ROUGH WORK

SA

KVPY QUESTION PAPER –STREAM SA
October 31, 2010

PART B

This booklet contains 12 questions with space provided for the answers

Name of the candidate: ....

Seat No. : ....

Signature of the candidate:

Signature of the Invigilator:

MATHEMATICS

1 In a triangle ABC, D and E are points on AB, AC respectively such that DE is parallel to BC. Suppose BE, CD intersect at O. If the areas of the triangles ADE and ODE are 3 and 1 respectively, find the area of the triangle ABC, with justification. (5 Marks)

Answer:

2 Leela and Madan pooled their music CD's and sold them. They got as many rupees for each CD as the total number of CD's they sold. They share the money as follows: Leela first takes 10 rupees, then Madan takes 10 rupees and they continue taking 10 rupees alternately till Madan is left out with less than 10 rupees to take. Find the amount that is left out for Madan at the end, with justification. (5 Marks)

Answer:

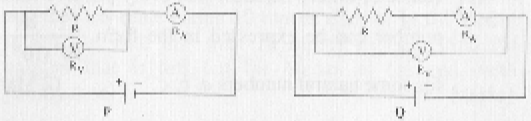
3 (a) Show that for every natural number n relatively prime to 10, there is another natural number m all of whose digits are 1's such that n divides m. (3 Marks)

(b) Hence or otherwise show that every positive rational number can be expressed in the form $\dfrac{a}{10^b(10^c-1)}$ for some natural numbers a, b, c. (2 Marks)

Answer:

PHYSICS

4 Consider the two circuits P and Q, shown below, which are used to measure the unknown resistance *R*.



In each case, the resistance is estimated by using Ohm's law: $R_{est} = V/I$, where V and I are the readings of the voltmeter and the ammeter respectively. The meter resistances, R_V and R_A are such that $R_A << R << R_V$. The internal resistance of the battery may be ignored. The absolute error in the estimate of the resistance is denoted by $\delta R = |R - R_{est}|$.

(a) Express δR_P in terms of the given resistance values. (2 marks)

(b) Express δR_Q in terms of the given resistance values. (2 marks)

(c) For what value of R will $\delta R_P \approx \delta R_Q$? (1 mark)

Answer:

5 A point source is placed 20 cm to the left of a concave lens of focal length 10 cm.

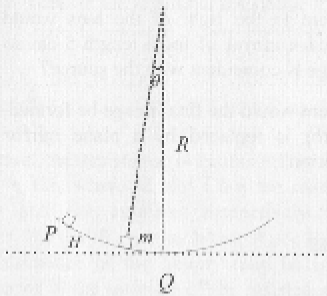
(a) Where is the image formed? (1 mark)

(b) Where to the right of the lens would you place a concave mirror of focal length 5 cm so that the final image is coincident with the source? (2 marks)

(c) Where would the final image be formed if the concave mirror is replaced by a plane mirror at the same position? (2 marks)

Answer:

6 A block of mass m is sliding on a fixed frictionless concave surface of radius R. It is released from rest at point P which is at a height of $H << R$ from the lowest point Q.



(a) What is the potential energy as a function of θ, taking the lowest point Q as the reference level for potential energy? (1 mark)

(b) What is the kinetic energy as a function of θ? (1 mark)

(c) What is the time taken for the particle to reach from point P to the lowest point Q? (2 marks)

(d) How much force is exerted by the block on the concave surface at the point Q? (1 mark)

Answer:

CHEMISTRY

7 Copper in an alloy is estimated by dissolving in conc. nitric acid. In this process copper is converted to cupric nitrate with the evolution of nitric oxide (NO). The mixture when treated with potassium iodide forms cupric iodide, which is unstable and decomposes to cuprous iodide and iodine. The amount of copper in the alloy is estimated by titrating the liberated iodine with sodium thiosulfate. The reactions are:

a Cu + b HNO_3 → c $Cu(NO_3)_2$ + d NO + e H_2O

f CuI_2 → g Cu_2I_2 + h I_2

i $Na_2S_2O_3$ + j I_2 → k $Na_2S_4O_6$ + l NaI

(fill up the blanks)

(a) The coefficients are: a = ____, b = ____, c = ____, d = ____ and e = ____. (1 mark, no partial marking)

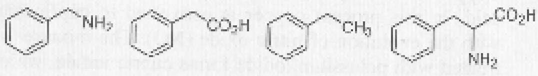
(b) The coefficients are: f = ____, g = ____ and h = ____. (1 mark, no partial marking)

(c) The coefficients are: i = ____, j = ____, k = ____ and l = ____. (1 mark, no partial marking)

(d) If 2.54 g of I_2 is evolved from a 2.0 g sample of the alloy, what is the percentage of copper in the alloy? (atomic weights of iodine and copper are 127 and 63.5, respectively). (2 marks, no partial marking)

Answer:

8 You have been given four bottles marked A, B, C and D each containing one of the organic compounds given below.



I II III IV

The following observations were made.

(i) The compound in the bottle A did not dissolve in either 1 N NaOH or 1 N HCl.

(ii) The compound in the bottle B dissolved in 1 N NaOH but not in 1 N HCl.

(iii) The compound in the bottle C dissolved in both 1 N NaOH and 1 N HCl.

(iv) The compound in the bottle D did not dissolve in 1N NaOH but dissolved in 1 N HCl.

(fill up the blanks)

(a) Indicate the compounds in: bottle A = ____, bottle B = ____, bottle C = ____and bottle D = ____.

(4 x 1 mark = 4 marks, no partial marking)

(b) The compound with the highest solubility in distilled water is ____. (1 mark, no partial marking)

(indicate the answers by the compound numbers)

9 Assume that a human body requires 2500 kcal of energy each day for metabolic activity and sucrose is the only source of energy, as per the equation

$C_{12}H_{22}O_{11}(s) + 12\,O_2(g) \rightarrow 12\,CO_2(g) + 11\,H_2O(l)$;

$\Delta H = -5.6 \times 10^6$ J.

(fill up the blanks)

(a) The energy requirement of the human body per day is _______kJ. (1 mark, no partial marking)

(b) The mass of sucrose required to provide this energy is________g and the volume of CO_2 (at STP) produced is ________litres.

(2 + 2 marks, no partial marking)

BIOLOGY

10 Mohini, a resident of Chandigarh went to Shimla with her parents. There she found the same plant that they have in their backyard, at home. However, she observed that while the plants in their backyard bore white flowers, those in Shimla had pink flowers. She brought home some seeds of the plant from Shimla and planted them in Chandigarh. Upon performing self-breeding for several generations she found that the plant from Shimla produced only white flowers.

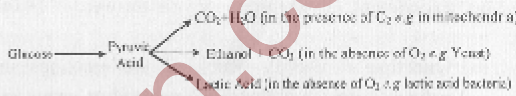
(a) According to you what might be the reason for this observation – genetic or environmental factors? (1 mark)

(b) Suggest a simple experiment to determine whether this variation is genetic in nature. (2 marks)

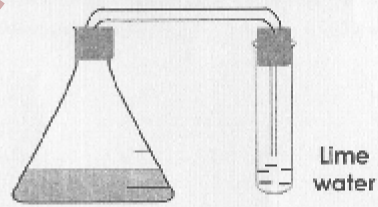
(c) Suggest another experiment to check whether this variation in flower color is due to environmental factors. (2 marks)

Answer:

11 The break-down of glucose in a cell occurs in any of the following pathways:



Three experiments (A, B, C) have been set up. In each experiment, a flask contains the organism in growth medium, glucose and a brown dye that changes its colour to



Organism in Culture Medium + Glucose + Dye

yellow when the pH decreases. The mouth of the flask is attached to a test tube containing lime water (Calcium Hydroxide; as shown in the figure). In C, but not in A and B, air is removed from the flask before beginning the experiment.

After a period of growth, the following observations were made:

A: Lime water turns milky; the dye colour remains the same.

B: The dye colour changes; lime water does not turn milky.

C: Lime water turns milky; the dye colour remains the same.

(a) **Question :** Identify which of the reactions in the pathways depicted above is taking place in each experiment. Give reasons for your answer. (4 marks)

(b) **Question :** Identify which of the reactions in the pathways depicted above is expected to occur in Red Blood Cells (RBCs). (1 mark)

Answer:

12 A scientist has a house just beside a busy highway. He collects leaves from some plants growing in his garden to do radio-carbon dating (to estimate the age of the plant by estimating the amount of a radioisotope of carbon in its tissues). Surprisingly the radio-carbon dating shows that the plant is a few thousand years old!

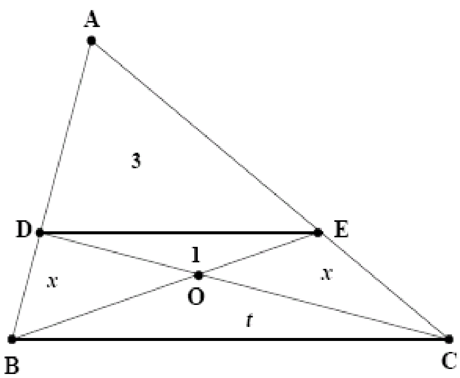
(a) Was the result of the radio-carbon dating wrong or can you propose a reason for such an observation? (3 marks)

(b) What simple experiment can be done to test the reason that you have proposed? (2 marks)

Answer:

Q.1

Solution:



We denote the area of traingle PQR by $[PQR]$. We see that $[BOD]$ and $[COE]$ are equal. Let the common value be x, and let $[BOC] = t$. Using the fact that the ratio of areas of two triangles having equal altitudes is the same as the ratio of their respective bases, we obtain

$$\frac{x}{1} = \frac{BO}{OE} = \frac{t}{x}.$$

This gives $t = x^2$. Now ADE and ABC are similar so that

$$\frac{[ADE]}{[ABC]} = \frac{DE^2}{BC^2} = \frac{[ODE]}{[OBC]},$$

since ODE and OCB are also similar. This implies that

$$\frac{3}{4+2x+t} = \frac{1}{t},$$

which simplifies to $t = 2+x$. Using $t = x^2$, we get a quadratic in x: $x^2 - x - 2 = 0$. Its solution are $x = 2$ and $x = -1$. Since x cannot be negative, $x = 2$ and $t = 4$. Thus $[ABC] = 4 + 2x + t = 4 + 4 + 4 = 12$.

Q.2

Solution: Let t be the total number of CD's that Leela and Madan together sold. Then they obtain t^2 rupees together. Since Leela is the first one to take 10 rupees and also the last one to take 10 rupees, we must have

$$t^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

Suppose $t = 10q + r$, where r is the remainder when t is divided by 10. Then $t^2 = 100q^2 + 20qr + r^2$. Comparing, we conclude that

$$r^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

But we know that $0 \leq r < 10$. Taking $r = 0, 1, 2, \ldots, 9$, we see that $r = 4$ or 6(for other values of r, tens place in r^2 is even). But then $r^2 = 16$ or 36. Hence the amount left for Madan at the end is 6 rupees.

Q.3

(a) Divide the $n+1$ numbers $1, 11, 111, \ldots, 111\cdots1$(all having only 1 as digits) by n. Among the $n+1$ remainders so obtained, two must be equal as the possibilities for remainders are $0, 1, 2, \ldots, n-1$ which are n in number. Thus there must be two numbers $x = 11\cdots1$ and $y = 11\cdots1$ having say j digits and k digits respectively which leave the same remainders after division by n. We may take $j < k$. Now we see that $y - x$ is divisible by n. But $y - x = 11\cdots100\cdots0$ where there are $k - j$ number of 1's and remaining zeros. Since n is coprime to 10, we see that n divides $m = 11\cdots1$, a number having only 1's as its digits.

(b) If p/q is any rational number $(p > 0, q > 0)$, then we may write $q = 2^r 5^s t$, where t is coprime to 10. Choose a number m having only 1's as its digits and is divisible by t. Consider $9m$, which has only 9 as its digits and is still divisible by t. Let $k = 9m/t$. We see that

$$qk = 9m2^r5^s = (10^c - 1)2^r5^s,$$

where c is the number of digits in m. Hence we can find d such that $qd = 10^b(10^c - 1)$(multiply by a suitable power of 2 if $s > r$ and by a suitable power of 5 if $r > s$). Then

$$\frac{p}{q} = \frac{pd}{qd} = \frac{a}{10^b(10^c - 1)},$$

where $a = pd$.

Q. 4

Solution: For P: $I = I_R + I_V = V/R + V/R_V$

$$R = \frac{V}{I}\left[\frac{R_V}{R_V - V/I}\right]$$
$$= R_{\text{est}}\left[\frac{1}{1 - R_{\text{est}}/R_V}\right]$$
$$\approx R_{\text{est}}\left[1 + R_{\text{est}}/R_V\right] \quad \text{(neglecting higher order terms in } R_{\text{est}}/R_V\text{)}$$
$$\delta R_{\text{P}} = |R_{\text{est}} - R| = R_{\text{est}}^2/R_V \approx \frac{R^2}{R_V}$$

Alternatively,

$$R_{\text{est}} = \frac{V}{I} = \frac{R_V R}{R_V + R}$$
$$\delta R_{\text{P}} = |R_{\text{est}} - R| = R\left[\frac{R_V}{R_V + R} - 1\right] \approx \frac{R^2}{R_V}$$

For Q: $V = I(R + R_A)$

$$R = V/I - R_A = R_{\text{est}} - R_A$$
$$\delta R_{\text{Q}} = |R_{\text{est}} - R| = R_A$$

If $R = \sqrt{R_A R_V}$, then $\delta R_{\text{P}}/\delta R_{\text{Q}} = R_{\text{est}}^2/(R_A R_V) = R_{\text{est}}^2/R^2 \approx 1$

Q. 5

Solution: (a) Object is at $2f$, so the image is formed at the same distance from the lens (20 cm) to the right. (b) Since light has to retrace its path, the mirror should be placed so that the previous image is at its center of curvature. Thus the mirror must be placed 30 cm to the right of the lens. (c) For the plane mirror, reflection forms an image 40 cm to the right of the lens. Using the lens formula, we see that the final image is formed at a distance of $40/3$ cm to the left of the lens.

Q.6

Solution: (a) $V(\theta) = mgR(1 - \cos\theta)$, (b) $mgH - mgR(1 - \cos\theta)$, (c) For $H \ll R$ the body executes SHM with a time period of $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ - the time taken for it to travel from P to Q will be a quarter of this, *i.e.* $\frac{1}{8\pi}\sqrt{\frac{R}{g}}$. (d) At the lowest point, the speed is given by $\frac{1}{2}mv^2 = mgH$. So, $T - mg = \frac{mv^2}{R} = \frac{2mgH}{R}$, and thus $T = mg\left(1 + \frac{2H}{R}\right)$.

Q. 7

Answers:

(a) a = 3, b = 8, c = 3, d = 2 and e = 4.

(b) f = 2, g = 1, h = 1.

(c) i = 2, j = 1, k = 1, l = 2

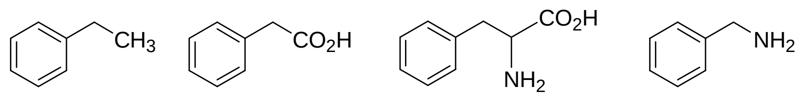
(d) 2.54 g of I_2 = 1/100 mole of I_2

= 2/100 gm atom of Cu

% Cu = (2/100) X (63.5/2) = 63.5%

Q. 8
Answers:

Bottle A = III, Bottle B = II, Bottle C = IV, Bottle D = I



Compound with the highest solubility in distilled water: IV

Q. 9
Answers:

(a) 2500 x 4.184 kJ = 10460 kJ

(b) 342 g of sucrose produces 5600 kJ of energy. To provide 10460 kJ we need 10460x 342/5600 g = 638 g

638 g/342 g x 12 x 22.4 L = 501 L

Q.10

Answers: (a) Difference in flower color is most likely due to environmental factors

(b) Perform cross breeding between the plants from Chandigarh and those from Shimla to find out whether we get any pink flower or flowers with any shade of color between pink and white in the F1 generation

(c) Grow the plants from Chandigarh in Shimla and check whether they still produce white flowers or bear pink flowers

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Q.11

Answers:

(a) In experiment A, ethanol fermentation occurs producing CO_2, turning lime water milky. Since acid is not produced the dye colour does not change.

In experiment B, lactic acid fermentation takes place, which produces acid but does not produce CO_2. Hence dye colour changes to yellow but the lime water does not turn milky .

In experiment C, since the lime water turns milky, ethanol fermentation is occurring. In addition, since removal of air did not affect the reaction, the fermentation is anaerobic and yeast must be the organism in the flask.

(b) In RBCs, lactic acid fermentation occurs.

Q. 12

Answers:

(a) The result of the radio-carbon dating was correct.

Reason: Vehicles running on the highway beside the house emitted carbon dioxide from the combustion of petrol or diesel, which are fossil fuels. The carbon in this carbon dioxide, coming from living material that has been converted into petroleum millions of years ago, would get assimilated into the tissues of the plant as it uses carbon dioxide from the surrounding atmosphere for photosynthesis. Therefore tissues of the plant, when used for radio-carbon dating, would show the age of the plant to be many thousands of years old.

(b) A simple experiment to test the validity of this explanation would be to collect seeds from the plant and grow them in a plot of land away from the highway or other sources of carbon dioxide coming from the burning of fossil fuels. Radio-carbon dating of plants growing from these seeds should show them as young plants.