Trains, Boats & Streams

Trains Formulae:

i) If a train of length *l*–metres moving with a speed of S–mps passes a pole or tower or tree in t– seconds,

then $t = \frac{l}{S} \Rightarrow l = S \times t$

ii)If a train of length of *l*- meters moving with a speed of "S" - mps passes a bridge or platform or tunnel of length p - metres in t - seconds, then

$t = \left(\frac{l+p}{S}\right) \quad \Rightarrow (l + p) = S \times t$

iii) If a train of length *l*-meters moving with a speed of S–mps crosses a man moving with a speed of S_M– mps, then the time taken by the train is given by

$$t = \frac{l}{(S \pm S_M)}$$

Use (+) sign if the train and the man move in opposite direction and (–) sign if the man and the train move in the same direction.

iv) Two trains of lengths l_1 and l_2 moving with respective speeds of S_1 and S_2 cross each other in a time t, then.

$$t = \frac{l_1 + l_2}{(S_1 \pm S_2)}$$

Use (+) sign if the two trains move in opposite direction and (–) sign if the two trains move in the same direction.

Boats & Streams Theory:

The speed at which the boat moves in still water is called the speed of the boat in still water (x). If there is movement for water then it is called a stream. The rate at which water flows is called rate of stream or rate of flow (y).

If the boat moves against a stream then the boat will be moving towards the upstream and rate at which it is moving then, is called upstream (U) = x – y

If the boat moves with the stream then the boat will be moving towards the downstream

and the rate at which it is moving then, is called downstream (D) = x+ y

$\therefore$ D + U = (x + y) + (x – y) = 2x

$\therefore x = \frac{1}{2}(D + U)$

D – U = (x + y) – (x – y) = 2y

$\Rightarrow y = \frac{1}{2}(D–U)$

Some Formulae:

v) If a boat travels a distance a k– km and returns in a time of t–hours then $k = \frac{t(x^2 - y^2)}{2x}$

vi) If the boat travels a certain distance and returns back then the average speed for the whole journey is

$S_A = \frac{(x^2 - y^2)}{xy}$

vii) If the time taken by a boat to travel a certain distance upstream is k-times the time taken by the boat to travel the same distance downstream then

$\frac{x}{y} = \frac{k+1}{k-1}$

PROBLEMS

1. A train crosses a pole in 17 seconds. If the length of the train is 425 metres, what will be the speed of the train in kmph?

1) 90 2) 93 3) 83 4) 87 5) 80

Ans: 1

l = 425 m, t = 17 sec, S = ?

When a train crosses a pole, then $S = \frac{l}{t} = \frac{425}{17} = 25$ mps

$\therefore$ S = 25 = 90 kmph

2. A train 800 metre long is running at the speed of 78 kmph. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is?

1) 77200 2) 500 3) 1300 4) 13 5) None of these

Ans: 2

l= 800 m,

$S = 78x\frac{5}{18} = \frac{65}{3}$mps

t = 1 min = 60 sec, P = ?

When a train crosses a tunnel

then t = $\frac{l+P}{S}$

$\Rightarrow$P = S × t – l = $\frac{65}{3}$ × 60 – 800

= 1300 – 800 = 500 m

3. If the speed of the train be 80 kmph and it takes 6 seconds to pass a platform. Next it takes 5 seconds to pass a cyclist walking at the rate of 12 kmph in the same direction. What is the length of the platform?

1) 38.56 metres 2) 35.56 metres 3) 48.56 metres

4) 45.56 metres 5) None of these

Ans: 5

Cyclist:

t = 5 sec, SR = 80–12 kmph

= 68 × $\frac{5}{18} = \frac{340}{18}$ mps, l = ?

t = $\frac{l}{S_R}$ $\Rightarrow$ l – t × S_R = 5 × $\frac{340}{18} = \frac{1700}{18}$ m

Platform

t = 6, $S = 80 \times \frac{5}{18} = \frac{200}{9}$ l + P = ?

$\therefore$ l + P = $\frac{200}{9} \times 6 = \frac{400}{3}$

P = $\frac{400}{3} - \frac{1700}{18} = \frac{700}{18}$ = 38.89

4. A man crosses a stationary train-A of 720 metres in 8 minutes. A woman crosses another stationery train–B in 2 minutes and 30 seconds. If the respective ratio between the speed of the man and that of the woman is 15:16, what is the length of train-B?

1) 260 metres 2) 360 metres 3) 340 metres

4) Cannot be determined 5) None of these

Ans: 5

$l_1$and l_2 be the lengths of Train-A and Train-B and S_M and S_W be the speeds of man and woman respectively.

Man

l_1 = 720, t_1 = 8 min = 480 sec,

$$S_M = \frac{l_1}{t_1} = \frac{720}{480} = \frac{3}{2} = 1.5 \text{ mps}$$

Woman

$$S_W = \frac{1.5}{15} \times 16 = 1.6 \quad (\because SM : SW = 15 : 16)$$

l_2= ?, t_2 = 2 min30 sec = 150 sec $\quad l_2 = S_W \times t_2 = 1.6 \times 150 = 240$ m

5. Train–A crossed a stationary train in 39 seconds. It also cros sed a man standing on a platform in 19 seconds. The length of the train-A is 456 metres. What is the length of the stationary train?

1) 460 metres 2) 480 metres 3) 490 metres

4) Cannot be determined 5) None of these

Ans: 2

l_1, l_2 and be the length of Train-A and the stationary train and S_1 be the speed of Train-A.

$$\frac{l_1}{S_1} = 19; \quad \frac{l_1 + l_2}{S_1} = 39$$

$$\Rightarrow \frac{19}{39} = \frac{\left(\frac{l_1}{S_1}\right)}{\left(\frac{l_1 + l_2}{S_1}\right)} = \frac{l_1}{l_1 + l_2}$$

$19l_1 + 19l_2 = 39l_1$

$19l_2 = 20l_1$

But l_1 = 456,

$$\therefore l_2 = \frac{20 \times 456}{19} = 20 \times 24 = 480\text{m}$$

6. Two trains, 80 metres and 120 metres long, are running at the speed of 25 kmph and 35 kmph respectively in the same direction on parallel tracks. How many seconds will they take to pass each other?

1) 48 2) 64 3) 70 4) 72 5) None of these

Ans: 4

$l_1 = 80$, $l_2 = 120$, $S_1 = 25$, $S_2 = 35$

$\therefore$ Trains are moving on the same direction t =

$$\frac{l_1 + l_2}{S_2 - S_1} = \frac{(80+120)\times 18}{(45-35)\times 5} = \frac{200\times 18}{50}$$

$= 72$

7. A man rows a boat 18 kilometres in 4 hours downstream and re turns upstream in 12 hours. The speed of the stream (in km per hour) is?

1) 1 2) 1.5 3) 2 4) 1.75 5) None of these

Ans: 2

Downstream Speed (D) $= \frac{18}{4} = 4.5$

Upstream speed (U) $= \frac{18}{12} = 1.5$, y = ?

Speed of stream (y) $= \frac{1}{2}(D-U) = \frac{1}{2}(4.5-1.5) = 1.5$

8. A man can row 30 km downstream and return in a total of 8 hours. If the speed of the boat in still water is four times the speed of the current, then the speed of the current is?

1) 1 km/hour 2) 2 km/hour 3) 4 km/hour 4) 3 km/hour 5) None of these

Ans: 2

Refer to formula (v) k = 30, t= 8, x = 4y 30 =

$$\frac{8(x^2 - y^2)}{2x} = \frac{8(16y^2 - y^2)}{8y} = \frac{8\times 15y^2}{8y} = 15y \qquad \Rightarrow y = 2$$

9. A boat takes 8 hours for travelling downstream from A to B and coming back to A upstr eam. If the velocity of the stream is 3 kmph and the speed of the boat in still water is 8kmph, what is the distance between A and B?

1) 55 km 2) 150 km 3) 48 km 4) 95 km 5) None of these

Ans: 5

$x = 8, \quad y = 3, \qquad D = x + y = 8 + 3 = 11, \qquad U = x - y = 8 - 3 = 5$

Let the distance between A and B be 'k', then

$\frac{k}{11} + \frac{k}{5} = 8 \qquad \frac{5k + 11k}{55} = 8 \qquad 16k = 8 \times 55 \qquad k = 27.5$ km

10. A man swims downstream a distance of 15 km in 1 hour. If the speed of the current is 5 kmph, the time taken by the man to swim the same distance upstream is?

1) 1 hour 30 minutes 2) 45 minutes 3) 2 hours 30 minutes

4) 3 hours 5) None of these

Ans: 4

$D = \frac{15}{1} = 15, \quad y = 5 \qquad \therefore x = D - y = 15 - 5 = 10 \qquad U = x - y = 10 - 5 = 5$

$\therefore t_{up} = \frac{15}{5} = 3$ hours

11. The current of a stream runs at the rate of 4 km an hour. A boat goes 6 km and comes back to the starting point in 2 hours. The speed of the boat in still water is?

1) 6 kmph 2) 8 kmph 3) 7.5 kmph 4) 6.8 kmph 5) None of these

Ans: 2

$y = 4, \quad k = 6, \quad t = 2 \qquad 6 = \frac{2(x^2 - 4^2)}{2x}$

$x^2 - 6x - 16 = 0 \qquad (x - 8)(x + 2) = 0 \qquad \therefore x = 8$

12. The rate at which a river flows is one-third the speed of a boat in still water. If that boat travels down the river for 2 hours and then back up river for 2 hours, it will be 16 km short of its starting point. The speed (kmph) of the boat in still water is?

1) 4 kmph 2) 6 kmph 3) 8 kmph 4) 12 kmph 5) None of these

Ans: 4

Let the speed of boat in still water be 3k, then river flows at $\frac{1}{3}(3k) = k$

$\therefore$ Downstream speed (D) $= 3k + k = 4k$

Upstream speed (U) $= 3k - k = 2k$

Given that $2 \times 4k - 2 \times 2k = 16 \qquad \Rightarrow k = 4$

Speed of boat in still water $= 3(4) = 12$ kmph