SIMPLE PENDULUM

1. The period of oscillation of a simple pendulum is independent of amplitude (for small values only), length being constant.

2. At constant length, the period of oscillation of a simple pendulum is independent of size, shape or material of the bob.

Time period of a simple pendulum (T) $= 2\pi\sqrt{\frac{L}{g}}$. Where

l = length of the simple pendulum

g = acceleration due to gravity at a place.

3. Tension in the string of simple pendulum

a. T_{min} = mg Cos θ (when bob is at extreme position)

b. T = mg (3 - 2 Cosθ) (When bob is at any position)

Where θ is any angular amplitude

4. I - T^2 graph of a simple pendulum is straight line passing through origin.

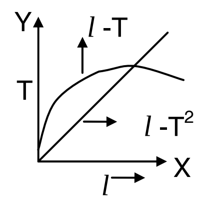
5. l-T graph of a simple pendulum is parabola.

6. At the point of intersection of l-T graph and l–T^2 graph of a simple pendulum

i) T = 1 second

ii) n = 1 Hz.

iii) $l = \frac{g}{4\pi^2} \cong 25cm$ On the surface of the earth



7. If $L = \infty$ (infinity). $T = 2\pi\sqrt{\frac{R}{g}} = 84.5$ min.

8. If $L = R$, $T = 2\pi\sqrt{\frac{R}{2g}} = \frac{84.5}{\sqrt{2}}$ min

9. If L is very small compared to Radius of the earth, $T = 2\pi\sqrt{\frac{\ell}{g}}$

10. Restoring Force on the bob of the pendulum is $F = mg\sin\theta$.

11. Seconds pendulum:

i) The simple pendulum whose time period equal to 2 seconds is called seconds pendulum.

ii) The length at place where g = 9.8 m/s^2 is 100 cm.

iii) Since T = 2 sec, L = $\frac{g}{\pi^2}$

iv) For two places, change in length = $\frac{g_1 \sim g_2}{\pi^2}$

APPLICATION

i) When the elevator is going up with an acceleration a, then its time period is given by T = $2\pi\sqrt{\frac{L}{g+a}}$

ii) When the elevator is moving down with an acceleration a, then its time period is given by T = $2\pi\sqrt{\frac{L}{g-a}}$

iii) When the elevator is at rest or moving up or down with constant velocity the time period is given by

T = 2 $T = 2\pi\sqrt{\frac{L}{g}}$

iv) When the elevator is moving down with an acceleration (-a) then its time period is given by

$$T = 2\pi\sqrt{\frac{L}{g+a}}$$

v) In case of downward accelerated motion is a > g the pendulum turns upside and oscillates about the highest point with T = $2\pi\sqrt{\frac{L}{a-g}}$

vi) If a simple pendulum of length 'L' suspended in a car that is travelling with a constant speed around a circle of radius 'r', Then its time period of oscillation is given by $T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}}$

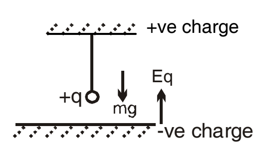
vii) If a simple pendulum of length 'L' suspended in car moving horizontally with acceleration 'a' is given by $T = 2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$.

The equilibrium position is inclined to the vertical by an angle 'θ'. Where θ = $\tan^{-1}\left(\frac{a}{g}\right)$ opposite to the acceleration.

viii) If the bob of a simple pendulum is given a charge 'q' and is arranged in an electric field of intensity 'E' to oscillate.

a) Opposite to g, → Electric force E_q will be opposite to the force mg. Hence g^1=g-$\frac{Eq}{m}$ .Then T_1=2π$\sqrt{\frac{l}{g-\frac{Eq}{m}}}$. So time period increases.

b) In the direction of g → Electric force Eq will be in the direction of force mg. Hence g^1 = g + $\frac{Eq}{m}$ then T_1 = 2π $\sqrt{\frac{l}{g+\frac{Eq}{m}}}$ so time period decreases.



c) Perpendicular to g → Electric force E_q will be perpendicular to the force mg. Hence$g^1 = \sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}$ Then $T_1 = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}}}$. So time period decreases.

ix) If a simple pendulum of length L is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination 'θ'. Then the time period of oscillations is given by $T = 2\pi\sqrt{\frac{L}{g\cos\theta}}$ since the effective acceleration changes from g to g cosθ.

x) A simple pendulum fitted with a metallic bob of density d_s has a time period T. When it is made to oscillate in a liquid of density d_1 then its time period increases. $T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{d_l}{d_s}\right)}}$

12. Time period of Torsion pendulum $T = 2\pi\sqrt{\frac{I}{C}}$ I = moment of Inertia about the suspension wire C = couple per unit twist.

13. When a hole is drilled along the diameter of the earth and if a body is dropped in it, it moves to and from about the centre of the earth and is in S.H M. with a time period of

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$