Wave Motion

Wave and Wave motion:

- Wave is a carrier of energy
- Wave is a form of disturbance which travels through a material medium due to the repeated periodic motion of the particles of the medium about their mean position. The disturbance is handed over from one particle to another particle of the medium.

Characteristics of wave motion:

- Wave motion is the disturbance traveling through the medium.
- When a wave travels through a medium, its particles execute SHM about their mean positions.
- Particles of medium hand over the energy to their next neighbors, but their displacement over one time period is zero.
- As the disturbance reaches a particle, it starts vibrating. The disturbance is transverse to the next particle a little latter. Hence there is a regular phase difference between consecutive particles.
- The wave travels with a uniform velocity where as the velocity of the particles is different at different positions.
- The wave velocity depends on the medium and the particle velocity is the function of time.
- In wave motion, the transfer of energy and momentum takes place from one point to another of the medium, but not matter.
- The properties of medium necessary for wave propagation :
  a) The medium should have the property of inertia.
  b) The medium should posses the property of elasticity.
  c) The medium should have low resistance (non viscous).

Types of Motion:

Wave motion is of two types

1) Transverse wave motion and 2) Longitudinal wave motion

Transverse wave motion:

- The wave motion in which the particles of the medium vibrate about their mean positions at right angles to the direction of propagation of the wave is called transverse waves.

Ex: Waves in a stretched string, ripples on water surface, Electromagnetic Waves etc.

- The region of elevation of the medium through which the wave propagates is called crest and the region of depression is called trough.
- The distance between two consecutive crests (or) troughs is called wavelength (λ).
- These travel in a medium which has the elasticity of shape.
- These can travel in solids.
- Density of the medium does not change during wave motion
- The phase difference between the particles at two consecutive crests (or) troughs is 2π radians (or) $360°$.
- These can be polarized.

Longitudinal wave motion:

- The wave motion in which the particles of the material medium vibrate back and forth about their mean position along the direction of the propagation of wave is called longitudinal waves Ex: Waves produced when a spring fixed at one end is placed and released, sound waves in air etc
- The region in which particles come close to a distance less than the normal distance is called compression. The region in which the particle get apart to a distance greater than the normal distance between them is called rare fraction
- The distance between two consecutive compressions (or) rarefactions is called wave length (λ)
- These travel in a medium which it has elasticity of volume.
- These can travel in solids, liquids and gases.
- Density of the medium changes during wave motion. In the compression region, the density of the media increases and in the rare fraction, the density of the medium decreases.
- The phase differences between the particles at two consecutive compressions has rare fraction is 2π radians or $360°$.
- These cannot be polarized.

Properties of Progressive waves

- These waves propagate in the forward direction of medium with finite velocity.
- Energy is propagated via these waves.
- In these waves all the particles of the medium execute S.H.M. with same amplitude and same frequency.
- In these waves all the particles of the medium pass through their mean position or positions of maximum displacements one after the other.
- In these waves the velocity of the particle and the strain are proportional to each other.
- This wave is an independent one.
- In these waves equal change in pressure and density occurs at all points of medium.
- In these waves equal strain is produced at all points.
- In these waves all the particles of the medium cross their mean position once in one time period.
- In these waves the average energy over one time period is equal to the sum of kinetic energy and potential energy.
- The energy per unit volume of a progressive wave is $\frac{1}{2}\rho A^2\omega^2$ where ρ is the density of the medium
- The equation of a progressive wave along the positive direction of x-axis is
  - $y = A\sin(\omega t - kx)$

  Or $y = A\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$

  Or $y = A\sin 2\pi n(t - \frac{x}{V})$ where y = displacement of a particle at an instant t; A = amplitude; ω = angular frequency = $2\pi n$; T = time period and k = propagation constant or angular wave number or wave vector and is equal to $\frac{2\pi}{\lambda}$.
- The time taken for one vibration of a particle is called time period or period of vibration ($T = \frac{1}{n}$) and Velocity of the wave $v = n\lambda$
- The maximum displacement of a vibrating particle from its mean position is called amplitude.
- The phase of vibration at any moment is the state of vibrating particle as regards its position and direction of motion at that moment.

- Phase difference $\Delta\phi = \frac{2\pi}{\lambda}$ x Path difference.
- The distance travelled by a wave in the time in which the particles of the medium complete one vibration or the distance between two nearest particles in the same state of vibration (i.e., same phase) is called wavelength (λ).

Reflection and Refraction of Waves

- **Rigid end:** When the incident wave reaches a fixed end, it exerts an upward pull on the end; according to Newton's law the fixed end exerts an equal and opposite down ward force on the string. It result an inverted pulse or phase change of π.

  Crest (C) reflects as trough (T) and vice-versa, Time changes by $\frac{T}{2}$ and Path changes by $\frac{\lambda}{2}$
- **Free end:** When a wave or pulse is reflected from a free end, then there is no change of phase (as there is no reaction force).

  Crest (C) reflects as crest (C) and trough (T) reflects as trough (T), Time changes by zero and Path changes by zero.

Principle of Superposition

- The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due each one of the waves at that point at the same time.
- If $\vec{y_1}, \vec{y_2}, \vec{y_3}$ .... are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement.

  $\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} + ...$

Properties of Stationary waves:

a) All the particles except a few (at nodes) execute S.H.M.
b) The period of each particle is the same but the amplitude of vibration varies from particle to particle.
c) The distance between any two successive nodes or antinodes is equal to $\lambda/2$.
d) The distance between a node and neighboring antinodes is equal to $\lambda/4$.
e) The wave is confined to a limited region and does not advance.
f) All the particles of a wave in a loop are in the same phase and the phase difference is zero.
g) Stationary waves are formed by combining two longitudinal progressive waves or two transverse progressive waves.
h) These waves do not transfer energy.
i) The change in pressure or density or strain will be maximum at nodes and minimum at antinodes.
j) The particle velocity at a node is zero and at antinodes it is maximum.
k) The phase difference between the particles in adjacent loops in a stationary wave is π.
l) The equation of a stationary wave is
   y = 2A sinkx.cosωt or y = 2A coskx.sinωt.

Types of vibrations:

- Whenever a body, capable of vibration, is displaced from its equilibrium position and then left to itself, the body begins to vibrate freely in its own natural way called the free or natural vibration of the body with a definite frequency. This frequency is called natural frequency.

- The free vibrations of a body have a unique frequency and it is dependent on the elasticity and inertia of the body and the mode of vibration.
- When a body is set into vibration with the help of strong periodic force having a frequency different from its natural frequency, then the vibrations of the body are called forced vibrations.
- If the amplitude of vibrations progressively decreases with time, then they are called damped vibrations. E.g. Vibrations of a tuning fork.
- Bells are made of metals and not of wood because wood dampens the vibrations while the metals are elastic.
- If the natural frequency of a vibrating body is equal to the frequency of the external periodic force and if they are in phase, the frequencies are said to be in resonance.
- Tuning a radio or television receiver is an example of electrical resonance.
- Optical resonance may also take place between the atoms in a gas at low pressure.

Vibrations of a string:

a) String can have only transverse vibrations that too when it is under tension.

b) The velocity of transverse wave propagating along a string or wire under tension is $V=\sqrt{\frac{T}{m}}$ where T is tension and m is linear density of the string or wire. $M=\frac{M}{l}=A.d=\pi r^2.d$ where M is total mass of wire of length 'l', A is area of cross-section of wire and r is its radius. Hence $V=\sqrt{\frac{T}{m}}=\sqrt{\frac{Tl}{M}}=\sqrt{\frac{T}{Ad}}=\sqrt{\frac{T}{\pi r^2 d}}$

c) If s is stress in the wire, S=T/A, hence $V=\sqrt{\frac{s}{d}}$; also $V=\sqrt{\frac{Y.Strain}{d}}$.

d) A wire held at the two ends by rigid support is just taut at temperature t_1. The velocity of transverse wave at a temperature t_2 is $V=\sqrt{\frac{Y\alpha(t_2 \sim t_1)}{d}}$ where α=co-efficient of linear expansion, Y=Young's modulus, d=density.

- **Frequency of a vibrating string :**
  a) The waves formed in a string under tension are transverse stationary.
  b) Always nodes are formed at fixed ends and antinodes at plucked points and free ends.
  c) A string can have number of frequencies depending on its mode of vibration.
- **Fundamental frequency:** When a string vibrates in a single loop, it is said to vibrate with fundamental frequency.
  a) Frequency is minimum and wavelength is maximum in this case.
  b) If l is the length of the string $l=\frac{\lambda}{2} \Rightarrow \lambda = 2l$.
  c) The fundamental frequency, $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$ where T=tension, m=linear density.
  d) The fundamental frequency is also given by $n=\frac{1}{2}\sqrt{\frac{T}{Ml}}=\frac{1}{2l}\sqrt{\frac{T}{Ad}}=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}}=\frac{1}{2l}\sqrt{\frac{s}{d}}$.
  e) For small change in tension in string, the fractional change in frequency is $\frac{\Delta n}{n}=\frac{1}{2}\frac{\Delta T}{T}$.
  f) The fundamental frequency is also called the first harmonic.
- **Overtones:** If string vibrates with more number of loops, higher frequencies are produced called overtones.

a) If string vibrates in p loops, it is called pth mode of vibration or pth harmonic or (p–1)th overtone. The corresponding frequency $n_p = \frac{p}{2l}\sqrt{\frac{T}{m}}$ =p.n Hence, for a string, $n_p \alpha p$; $\frac{n_1}{n_2} = \frac{p_1}{p_2}$ when other, quantities are constant.

b) The fundamental and overtone frequencies are in the ratio 1:2:3:4:….

c) The wavelength is above case is $\lambda_p = \frac{2l}{p}$ i.e., wavelengths are in the ratio $1 : \frac{1}{2} : \frac{1}{3} : ...$

- **Laws of transverse waves along stretched string :**
  a) Law of length: The frequency of a stretched string is inversely proportional to the length of the string $n \alpha 1/l$ where T & m are constants, nl=constant, $n_1 l_1 = n_2 l_2$.

  b) Law of tension: The frequency of a stretched string is inversely proportional to square root of tension. $n \alpha \sqrt{T}$ When l & T are constant. $\frac{n}{\sqrt{T}}$ =constant, $\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}}$

  c) Law of mass: The frequency of a stretched string is inversely proportional to square root of linear density $n \alpha \frac{1}{\sqrt{m}}$ when l & T are constants. $n\sqrt{m}$ =constant; $n_1\sqrt{m_1} = n_2\sqrt{m_2}$

- Sonometer is used to determine the velocity of transverse waves in strings and to verify the laws of transverse waves.

  $RD = \frac{l_1^2}{l_1^2 - l_2^2}$

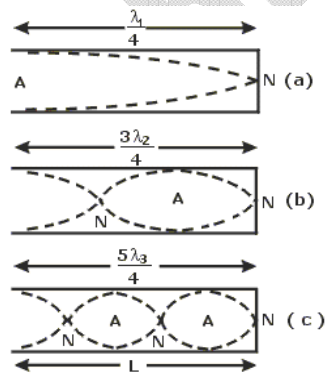
Stationary waves in Organ pipes:

- An organ pipe is a cylindrical tube having an air column. The vibration of a cylindrical air column are made of two progressive longitudinal vibrations moving in opposite directions with equal and opposite speed superposed on each other. Hence the waves are longitudinal stationary waves.

- The possible frequencies in which standing waves are formed are called harmonics.

Closed pipes:

- A Pipe whose one end is closed and the other end is open is called closed pipe. At the closed end of the pipe always a node is formed.



- If ℓ is the length of the pipe

  $n_1 = \frac{V}{4\ell}$ This is called fundamental frequency (or) 1st harmonic

- In the 1st overtone (or) 2nd harmonic, two nodes and two antinodes are formed in the pipe

$$n_2 = \frac{3V}{4\ell} = 3n_1$$

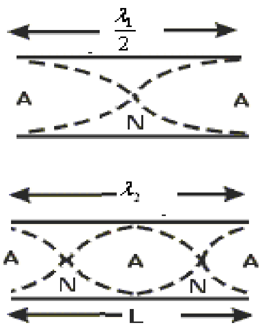
- Similarly for the 2nd overtone (or) 3rd harmonic three nodes and three antinodes are formed in the pipe

$$\therefore n_3 = \frac{5V}{4\ell} = 5n_1$$

$$\therefore n_1 : n_2 : n_3 : .... = 1:3:5: ....$$

Open pipes:

- A pipe whose both ends are open is called open pipe. At the open ends of the pipe always antinodes are formed



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- If ℓ is the length of the pipe in the simplest mode of vibration two antinodes are formed, one at each end with a node at the middle of the pipe

$n_1 = \frac{V}{2\ell}$ This is called the fundamental frequency (or) 1st harmonic.

- In the 1st overtone (or) 2nd harmonic, two nodes and three antinodes are formed in the pipe.

$$n_2 = \frac{V}{\ell} = 2n_1$$

- Similarly for the 2nd overtone (or) 3rd harmonic, three nodes and four antinodes are formed in the pipe.

$$n_3 = \frac{3V}{2\ell} = 3n_1$$

$$\therefore n_1 : n_2 : n_3 = 1:2:3: ....$$

- It is observed that in an open pipe all harmonics are formed where as in a closed pipe only odd harmonics are formed.

- In General Antinodes are formed nearer to the open end outside the pipe. The exact distance between the position of the antinode and the mouth of the pipe is measured as end correction (e). The end correction is double for open pipe.

- End correction = $e = 0.3d = 0.6r$ where d is the diameter and r is the radius of the pipe.
- The distance of the exact antinode from the brim of the pipe is called end correction.

For a closed pipe, $\ell_c + e = \frac{\lambda}{4}$

$$e = \left(\frac{\lambda - 4\ell_c}{4}\right)$$

For an open pipe, $\ell_o + 2e = \frac{\lambda}{2}$

$$e = \frac{\lambda - 2\ell_o}{4}$$

BEATS :

- When two sounds of slightly different frequencies superimpose, the resultant sound consists of alternate waxing and waxing. This phenomenon is called beats.
- One waxing and one waning together is called one beat.
- If simple harmonic progressive waves of frequencies n_1 & n_2 travelling in same direction superimpose, the resultant wave is represented by y= $\left\{2a\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t\right\}\sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t$.
- The amplitude of resultant wave is $2a\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t$.
- The maximum amplitude is 2a and minimum amplitude zero.
- The frequency of resultant wave is $\left(\frac{n_1 + n_2}{2}\right)$.
- The number of beats produced per second or beat frequency is equal to the difference of frequencies of nodes producing beats. $n=n_1 \sim n_2$.
- If two sound waves of wavelengths λ_1 and λ_2 produce n beats per second, then velocity of sound can be determined by n= $\frac{V}{\lambda_1} \sim \frac{V}{\lambda_2}$ or $V = \frac{\lambda_1\lambda_2}{(\lambda_2 \sim \lambda_1)}$.
- The maximum number of beats heard by a person is 10, since persistence of hearing is 1/10 sec.
- The time internal between two consecutive maxima or minima is $\frac{1}{(n_1 \sim n_2)}$.
- The time interval between consecutive maxima and minima is $\frac{1}{2(n_1 \sim n_2)}$.
- Beats can be produced by taking two identical tuning fork and loading or filing either of them and vibrating them together.
- When a tuning fork is loaded its frequency decreases and when it is filed frequency increases.

DOPPLER EFFECT :

The apparent change in frequency due to relative motion between the source and the listener is called **Doppler Effect**.

- Let V_O and V_s represents the velocities of a listener and a source respectively. Let V be the velocity of sound and n and n_1 be the true and apparent frequencies of the sound. Then if

a) The source alone is in motion towards the observer, $n_1 = n\left(\frac{V}{V - V_s}\right)$. Clearly $n_1 > n$.

b) the source alone is in motion away from the observer, $n_1 = n\left(\frac{V}{V + V_s}\right)$. Clearly $n_1 < n$.

c) the observer alone is in motion towards the source, $n_1 = n\left(\frac{V + V_O}{V}\right)$. Clearly $n_1 > n$.

d) the observer alone is in motion away from the source, $n_1 = n\left(\frac{V - V_O}{V}\right)$. Clearly $n_1 < n$.

e) the source and the observer both are in motion towards each other, $n_1 = n\left(\frac{V + V_O}{V - V_s}\right)$.

f) the source and the observer both are in motion away from each other, $n_1 = n\left(\frac{V - V_O}{V + V_s}\right)$.

g) the source and the observer both are in motion, source following the observer, $n_1 = n\left(\frac{V - V_O}{V - V_s}\right)$.

h) the source and the observer both are in motion, observer following the source, $n_1 = n\left(\frac{V + V_O}{V + V_s}\right)$.

i) the source, observer and the medium all are moving in the same direction as the sound, $n_1 = n\left(\frac{V + V_w - V_O}{V + V_w - V_s}\right)$ where V_w = velocity of wind.

j) the source and the observer are moving in the direction of the sound but the direction of wind is opposite to the direction of the propagation of sound,
$n_1 = n\left(\frac{V - V_w - V_O}{V - V_w - V_s}\right)$.

- If the source of sound is moving towards a wall and the observer is standing between the source and the wall, no beats are heard by the observer.
- When source and observer are not moving along the same line then $n^{l} = n\left[\frac{V}{V - V_s \cos\theta}\right]$ where θ is angle between source velocity and line joining source and observer
- When source and observer do not move along the line joining them, then components of their velocities along the line joining them must be taken as velocity of observer and velocity of source in Doppler is formula n = $n_0\left(\frac{V + V_0 \cos\theta_2}{V - V_s \cos\theta_1}\right)$.
- If $\vec{r}$ is unit vector along line joining source and observer, $\vec{v}$ is velocity of sound (taken from the source to observer), $\vec{v}_0$ is velocity of observer and $\vec{v}_s$ is velocity of source then Doppler's effect in vector form is $n^{l} = \frac{(\vec{v}.\vec{r} - \vec{v}_0.\vec{r})}{(\vec{v}.\vec{r} - \vec{v}_s.\vec{r})}n$.
- Doppler Effect in sound is asymmetric. This means the change in frequency depends on whether the source is in motion or observer is in motion even though relative velocities are same in both cases.

- Motion of source produces greater change than motion of observer even though the relative velocities are same in both cases.

  Eg : $n^{I} = \left(\frac{v}{v-u}\right)n$

  $n^{II} = \left(\frac{v+u}{v}\right)n$

  $n^{I} > n^{II}$
- Doppler Effect in sound is asymmetric because sound is mechanical wave requiring material medium and v, v_0, v_s are taken with respect to the medium.
- Doppler effect in light is symmetric because light waves are electromagnetic (do not require medium)
- Doppler effect is not applicable if
  1) $V_0=V_s=0$ (both are at rest)
  2) $V_0=V_s=0$ and medium is alone in motion
  3) $V_0=V_s=u$ (V_0, V_s are in same direction)
  4) V_s is $\perp$ to line of sight
- Doppler effect is applicable only when, $V_0<<v$ and $V_s<<v$. (v=velocity of sound)
- a) Doppler effect in sound is asymmetric.

  b) Doppler Effect holds good for light also. An increase of frequency is called blue shift and it indicates that the source is approaching the observer. Red shift indicates that the source is receding from the observer.

  Red shift $(\Delta\lambda) = \frac{V}{c} \times \lambda$

  c) Doppler Effect in light is symmetric.

  d) The red shift observed by Hubble in many stars supports the 'Big Bang Theory' of the universe.
- **Uses of Doppler effect :**

  It is used in

  a) SONAR

  b) RADAR (Radio detection and Ranging used to determine speed of objects in space) (Radio waves)

  c) To determine speeds of automobiles by traffic police.

  d) To determine speed of rotation of sun and to explain Saturn's rings.

  e) Led to the discovery of double stars/Binary stars.

  f) In accurate navigation and accurate target bombing techniques.

  g) In tracking earth's satellite.
- Doppler's effect is used in the estimation of the velocities of aero planes and submarines, the velocities of stars and galaxies and the velocities of satellites.
- If the observer is standing behind the source moving towards a wall with a velocity V_s, then the number of beats heard is equal to $n\left[\frac{V}{V-V_s} - \frac{V}{V+V_s}\right]$ and is approximately equal to $\frac{2nV_s}{V}$.