

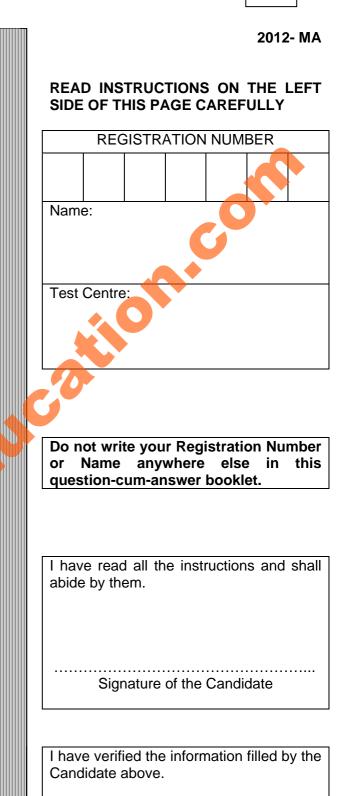
2012- MA Test Paper Code: MA

Time: 3 Hours

Maximum Marks: 300

INSTRUCTIONS

- 1. This question-cum-answer booklet has **36** pages and has **29** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
- 2. Write your **Registration Number**, **Name and the name of the Test Centre** in the appropriate space provided on the right side.
- 3. Write the answers to the objective questions against each Question Number in the **Answer Table for Objective Questions**, provided on Page **7**. Do not write anything else on this page.
- 4. Each objective question has 4 choices for its answer: (A), (B), (C) and (D). Only ONE of them is the correct answer. There will be negative marking for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
 - (a) For each correct answer, you will be awarded 6 (Six) marks.
 - (b) For each wrong answer, you will be awarded **-2 (Negative two)** mark.
 - (c) Multiple answers to a question will be treated as a wrong answer.
 - (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
 - (e) Negative marks for objective part will be carried over to total marks.
- 5. Answer the subjective question only in the space provided after each question.
- Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
- 7. All answers must be written in blue/black/blueblack ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 8. All rough work should be done in the space provided and scored out finally.
- 9. No supplementary sheets will be provided to the candidates.
- 10. Clip board, log tables, slide rule, calculator, cellular phone and electronic gadgets in any form are NOT allowed.
- 11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
- 12. Refer to special instructions/useful data on the reverse.



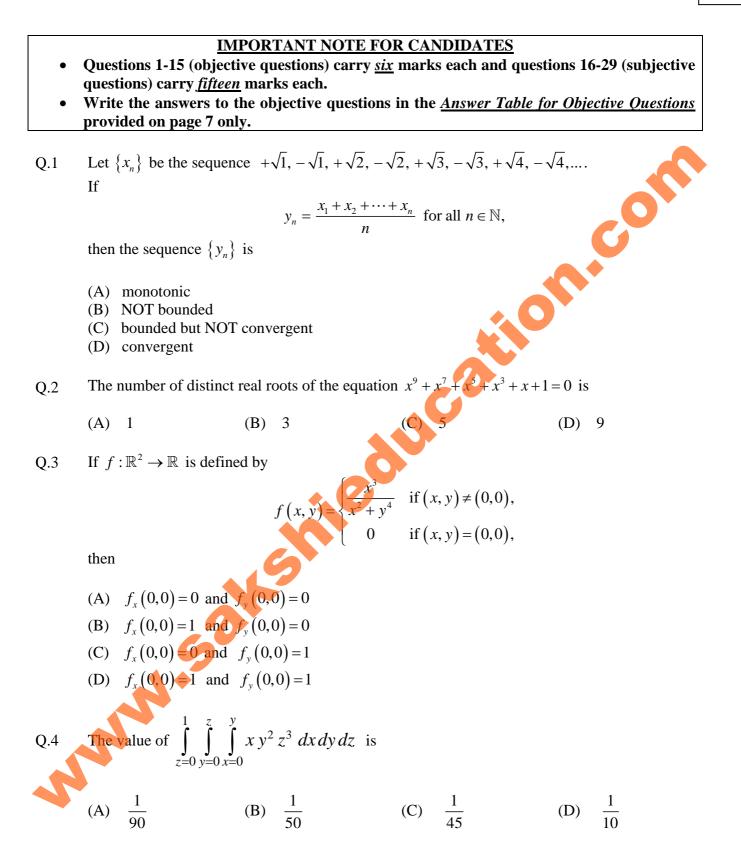
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Signature of the Invigilator

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		Special Instructions/ Useful Data
	N	: The set of all natural numbers, that is, the set of all positive integers 1, 2, 3,
	Z	: The set of all integers
	Q	: The set of all rational numbers
	\mathbb{R}	: The set of all real numbers
	$\{e_1, e_2, \dots, e_n\}$: The standard basis of the real vector space \mathbb{R}^n
	f', f''	: First and second derivatives respectively of a real function f
	$f_x(a,b), f_y(a,b)$: Partial derivatives with respect to x and y
		respectively of $f : \mathbb{R}^2 \to \mathbb{R}$ at (a, b)
	$R \times S$: Product ring of rings <i>R</i> , <i>S</i> with component-
		wise operations of addition and multiplication
~		

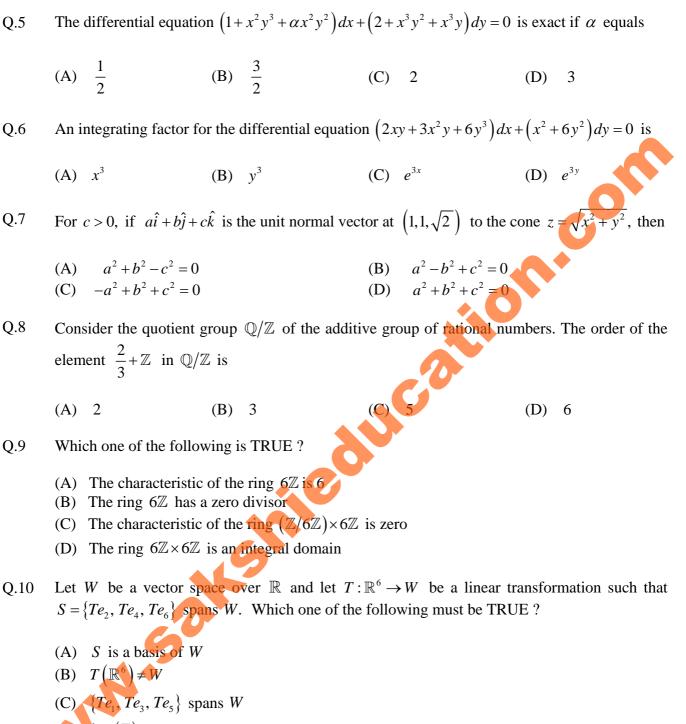
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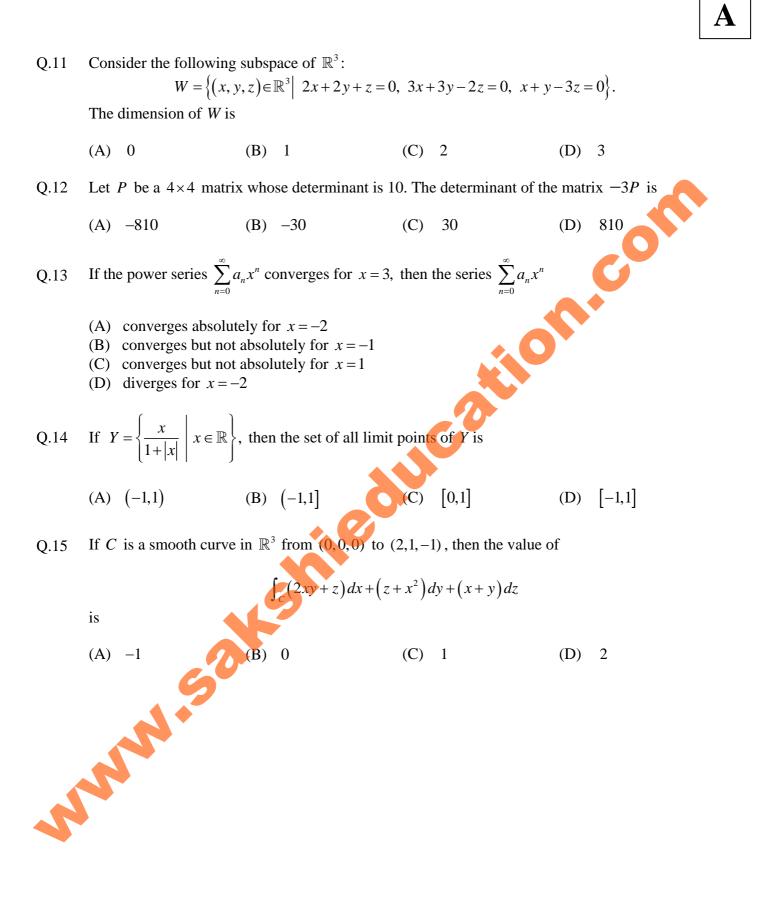
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A



 \mathbf{D} ker(T) contains more than one element



Space for rough work

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Space for rough work

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Write the Code of your chosen answer only in the 'Answer' column against each Question Number. Do not write anything else on this page.

Question Number	Answer	Do not write in this column	
01			
02			
03			
04			0
05			
06		~	
07		6	
08			
09			
10		0	
11			
12			
13	9		
14			
15			



FOR EVALUATION ONLY					
Number of Correct Answers	Marks	(+)			
Number of Incorrect Answers	Marks	(–)			
Total Marks in Question	()				

Q.16 (a) Examine whether the following series is convergent:

$$\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}.$$
 (6)

. that the (9) (b) For each $x \in \mathbb{R}$, let [x] denote the greatest integer less than or equal to x. Further,

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Q.17 (a) Evaluate
$$\lim_{x \to 0} \frac{\int_{0}^{x^{2}} \sqrt{4+t^{3}} dt}{x^{2}}$$
. (6)

(b) For $a, b \in \mathbb{R}$ with a < b, let $f:[a,b] \to \mathbb{R}$ be continuous on [a,b] and twice differentiable on (a,b). Further, assume that the graph of f intersects the straight line segment joining the points (a, f(a)) and (b, f(b)) at a point (c, f(c)) for a < c < b. Show that there exists a real number $\xi \in (a,b)$ such that $f''(\xi) = 0$. (9)

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- Q.18 (a) Show that the point (0,0) is neither a point of local minimum nor a point of local maximum for the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = 3x^4 4x^2y + y^2$ for $(x, y) \in \mathbb{R}^2$. (6)
- the critical points of the function $f: \mathbb{R}^2 \to \mathbb{R}$ (b) Find given all by e .efu .al point. $f(x, y) = x^3 + y^3 - 3x - 12y + 40$ for $(x, y) \in \mathbb{R}^2$. Also, examine whether the function

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Q.19 (a) Evaluat
$$\int_{x=0}^{0} \int_{y=\sqrt{4-x}}^{0} e^{y^2} dy dx$$
. (b) Using multiple integral, find the volume of the solid region in \mathbb{R}^3 bounded above to the hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$. (c)

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- A
- Q.20 Find the area of the portion of the surface $z = x^2 y^2$ in \mathbb{R}^3 which lies inside the solid (5) cylinder $x^2 + y^2 \le 1$. (15)

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Let y(x) be the solution of the differential equation $\frac{d^2y}{dx^2} - y = 0$ such that y(0) = 2st station contraction contractico contractico contractico contractico contractico contrac Q.21

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(9)

- Q.22 (a) Assume that $y_1(x) = x$ and $y_2(x) = x^3$ are two linearly independent solutions of the homogeneous differential equation $x^2 \frac{d^2 y}{dx^2} 3x \frac{dy}{dx} + 3y = 0$. Using the method of variation of parameters, find a particular solution of the differential equation $x^2 \frac{d^2 y}{dx^2} 3x \frac{dy}{dx} + 3y = x^5$.
- idita ; Solve the differential equation $\frac{dy}{dx} + \frac{5y}{6x} = \frac{5x^4}{y^5}$ subject to the condition y(1) = 1.

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(9)

- Q.23 (a) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector field in \mathbb{R}^3 and let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function. Show that $\vec{\nabla} \times \{f(|\vec{r}|)\vec{r}\} = \vec{0} \text{ for } \vec{r} \neq \vec{0}.$ (6)
- (b) Let W be the region inside the solid cylinder $x^2 + y^2 \le 4$ between the plane z = 0Gaussi Gaussi Control and the paraboloid $z = x^2 + y^2$. Let S be the boundary of W. Using Gauss's

$$\vec{F} = (x^2 + y^2 - 4)\hat{i} + (3xy)\hat{j} + (2xz + z^2)\hat{k}$$

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- Q.24 (a) Let G be a finite group whose order is not divisible by 3. Show that for every $g \in G$, there exists an $h \in G$ such that $g = h^3$. (6)
- and control of the second seco (b) Let A be the group of all rational numbers under addition, B be the group of all

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Α

(6)

(9)

Q.25 (a) Let I be an ideal of a commutative ring R. Define $A = \left\{ r \in R \mid r^n \in I \text{ for some } n \in \mathbb{N} \right\}.$ Show that A is an ideal of R.

(b) Let F be a field. For each $p(x) \in F[x]$ (the polynomial ring in x over F) define $\varphi: F[x] \to F \times F$ by $\varphi(p(x)) = (p(0), p(1)).$ aring F2

- Prove that φ is a ring homomorphism. (i)
- Prove that the quotient ring $F[x]/(x^2 x)$ is isomorphic to the ring $F \times F$.

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- (a) Let P, D and A be real square matrices of the same order such that P is invertible, Q.26 *D* is diagonal and $D = PAP^{-1}$. If $A^n = 0$ for some $n \in \mathbb{N}$, then show that A = 0. (6)
 - (b) Let $T: V \to W$ be a linear transformation of vector spaces. Prove the following:
 - (i) If $\{v_1, v_2, \dots, v_k\}$ spans V, and T is onto, then $\{Tv_1, Tv_2, \dots, Tv_k\}$ spans W.
 - (ii) If $\{v_1, v_2, \dots, v_k\}$ is linearly independent in V, and T is one-one, then $\{Tv_1, Tv_2, \dots, Tv_k\}$ is linearly independent in W.
- (iii) If $\{v_1, v_2, \dots, v_k\}$ is a basis of V, and T is bijective, then $\{Tv_1, Tv_2, \dots, Tv_k\}$ is (9)

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(6)

Q.27 (a) Let $\{v_1, v_2, v_3\}$ be a basis of a vector space *V* over \mathbb{R} . Let $T: V \to V$ be the linear transformation determined by

$$Tv_1 = v_1$$
, $Tv_2 = v_2 - v_3$ and $Tv_3 = v_2 + 2v_3$.

Find the matrix of the transformation T with $\{v_1 + v_2, v_1 - v_2, v_3\}$ as a basis of both the domain and the co-domain of T.

(b) Let W be a three dimensional vector space over \mathbb{R} and let $S: W \to W$ be a linear transformation. Further, assume that every non-zero vector of W is an eigenvector of el:w S. Prove that there exists an $\alpha \in \mathbb{R}$ such that $S = \alpha I$, where $I: W \to W$ is the (9)

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- Q.28 (a) Show that the function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^2$ for $x \in \mathbb{R}$, is not uniformly continuous. (6)
- o, . (f is . (f is) . (

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- Q.29 (a) Let A be a nonempty bounded subset of \mathbb{R} . Show that $\{x \in \mathbb{R} \mid x \ge a \text{ for all } a \in A\}$ is a closed subset of \mathbb{R} . (6)
- .te .te (b) Let $\{x_n\}$ be a sequence in \mathbb{R} such that $|x_{n+1} - x_n| < \frac{1}{n^2}$ for all $n \in \mathbb{N}$. Show that the

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Space for rough work

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2012 - MA			
Objective Part			
(Question Number 1 – 15)			
Total Marks	Signature		

	\$	Subjective Part			0
Question Number	Marks	Question Number	Marks	C	
16		23			
17		24			
18		25			
19		26			
20		27			
21		28			
22		29			
	Total Marks	in Subjective Part			

Total (Objective Part)	
Total (Subjective Part)	:
Grand Total	:
Total Marks (in words)	:
Signature of Examiner(s)	:
Signature of Head Examiner(s)	:
Signature of Scrutinizer	:
Signature of Chief Scrutinizer	:
Signature of Coordinating Head Examiner	:

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