time graph of a particle moving along a solid curve is shown below. The distance traversed by the particle from $t = 0\text{s}$ to $t = 3\text{s}$ is



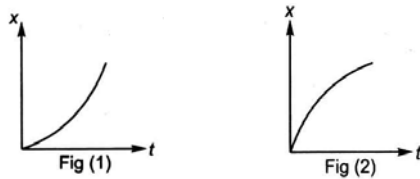
- a) $\frac{10}{2}\text{s}$ b) $\frac{10}{4}\text{s}$ c) $\frac{10}{3}\text{s}$ d) $\frac{10}{5}\text{s}$

2010

5. A boat is sent across a river with a velocity of 8km/h. If the resultant velocity of boat is 10km/h, then velocity of the river is

- a) 10km/h b) 8 km/h c) 6 km/h d) 4 km/h

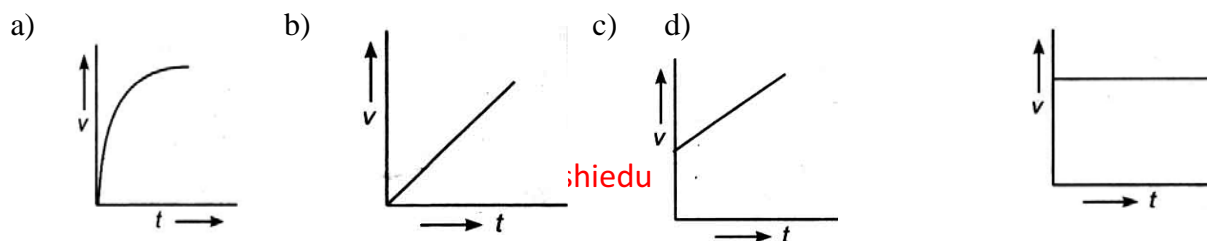
15. Figure (1) and (2) show the displacement-time graphs of two particles moving along the x-axis. We can say that



- a) both the particles are having an uniformly accelerated motion
- b) both the particles are having an uniformly retarded motion
- c) particle (1) is having on uniformly accelerated motion which particle (2) is having an uniformly retarded motion
- d) particle (1) is having an uniformly retarded motion while particle (2) is having an uniformly accelerated motion.

2008

16. Which of the following can be zero, when a particle is in motion for some time?
- a) distance
 - b) displacement
 - c) speed
 - d) none of these
17. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}ms^{-2}$, in the third second is
- a) 6m
 - b) 4m
 - c) $\frac{10}{3}m$
 - d) $\frac{19}{3}m$
18. A particle moves in a straight line with a constant acceleration. It changes its velocity from $10ms^{-1}$ to $20ms^{-1}$ while passing through a distance 135m in t second. The value of t is
- a) 10
 - b) 1.8
 - c) 12
 - d) 9
19. A parachutist after bailing out falls 50m without friction. When parachute opens, it decelerates at $2ms^{-2}$. He reaches the ground with a speed of $3ms^{-1}$. At what height, did he bail out ?
- a) 91 m
 - b) 182 m
 - c) 293 m
 - d) 111m
20. A car, starting from rest, acceleration at the rate f through a distance S, then continues at constant speed for time t and then decelerates as the rate $f/2$ to come to rest. If the total distance travelled is 15S, then
- a) $S = ft$
 - b) $S = \frac{1}{6}ft^2$
 - c) $S = \frac{1}{72}ft^2$
 - d) $S = \frac{1}{4}ft^2$
21. A body starts from rest and moves with uniform acceleration. Which of the following graphs represent its motion?



2007

22. A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d . The average speed for this round trip is
- a) $\frac{2v_d v_u}{v_d + v_u}$ b) $\sqrt{v_u v_d}$ c) $\frac{v_d v_u}{v_d + v_u}$ d) $\frac{v_u + v_d}{2}$
23. The position x of a particle with respect to time t along X-axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. What will be position of this particle when it achieves maximum speed along the +x direction?
- a) 32m b) 54m c) 81m d) 24m
24. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the X-axis an angle of
- a) 30° b) 45° c) 60° d) 0°
25. A particle moving along X-axis has acceleration f , at time t , given by $f = f_0 \left(1 - \frac{t}{T}\right)$, where f_0 and T are constant. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) is
- a) $f_0 T$ b) $f_0 T^2$ c) $f_0 T^3$ d) $\frac{1}{2} f_0 T$
26. A man throws balls with the same speed vertically upwards one after the other at an interval of 2s. What should be the speed of the throw so that more than two balls are in the sky at any time? (Given $g = 9.8 \text{ms}^{-2}$)
- a) Any speed less than 19.6ms^{-1} b) Only with speed 19.6ms^{-1}
 c) More than 19.6ms^{-1} d) At least 19.6ms^{-1}
27. A conveyor belt is moving horizontally at a speed of 4ms^{-1} . A box of mass 20 kg is gently laid on it. It takes 0.1 s for the box to come to rest. If the belt continues to move uniformly, then the distance moved by the box on the conveyor belt is
- a) Zero b) 0.2 m c) 0.4 m d) 0.8 m
28. The acceleration of a particle is increasing linearly with time t as bt . The particle starts from the origin with an initial velocity v_o . The distance travelled by the particle in time t will be
- a) $v_o t + \frac{1}{3} bt^2$ b) $v_o t + \frac{1}{3} bt^3$ c) $v_o t + \frac{1}{6} bt^3$ d) $v_o t + \frac{1}{2} bt^2$

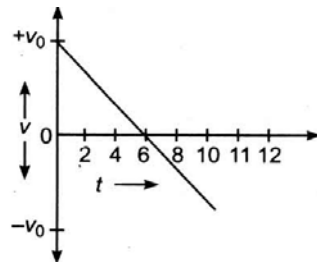
2006

29. Two spheres of same size, one of mass 2kg and another of mass 4kg, are dropped simultaneously from the top of Qutab Minar (height = 72m). When they are 1m above the ground, the two spheres have the same
- a) momentum b) kinetic energy c) potential energy d) acceleration

30. The velocity of a particle at an instant is 10ms^{-1} . After 3s its velocity will become 16ms^{-1} . The velocity at 2s, before the given instant would have been
- a) 6ms^{-1} b) 4ms^{-1} c) 2ms^{-1} d) 1ms^{-1}
31. A body falls from a height $h = 200\text{m}$. The ratio of distance travelled in each 2s, during $t = 0$ to $t = 6\text{s}$ of the journey is
- a) 1 : 4 : 9 b) 1 : 2 : 4 c) 1 : 3 : 5 d) 1 : 2 : 3

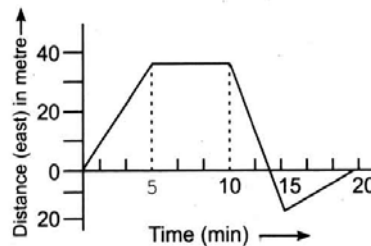
2006

32. Consider the given velocity-time graph



It represents the motion of

- a) a projectile projected vertically upward, from a point
 b) an electron in the hydrogen atom
 c) a bullet fired horizontally from the top of a tower
 d) an object in the positive direction with decreasing speed
33. A body begins to walk eastward along a street in front of his house and the graph of his displacement from home is shown in the following figure. His average speed for the whole time interval is equal to



- a) 8m min^{-1} b) 6m min^{-1} c) $\frac{8}{3}\text{m min}^{-1}$ d) 2m min^{-1}

KEY

1) c	2) a	3) d	4) b	5) c	6) d				
7) b	8) b	9) c	10) d	11) b	12) a	13) b	14) d	15) c	16) b
17) c	18) d	19) c	20) c	21) b	22) a	23) b	24) c	25) d	26) c
27) b	28) c	29) d	30) a	31) c	32) a	33) b	34) d	35) a	36) d
37) b	38) a	39) c	40) b						

HINTS

1. Average acceleration = $\frac{\text{change in velocity}}{\text{total time}}$

$$a = \frac{v_f - v_i}{\Delta t}$$

$$= \frac{\sqrt{30^2 + 40^2}}{10} = \frac{\sqrt{900 + 1600}}{10} = 5 \text{ ms}^{-2}$$

2. Given, $g = 10 \text{ ms}^{-2}$ and $h = 20 \text{ m}$

We have $v = \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 20} = \sqrt{400} = 20 \text{ ms}^{-1}$$

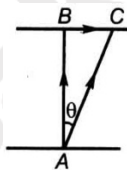
4. Distance = $\frac{1}{2} \times 1 \times 1 \times \frac{1}{2} (1.5 + 1) \times 1 + \frac{1}{2} (1.5 \times 1) = \frac{10}{4}$

5. Given

AB = Velocity of boat = 8 km/h

AC = resultant velocity of boat = 10 km/h

BC = velocity of river = $\sqrt{AC^2 - AB^2}$



$$= \sqrt{10^2 - 8^2}$$

$$= 6 \text{ km/h}$$

6. Relative velocity of the passenger with respect to train

$$= v_{\text{passenger}} - v_{\text{train}} = 0$$

∴ Relative velocity of the passenger with respect to the observer is zero.

7. Displacement can be defined as the distance between initial and final positions of the ball. Since the ball returns back to its initial position, the displacement is zero.

$$8. \quad S_{3rd} = 10 + \frac{10}{2}(2 \times 3 - 1) = 35 \text{ m}$$

$$S_{2nd} = 10 + \frac{10}{2}(2 \times 2 - 1) = 25 \text{ m}$$

$$\Rightarrow \frac{S_{3rd}}{S_{2nd}} = \frac{7}{5}$$

$$9. \quad x = at^2 - bt^3$$

$$\text{Velocity, } v = \frac{dx}{dt} = 2at - 3bt^2$$

$$\text{Acceleration, } a' = \frac{d^2x}{dt^2} = 2a - 6bt$$

Substituting for acceleration given, $2a - 6bt = 0$

$$\Rightarrow t = \frac{a}{3b}$$

$$10. \quad \text{As } v = 0 + na \Rightarrow a = \frac{v}{n}$$

$$\Rightarrow S_n = \frac{1}{2}an^2 \text{ and distance travelled in } (n - 2) \text{ second is}$$

$$S_{n-2} = \frac{1}{2}a(n-2)^2$$

So the distance travelled in the last 2 s is

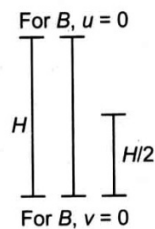
$$S_n - S_{n-2} = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= \frac{a}{2}(n^2 - (n-2)^2)$$

$$= \frac{a}{2}\{n + (n-2)\}\{n - (n-2)\}$$

$$= \frac{2v(n-1)}{n}$$

11. Suppose the two bodies A and B meet at time t , at height $\frac{H}{2}$ from the ground.



$$\text{For body B, } u = 0, h = \frac{H}{2}$$

$$\Rightarrow h = ut + \frac{1}{2}gt^2$$

Hence, $\frac{H}{2} = \frac{1}{2}gt^2$

For body A, $u = v_0$, $h = \frac{H}{2}$

$$\Rightarrow h = v_0t - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{H}{2} = v_0t - \frac{1}{2}gt^2$$

So, from Eqs. (i) and (ii),

$$v_0t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$\Rightarrow v_0t = gt^2 \Rightarrow t = \frac{v_0}{g}$$

Thus, we get $\frac{H}{2} = \frac{1}{2}g \times \frac{v_0^2}{g^2}$

$$\Rightarrow H = \frac{v_0^2}{g}$$

$$\Rightarrow v_0 = \sqrt{gH}$$

12. Initially $u = 0$

Distance travelled in the n th second is given by

$$h_n = u + \frac{g}{2}(2n - 1)$$

Distance travelled in the 1st second is

$$h_1 = 0 + \frac{g}{2}(2 \times 1 - 1) = \frac{g}{2}$$

Distance travelled in the 2nd second is

$$h_2 = 0 + \frac{g}{2}(2 \times 2 - 1) = \frac{3g}{2}$$

Distance travelled in 3rd second

$$h_3 = 0 + \frac{g}{2}(2 \times 3 - 1) = \frac{5g}{2}$$

The ratio of distances

$$h_1 : h_2 : h_3 : h_4 : h_5 : \dots = 1 : 3 : 5 : 7 : \dots$$

13. Given $s = 6t^2 - t^3$

Velocity $v = \frac{ds}{dt} = 12t - 3t^2$

If velocity is zero, then $0 = 12t - 3t^2 \Rightarrow t = 4s$

14. Let u be the initial velocity and h be the maximum height attained by the stone

$$\text{So, } v_1^2 = u^2 - 2gh_1$$

$$\therefore (10)^2 = u^2 - 2 \times 10 \times \frac{h}{2} \quad \left(\because h_1 = \frac{h}{2}, v_1 = 10 \text{ m/s} \right)$$

$$\text{Or } 100 = u^2 - 10h \dots\dots\dots(i)$$

$$\text{Again at height } h, v_2^2 = u^2 - 2gh$$

$$\Rightarrow 0^2 = u^2 - 2 \times 10 \times h \quad (\because v_2 = 0)$$

$$\Rightarrow u^2 = 20h \dots\dots\dots(ii)$$

So, from eqs. (i) and (ii), we have $100 = 10h$

$$\Rightarrow h = 10 \text{ m}$$

17. Distance travelled by the particle in n th second is

$$S_{nth} = u + \frac{1}{2}a(2n-1)$$

Where u is initial speed and a is acceleration of the particle

$$\text{Hence, } n = 3, u = 0, a = \frac{4}{3} \text{ ms}^{-2}$$

$$S_{3rd} = 0 + \frac{1}{2} \times \frac{4}{3} \times (2 \times 3 - 1)$$

$$= \frac{4}{6} \times 5 = \frac{10}{3} \text{ m}$$

18. Let u and v be the first and final velocities of particle and a and s be the constant acceleration and distance covered by it. From third equation of motion

$$v^2 = u^2 + 2as$$

$$\Rightarrow (20)^2 = (10)^2 + 2a \times 135$$

$$\text{Or } a = \frac{300}{2 \times 135} = \frac{10}{9} \text{ ms}^{-2}$$

Now using first equation of motion,

$$V = u + at$$

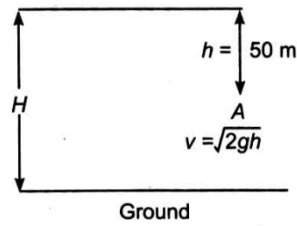
$$\text{Or } t = \frac{v-u}{a} = \frac{20-10}{(10/9)} = \frac{10 \times 9}{10} = 9 \text{ s}$$

19. Parachute bails out at height H from ground. Velocity at A

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50} = \sqrt{980} \text{ ms}^{-1}$$

$$\text{The velocity at ground } v_1 = 3 \text{ ms}^{-1} \quad (\text{given})$$

$$\text{Acceleration} = -2 \text{ ms}^{-1} \quad (\text{given})$$



$$H - h = \frac{v^2 - v_1^2}{2 \times 2}$$

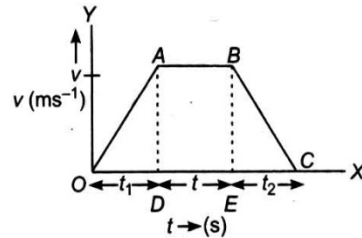
$$= \frac{980 - 9}{4}$$

$$= \frac{971}{4} = 242.75$$

$$\therefore H = 242.75 + h$$

$$= 242.75 + 50 \approx 293 \text{ m}$$

20. The velocity-time graph for the given situation can be drawn as below. Magnitudes of slope of OA = f



And slope of $BC = \frac{f}{2}$

$$v = ft_1 = \frac{f}{2} t_2$$

$$\therefore t_2 = 2t_1$$

In graph area of ΔOAD gives

Distances, $S = \frac{1}{2} ft_1^2$ (i)

Area of rectangle ABED gives distance travelled in time t.

$$S_2 = (ft_1)t$$

Distance travelled in time t_2

$$= S_3 = \frac{1}{2} f(2t_1)^2$$

Thus, $S_1 + S_2 + S_3 = 15S$

$$S + (ft_1)t + ft_1^2 = 15S$$

$$S + (ft_1)t + 2S = 15S \quad \left(S = \frac{1}{2} ft_1^2 \right)$$

$$(ft_1)t = 12S \quad \text{.....(ii)}$$

From eqs (i) and (ii), we have

$$\frac{12S}{S} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

$$\therefore t_1 = \frac{t}{6}$$

From eq (i), we get

$$\therefore S = \frac{1}{2} f(t_1)^2$$

$$\therefore S = \frac{1}{2} f \left(\frac{t}{6} \right)^2 = \frac{1}{72} ft^2$$

22. Average speed = $\frac{\text{distance travelled}}{\text{time taken}}$

Let t_1 and t_2 be times taken by the car to go from X to Y and then from Y to X respectively.

$$\text{Then, } t_1 + t_2 = \frac{XY}{v_u} + \frac{XY}{v_d} = XY \left(\frac{v_u + v_d}{v_u v_d} \right)$$

Total distance travelled

$$= XY + XY = 2XY$$

Therefore, average speed of the car for this round trip is

$$v_{av} = \frac{2XY}{XY \left(\frac{v_u + v_d}{v_u v_d} \right)} \text{ or } v_{av} = \frac{2v_u v_d}{v_u + v_d}$$

23. The position x of a particle with respect to time t along X-axis

$$x = 9t^2 - t^3 \dots\dots\dots(i)$$

Differentiating eq (i), with respect to time, we get speed, i.e

$$v = \frac{dx}{dt} = \frac{d}{dt} (9t^2 - t^3)$$

$$\text{Or } v = 18t - 3t^2 \dots\dots\dots(ii)$$

Again differentiating eq. (ii), with respect to time, we get acceleration i.e.

$$a = \frac{dv}{dt} = \frac{d}{dt} (18t - 3t^2)$$

$$\text{Or } a = 18 - 6t \dots\dots\dots(iii)$$

Now, when speed of particle is maximum, its acceleration is zero, i.e

$$A = 0$$

$$\text{I.e, } 18 - 6t = 0$$

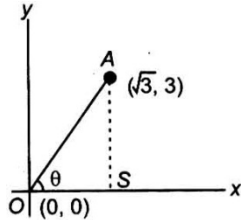
$$\text{Or } t = 3\text{s}$$

Putting in eq (i), we obtain position of particle at that time

$$x = 9(3)^2 - (3)^3 = 9(9) - 27$$

$$= 81 - 27 = 54\text{m}$$

24. Draw the situation as shown. OA represents the path of the particle starting from origin O (0, 0). Draw a perpendicular from point A to X-axis. Let path of the particle makes an angle θ with the X-axis, then



$\tan \theta = \text{slope of line OA}$

$$= \frac{AB}{OB} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Or $\theta = 60^\circ$

25. Acceleration $f = f_0 \left(1 - \frac{t}{T}\right)$

Or $f = \frac{dv}{dt} = f_0 \left(1 - \frac{t}{T}\right) \left[\because f = \frac{dv}{dt} \right]$

Or $dv = f_0 \left(1 - \frac{t}{T}\right) dt \dots\dots\dots (i)$

Integrating eq. (i) on both sides

$$\int dv = \int f_0 \left(1 - \frac{t}{T}\right) dt$$

$$\therefore v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} + C \dots\dots\dots (ii)$$

Where C is constant of integration.

Now, when $t = 0, v = 0$

So, from eq. (ii), we get, $C = 0$

$$\therefore v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} \dots\dots\dots (iii)$$

As, $f = f_0 \left(1 - \frac{t}{T}\right)$

When, $f = 0, 0 = f_0 \left(1 - \frac{t}{T}\right)$

As $f_0 \neq 0$, so, $1 - \frac{t}{T} = 0$

$\therefore t = T$

Substituting $t = T$ in eq. (iii) then velocity

$$v_x = f_0 T - \frac{f_0}{T} \cdot \frac{T^2}{2} = f_0 T - \frac{f_0 T}{2} = \frac{1}{2} f_0 T$$

27. From first equation of motion

$$v = u + at$$

$$0 = -4 + a \times (0.1)$$

$$\Rightarrow a = 40ms^{-2}$$

$$\therefore s = \frac{v^2}{2a}$$

$$\Rightarrow s = \frac{(4)^2}{2 \times 40}$$

$$\Rightarrow s = \frac{16}{80} \Rightarrow s = 0.2m$$

28. $\frac{dv}{dt} = bt \Rightarrow dv = dt dt \Rightarrow v = \frac{bt^2}{2} + k_1$

At $t = 0$, $v = v_0 \Rightarrow k_1 = v_0$

We get $v = \frac{1}{2}bt^2 + v_0$

Now $\frac{dx}{dt} = \frac{1}{2}bt^2 + v_0 \Rightarrow x = \frac{1}{2} \frac{bt^3}{3} + v_0 t + k_2$

At $t = 0$, $x = 0 \Rightarrow k_2 = 0$

$$\therefore x = \frac{1}{6}bt^3 + v_0 t$$

30. From equation of motion $v = u + at$

$$16 = 19 + 3a$$

[here $u = 10ms^{-1}$, $v = 16ms^{-1}$, $t = 3s$, $a = 2ms^{-2}$]

And $10 = u + 2 \times 2$ ($u =$ required velocity)

$$u = 6ms^{-1} (\because t = 2s)$$

31. From $s = ut + \frac{1}{2}gt^2$

As the body is falling from rest, $u = 0$

$$s = \frac{1}{2}gt^2$$

Suppose the distance travelled in $t = 2s$, $t = 4s$, $t = 6s$ are s_2, s_4 and s_6 respectively

Now $s_2 = \frac{1}{2}g(2)^2 = 2g$

$$s_4 = \frac{1}{2}g(4)^2 = 8g$$

$$s_6 = \frac{1}{2} g(6)^2 = 18g$$

Hence, the distance travelled in first two seconds

$$(s_i)_2 = s_2 - s_0 = 2g$$

$$(s_m)_2 = s_4 - s_2 = 8g - 2g = 6g$$

$$(s_f)_2 = s_6 - s_4 = 18g - 8g = 10g$$

Now, the ratio becomes

$$= 2g : 6g : 10g = 1 : 3 : 5$$

33. Distance from 0 to 5s = 40 m

Distance from 5 to 10s = 0 m

Distance from 1 to 15s = 60 m

Distance from 15 to 20s = 20 m

So, net distance

$$= 40 + 0 + 60 + 20 = 120 \text{ m}$$

Total time taken = 20 min.

Hence, average speed

$$= \frac{\text{distance (m)}}{\text{time (min)}} = \frac{120}{20} = 6 \text{ m min}^{-1}$$

34. Given $x = ae^{-\alpha t} + be^{\beta t}$

So, velocity $v = \frac{dx}{dt}$

$$= -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

$$= A + B$$

Where $A = -a\alpha e^{-\alpha t}$, $B = b\beta e^{\beta t}$

The value of term $A = -a\alpha e^{-\alpha t}$ decreases and of term $B = b\beta e^{\beta t}$ increases with increase in time.

As a result, velocity goes on increasing with time.

35. $v^2 = u^2 - 2hg$

$$\text{Or } u^2 \propto h$$

$$\therefore \frac{u_1}{u_2} = \sqrt{\frac{h_1}{h_2}}$$

$$\text{Or } \frac{v_0}{u_2} = \sqrt{\frac{h}{3h}} \text{ or } u_2 = \sqrt{3}v_0$$