

**2011 (II)  
PHYSICAL SCIENCES  
TEST BOOKLET**

**5**

**B**

*Time : 3:00 Hours*

*Maximum Marks: 200*

SAKSHI

## उपयोगी मूलभूत स्थिरांक

m	इलेक्ट्रॉन का द्रव्यमान	$9.11 \times 10^{-31}$ Kg
h	प्लांक स्थिरांक	$6.63 \times 10^{-34}$ J sec
e	इलेक्ट्रॉन का आवेश	$1.6 \times 10^{-19}$ C
k	बोल्ट्समैन स्थिरांक	$1.38 \times 10^{-23}$ J/K
c	प्रकाश का वेग	$3.0 \times 10^8$ m/Sec
1 eV	$1.6 \times 10^{-19}$ J	
amu	$1.67 \times 10^{-27}$ kg	
G	$6.67 \times 10^{-11}$ Nm <sup>2</sup> kg <sup>-2</sup>	
R <sub>y</sub>	रिजबर्ग स्थिरांक	$1.097 \times 10^7$ m <sup>-1</sup>
N <sub>A</sub>	आवोगाद्रो संख्या	$6.023 \times 10^{23}$ mole <sup>-1</sup>
ε <sub>0</sub>	$8.854 \times 10^{-12}$ Fm <sup>-1</sup>	
μ <sub>0</sub>	$4\pi \times 10^{-7}$ Hm <sup>-1</sup>	
R	मोलर गैस स्थिरांक	$8.314$ J K <sup>-1</sup> mole <sup>-1</sup>

USEFUL FUNDAMENTAL  
CONSTANTS

m	Mass of electron	$9.11 \times 10^{-31}$ Kg
h	Planck's constant	$6.63 \times 10^{-34}$ J sec
e	Charge of electron	$1.6 \times 10^{-19}$ C
k	Boltzmann constant	$1.38 \times 10^{-23}$ J/K
c	Velocity of Light	$3.0 \times 10^8$ m/Sec
1 eV	$1.6 \times 10^{-19}$ J	
amu	$1.67 \times 10^{-27}$ kg	
G	$6.67 \times 10^{-11}$ Nm <sup>2</sup> kg <sup>-2</sup>	
R <sub>y</sub>	Rydberg constant	$1.097 \times 10^7$ m <sup>-1</sup>
N <sub>A</sub>	Avogadro number	$6.023 \times 10^{23}$ mole <sup>-1</sup>
ε <sub>0</sub>	$8.854 \times 10^{-12}$ Fm <sup>-1</sup>	
μ <sub>0</sub>	$4\pi \times 10^{-7}$ Hm <sup>-1</sup>	
R	Molar Gas constants	$8.314$ J K <sup>-1</sup> mole <sup>-1</sup>

## LIST OF THE ATOMIC WEIGHTS OF THE ELEMENTS

Element	Symbol	Atomic Number	Atomic Weight	Element	Symbol	Atomic Number	Atomic Weight
Actinium	Ac	89	(227)	Mercury	Hg	80	200.59
Aluminium	Al	13	26.98	Molybdenum	Mo	42	95.94
Americium	Am	95	(243)	Neodymium	Nd	60	144.24
Antimony	Sb	51	121.75	Neon	Ne	10	20.183
Argon	Ar	18	39.948	Neptunium	Np	93	(237)
Arsenic	As	33	74.92	Nickel	Ni	28	58.71
Astatine	At	85	(210)	Niobium	Nb	41	92.91
Barium	Ba	56	137.34	Nitrogen	N	7	14.007
Berkelium	Bk	97	(249)	Nobelium	No	102	(253)
Beryllium	Be	4	9.012	Osmium	Os	76	190.2
Bismuth	Bi	83	208.98	Oxygen	O	8	15.9994
Boron	B	5	10.81	Palladium	Pd	46	106.4
Bromine	Br	35	79.909	Phosphorus	P	15	30.974
Cadmium	Cd	48	112.40	Platinum	Pt	78	195.09
Calcium	Ca	20	40.08	Plutonium	Pu	94	(242)
Californium	Cf	98	(251)	Polonium	Po	84	(210)
Carbon	C	6	12.011	Potassium	K	19	39.102
Cerium	Ce	58	140.12	Praseodymium	Pr	59	140.91
Cesium	Cs	55	132.91	Promethium	Pm	61	(147)
Chlorine	Cl	17	35.453	Protactinium	Pa	91	(231)
Chromium	Cr	24	52.00	Radium	Ra	88	(226)
Cobalt	Co	27	58.93	Radon	Rn	86	(222)
Copper	Cu	29	63.54	Rhenium	Re	75	186.23
Curium	Cm	96	(247)	Rhodium	Rh	45	102.91
Dysprosium	Dy	66	162.50	Rubidium	Rb	37	85.47
Einsteinium	Es	99	(254)	Ruthenium	Ru	44	101.1
Erbium	Er	68	167.26	Samarium	Sm	62	150.35
Europium	Eu	63	151.96	Scandium	Sc	21	44.96
Fermium	Fm	100	(253)	Selenium	Se	34	78.96
Fluorine	F	9	19.00	Silicon	Si	14	28.09
Francium	Fr	87	(223)	Silver	Ag	47	107.870
Gadolinium	Gd	64	157.25	Sodium	Na	11	22.9898
Gallium	Ga	31	69.72	Strontium	Sr	38	87.62
Germanium	Ge	32	72.59	Sulfur	S	16	32.064
Gold	Au	79	196.97	Tantalum	Ta	73	180.95
Hafnium	Hf	72	178.49	Technetium	Tc	43	(99)
Helium	He	2	4.003	Tellurium	Te	52	127.60
Holmium	Ho	67	164.93	Terbium	Tb	65	158.92
Hydrogen	H	1	1.0080	Thallium	Tl	81	204.37
Indium	In	49	114.82	Thorium	Th	90	232.04
Iodine	I	53	126.90	Thulium	Tm	69	168.93
Iridium	Ir	77	192.2	Tin	Sn	50	118.69
Iron	Fe	26	55.85	Titanium	Ti	22	47.90
Krypton	Kr	36	83.80	Tungsten	W	74	183.85
Lanthanum	La	57	138.91	Uranium	U	92	238.03
Lawrencium	Lr	103	(257)	Vanadium	V	23	50.94
Lead	Pb	82	207.19	Xenon	Xe	54	131.30
Lithium	Li	3	6.939	Ytterbium	Yb	70	173.04
Lutetium	Lu	71	174.97	Yttrium	Y	39	88.91
Magnesium	Mg	12	24.312	Zinc	Zn	30	65.37
Manganese	Mn	25	54.94	Zirconium	Zr	40	91.22
Mendelevium	Md	101	(256)				

\*Based on mass of  $C^{12}$  at 12.000... The ratio of these weights of those on the order chemical scale (in which oxygen of natural isotopic composition was assigned a mass of 16.0000...) is 1.000050. (Values in parentheses represent the most stable known isotopes.)

## PART A

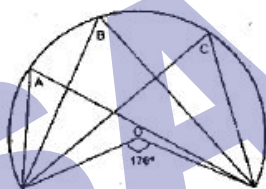
1. The most abundant element by mass in the human body is

- (1) carbon    (2) hydrogen  
 (3) calcium     (4) oxygen

2. A number system consists of digits 0, 1, 2, 3, 4 and 5. What is the decimal equivalent of 15 in this number system?

- (1) 15                      (2) 13  
 (3) 11                      (4) 12

3. A segment of a circle (slightly greater than a semicircle, whose centre is O) is given below. Identify the correct statement regarding the three angles A, B and C.



1. A is equal to B but not equal to C.  
 2. A, B and C are equal and have a value of  $85^\circ$ .  
 3. A, B and C are unequal.  
 4. A, B and C are each equal to  $95^\circ$ .

4. After bubbling air through pure water ( $\text{pH} = 7.0$ ), its pH decreased. Which of the following is responsible for the pH change?

- (1) Nitrogen     (2) Carbon dioxide  
 (3) Oxygen    (4) Helium

5

5. Which of the following caused disruption of air-traffic in Europe after the volcanic eruption in Iceland in the year 2010?

1. Risk of engine damage by volcanic ash.  
 2. Increased local temperature.  
 3. Reduced visibility due to volcanic gases.  
 4. Presence of toxic gases.

6. Three boys A, B and C kicked three balls horizontally from the edge of the roof of a building. The horizontal distances traversed by these balls before hitting the ground are  $d_A$ ,  $d_B$ ,  $d_C$  respectively, with  $d_A > d_B > d_C$ . If  $t_A$ ,  $t_B$  and  $t_C$  are the times taken to hit the ground respectively, then

- (1)  $t_A > t_B > t_C$     (2)  $t_A < t_B < t_C$   
 (3)  $t_A > t_B < t_C$      (4)  $t_A = t_B = t_C$

7. What is the minimum number of cards you need to uncover from the top of a well-shuffled deck of 52 playing cards, to ensure that you have two cards of a suit?

- (1) 41                      (2) 15  
 (3) 7                        (4) 5

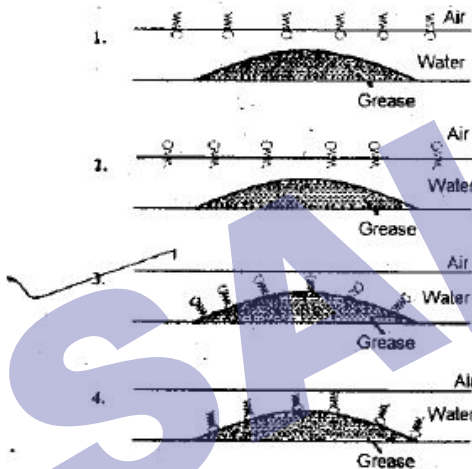
8. A swinging door is to be stopped by driving a wedge between the floor and the door. The most favourable position of the wedge is close to the

1. outer edge of the door because even a small frictional force can provide adequate torque to stop the door swing.  
 2. outer edge of the door because the frictional force is the largest at the outer edge.

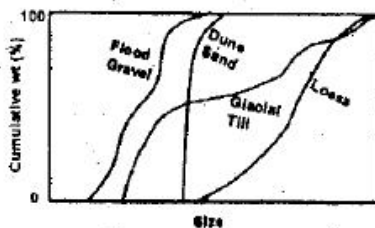
3. hinged edge of the door because the moment of force is smallest near the hinge.
4. hinged edge of the door because there is friction in the hinge.
9. The simple representation of a detergent molecule is given below:



Which one of the following representations describes the interaction of the detergent molecule in a system composed of grease and water?



10. The figure shows cumulative weight percent curves for different types of sediments. Which type of sediment has the narrowest size distribution?



- (1) Flood gravel (2) Glacial till  
(3) Dune sand (4) Loess

11. Which of the following animals does not have modified legs used for flight?

(1) Sparrow (2) Bat  
(3) Flying squirrel (4) Butterfly

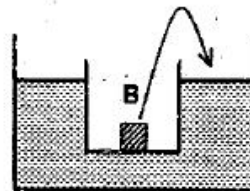
12. A polypeptide of 300 amino acids has tyrosine at the 157<sup>th</sup> position. If the tyrosine codon mutates to a nonsense codon, what would be the size of the polypeptide in this mutated organism?

(1) 157 (2) 156  
(3) 299 (4) 144

13. A room has a west-facing window with glass panes. Which of the following is the most effective way to prevent the room from getting hot in summer?

1. Cover the inside of the glass pane by a black paper.  
2. Cover the outside of the glass pane by an aluminum foil.  
3. Cover the outside of the glass pane by a white thermocol sheet.  
4. Cover the inside of the glass pane by a white thermocol sheet.

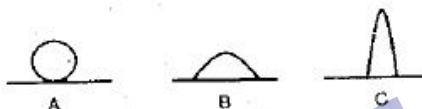
14. A jar containing an iron block B was floating on water in a bigger container. The block was taken out and put into water. As a result, the level of water in the container will



1. rise
  - ~~2. fall~~
  3. remain the same
  4. rise or fall depending upon the mass of the block
15. Suppose the Sun somehow becomes a black hole without change in its mass. Then this black hole will pull into itself
1. all the planets
  2. only Mercury
  3. all planets from Mercury to Mars
  - ~~4. none of the planets~~
16. Absolute water content in the air in the equatorial region is measured to be  $18 \text{ g m}^{-3}$  and the same is  $4 \text{ g m}^{-3}$  in the polar region. However, the values of relative humidity reported are 60% and 78%, respectively. This could be because
1. equatorial region is warmer and therefore the atmosphere has a higher water holding capacity.
  2. winds are stronger in the polar region.
  3. polar region is ice-covered and therefore its atmosphere has a higher water content.
  4. of higher snowfall in the polar region.
17. Oxidation of alcohols to acids involves formation and cleavage of bonds. Which of the following possibilities is valid in the process?
1. Formation of C=O bond and cleavage of O-H and C-H bonds.
  2. Formation of C=O bonds and cleavage of O-H bond.

- ~~3. Formation of C=O bonds and cleavage of C-H bonds.~~
4. Formation of C=O bond and cleavage of O-H and C-H bonds.

18. Droplets of a herbicide solution form various shapes on a leaf as shown.



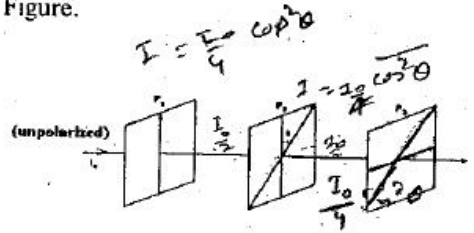
Assuming that the droplets have the same volume, the trend in the rates of herbicide uptake would be

1.  $A > B > C$ .
  2.  $B > A > C$ .
  - ~~3.  $B > C > A$ .~~
  4.  $C > A > B$ .
19. Parents with blood groups A and AB have two children. Which of the following cannot be the blood groups of their children?
- (1) A and AB
  - (2) B and AB.
  - (3) A and B
  - ~~(4) O and B.~~
20. A long cylinder has an axially placed two-bladed fan spinning inside it. Bullets are shot through the cylinder at a constant rate. If the number of blades is increased to four, the number of bullets
1. missing the blades is halved.
  2. missing the blades is reduced by one-fourth.
  3. hitting the blades is doubled.
  - ~~4. hitting the blades remains the same.~~

$$\frac{I_0}{2} = \left(\frac{I_0}{4}\right) \cos^2 \theta \quad \frac{I_0}{8} (\cos^2 \theta \cos^2 \theta)$$

**PART B**

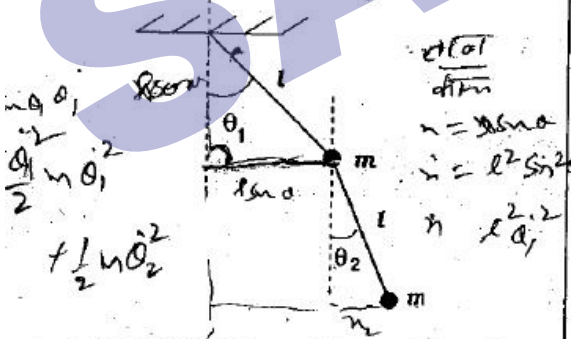
21. Consider three polarizers  $P_1, P_2$  and  $P_3$  placed along an axis as shown in the Figure.



The pass axis of  $P_1$  and  $P_3$  are at right angles to each other while the pass axis of  $P_2$  makes an angle  $\theta$  with that of  $P_1$ . A beam of unpolarized light of intensity  $I_0$  is incident on  $P_1$  as shown. The intensity of light emerging from  $P_3$  is

- (1) 0
- (2)  $\frac{I_0}{2}$
- (3)  $\frac{I_0}{8} \sin^2 2\theta$
- (4)  $\frac{I_0}{4} \sin^2 2\theta$

22. A double pendulum consists of two point masses  $m$  attached by massless strings of length  $l$  as shown in the figure:

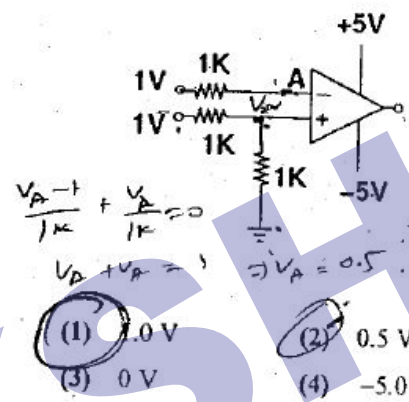


The kinetic energy of the pendulum is:

- (1)  $\frac{1}{2} ml^2 [\dot{\theta}_1^2 + \dot{\theta}_2^2]$
  - (2)  $\frac{1}{2} ml^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)]$
- Handwritten notes:  $I = I_0 \cos^2 \theta \cos^2 \theta$ ,  $I = \frac{I_0}{2} \cos^2 \theta (1 - \cos 2\theta)$ ,  $\frac{I_0}{4} \cos^2 \theta$ ,  $\frac{I_0}{8}$

- 3.  $\frac{1}{2} ml^2 [\dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2]$
- 4.  $\frac{1}{2} ml^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 + \theta_2)]$

23. In the operational amplifier circuit below, the voltage at point A is



- (1) 1.0 V
- (2) 0.5 V
- (3) 0 V
- (4) -5.0 V

24. A point particle of mass  $m$  carrying an electric charge  $q$  is attached to a spring of stiffness constant  $k$ . A constant electric field  $E$  along the direction of the spring is switched on for a time interval  $T$  (where  $T \ll \sqrt{m/k}$ ). Neglecting radiation loss, the amplitude of oscillation after the field is switched off is:

- (1)  $qE/k$
- (2)  $qET^2/m$
- (3)  $qE\sqrt{m}/Tk^{3/2}$
- (4)  $qET/\sqrt{mk}$

25. A constant force  $F$  is applied to a relativistic particle of rest mass  $m$ . If the particle starts from rest at  $t=0$ , its speed after a time  $t$  is

- (1)  $Ft/m$
- (2)  $c \tanh\left(\frac{Ft}{mc}\right)$
- (3)  $c(1 - e^{-Ft/mc})$
- (4)  $\frac{Fct}{\sqrt{F^2 t^2 + m^2 c^2}}$

Handwritten notes:  $\frac{1}{2} m \dot{\theta}_1^2 + \frac{1}{2} m \dot{\theta}_2^2$ ,  $\frac{1}{2} m (\dot{\theta}_1^2 + \dot{\theta}_2^2)$

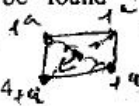
26. The potential of a diatomic molecule as a function of the distance  $r$  between the atoms is given by  $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$ . The value of the potential at equilibrium separation between the atoms is:

- (1)  $-4a^2/b$  (2)  $-2a^2/b$   
 (3)  $-a^2/2b$  (4)  $-a^2/4b$



27. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e., points where the electric field vanishes) will be found inside the square?

- (1) 3 (2) 4  
 (3) 5 (4) 7

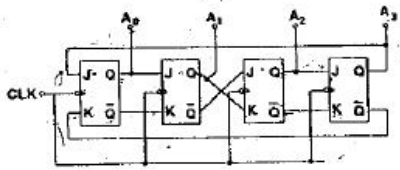


Handwritten notes: "Electric field", "is not correct"

28. A static charge distribution gives rise to an electric field of the form  $\vec{E} = \alpha(1 - e^{-r/R})\hat{r}$ , where  $\alpha$  and  $R$  are positive constants. The charge contained within a sphere of radius  $R$ , centred at the origin is

- (1)  $\pi\alpha\epsilon_0 \frac{e}{R^2}$  (2)  $\pi\alpha\epsilon_0 \frac{e^2}{R^2}$   
 (3)  $4\pi\alpha\epsilon_0 \frac{R}{e}$  (4)  $\pi\alpha\epsilon_0 \frac{R^2}{e}$

29. A counter consists of four flip-flops connected as shown in the figure.



If the counter is initialized as  $A_0 A_1 A_2 A_3 = 0110$ , the state after the next clock pulse is

Handwritten calculations for question 28:

$$\int_0^R \rho(r) 4\pi r^2 dr = \int_0^R \epsilon_0 \text{div}(\vec{E}) 4\pi r^2 dr$$

$$= \int_0^R \epsilon_0 \alpha (1 - e^{-r/R}) 4\pi r^2 dr$$

$$= 4\pi\epsilon_0 \alpha \left[ \int_0^R r^2 dr - \int_0^R r^2 e^{-r/R} dr \right]$$

$$= 4\pi\epsilon_0 \alpha \left[ \frac{R^3}{3} - \left( -R^3 \frac{e^{-r/R}}{R} + \int_0^R e^{-r/R} dr \right) \right]$$

$$= 4\pi\epsilon_0 \alpha \left[ \frac{R^3}{3} - \left( -R^2 e^{-r/R} + R e^{-r/R} \right) \right]_0^R$$

$$= 4\pi\epsilon_0 \alpha \left[ \frac{R^3}{3} - \left( -R^2 + R \right) + \left( 0 - R \right) \right]$$

$$= 4\pi\epsilon_0 \alpha \left[ \frac{R^3}{3} - R^2 + R \right]$$

- (1) 1000 (2) 0001  
 (3) 0011 (4) 1100

30. In a Young's double slit interference experiment, the slits are at a distance  $2L$  from each other and the screen is at a distance  $D$  from the slits. If a glass slab of refractive index  $\mu$  and thickness  $d$  is placed in the path of one of the beams, the minimum value of  $d$  for the central fringe to be dark is

1.  $\frac{\lambda D}{(\mu-1)\sqrt{D^2+L^2}}$   
 2.  $\frac{\lambda D}{(\mu-1)L}$   
 3.  $\frac{\lambda}{(\mu-1)}$   
 4.  $\frac{\lambda}{2(\mu-1)}$

Handwritten calculations for question 30:

$$2\mu t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{4(\mu-1)}$$

31. The pins 0, 1, 2 and 3 of part A of a microcontroller are connected with resistors to drive an LED at various intensities as shown in the figure.

For  $V_{cc} = 4.2V$  and a voltage drop of  $1.2V$  across the LED, the range (maximum current) and resolution (step size) of the drive current are, respectively,

1. 4.0 mA and 1.0 mA  
 2. 15.0 mA and 1.0 mA  
 3. 7.5 mA and 0.5 mA  
 4. 4.0 mA and 0.5 mA

Handwritten note: "Probability"

32. An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is

Handwritten calculations for question 32:

$$P(\text{sum} = 16) = \frac{3}{6^3} = \frac{1}{120}$$



$$F_3(x) = (1 - 2x + x^2)^2$$

$$V(r, \theta) = \frac{1}{2} m \omega^2 r^2 + \frac{1}{2} m g z$$

- (1)  $\frac{1}{16}$  (2)  $\frac{1}{36}$   
 (3)  $\frac{1}{108}$  (4)  $\frac{1}{216}$

33. The generating function

$$F(x, t) = \sum_{n=0}^{\infty} P_n(x) t^n$$

for the Legendre polynomials  $P_n(x)$  is  $F(x, t) = (1 - 2xt + t^2)^{-1/2}$ . The value of  $P_3(-1)$  is

- (1)  $5/2$  (2)  $3/2$   
 (3)  $+1$  (4)  $-1$

34. Given that the ground state energy of the hydrogen atom is  $-13.6$  eV, the ground state energy of positronium (which is a bound state of an electron and a positron) is

- (1)  $+6.8$  eV (2)  $-6.8$  eV  
 (3)  $-13.6$  eV (4)  $-27.2$  eV

35. Two particles of identical mass move in circular orbits under a central potential

$V(r) = \frac{1}{2} k r^2$ . Let  $\ell_1$  and  $\ell_2$  be the angular momenta and  $r_1, r_2$  be the radii of the orbits respectively. If  $\ell_1 / \ell_2 = 2$ , the value of  $r_1 / r_2$  is:

- (1)  $\sqrt{2}$  (2)  $1/\sqrt{2}$   
 (3)  $2$  (4)  $1/2$

36. The equation of the plane that is tangent to the surface  $xyz=8$  at the point  $(1, 2, 4)$  is

1.  $x+2y+4z=12$   
 2.  $4x+2y+z=12$   
 3.  $x+4y+2z=0$   
 4.  $x+y+z=7$

$l_1 = m v_1 r_1$   
 $l_2 = m v_2 r_2$   
 $2 l_2 = l_1$   
 $2 m v_2 r_2 = m v_1 r_1$   
 $2 v_2 r_2 = v_1 r_1$   
 $2 \sqrt{2} r_2 = r_1$   
 $r_1 = 2\sqrt{2} r_2$

37. The energy of the first excited quantum state of a particle in the two-dimensional potential  $V(x, y) = \frac{1}{2} m \omega^2 (x^2 + 4y^2)$  is

- 1)  $2 \hbar \omega$  (2)  $3 \hbar \omega$   
 (3)  $\frac{3}{2} \hbar \omega$  (4)  $\frac{5}{2} \hbar \omega$

38. The internal energy  $E$  of a system is given by  $E = \frac{bS^3}{VN}$ , where  $b$  is a constant and other symbols have their usual meaning. The temperature of this system is equal to

- (1)  $\frac{bS^2}{VN}$  (2)  $\frac{3bS^2}{VN}$   
 (3)  $\frac{bS^3}{V^2N}$  (4)  $\left(\frac{S}{N}\right)^2$

39. Consider a particle in a one dimensional potential that satisfies  $V(x) = V(-x)$ .

Let  $|\psi_0\rangle$  and  $|\psi_1\rangle$  denote the ground and the first excited states, respectively, and let  $|\Psi\rangle = \alpha_0 |\psi_0\rangle + \alpha_1 |\psi_1\rangle$  be a normalized state with  $\alpha_0$  and  $\alpha_1$  being real constants.

The expectation value  $\langle x \rangle$  of the position operator  $x$  in the state  $|\Psi\rangle$  is given by

1.  $\alpha_0^2 \langle \psi_0 | x | \psi_0 \rangle + \alpha_1^2 \langle \psi_1 | x | \psi_1 \rangle$   
 2.  $\alpha_0 \alpha_1 [\langle \psi_0 | x | \psi_1 \rangle + \langle \psi_1 | x | \psi_0 \rangle]$   
 3.  $\alpha_0^2 + \alpha_1^2$   
 4.  $2\alpha_0 \alpha_1$

40. A  $3 \times 3$  matrix  $M$  has  $\text{Tr}[M] = 6$ ,  $\text{Tr}[M^2] = 26$  and  $\text{Tr}[M^3] = 90$ . Which of the following can be a possible set of eigenvalues of  $M$ ?

$l_1 = 2 l_2$   
 $l_1 = m v_1 r_1$   
 $l_2 = m v_2 r_2$   
 $2 l_2 = l_1$   
 $2 m v_2 r_2 = m v_1 r_1$   
 $2 v_2 r_2 = v_1 r_1$   
 $2 \sqrt{2} r_2 = r_1$   
 $r_1 = 2\sqrt{2} r_2$

$E_n = (n + \frac{1}{2}) h \omega$   
 $[1, 4]$  36

$\frac{\omega}{k} = \frac{k = m \omega}{k = 4 \pi m \omega}$

- (1) ~~{1, 1, 4}~~ (2) {-1, 0, 7}  
 (3) ~~{-1, 3, 4}~~ (4) {2, 2, 2}

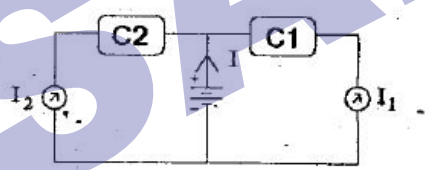
41. The perturbation  $H' = bx^4$ , where  $b$  is a constant, is added to the one dimensional harmonic oscillator potential  $V(x) = \frac{1}{2} m \omega^2 x^2$ . Which of the following denotes the correction to the ground state energy to first order in  $b$ ?

[Hint: The normalized ground state wave function of the one dimensional harmonic oscillator potential is

$\psi_0 = \left(\frac{m\omega}{h\pi}\right)^{1/4} e^{-m\omega x^2/2h}$ . You may use the following integral  
 $\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = a^{-n-1/2} \Gamma(n + \frac{1}{2})$ .

- (1)  $\frac{3b\hbar^2}{4m^2\omega^2}$  (2)  $\frac{3b\hbar^2}{2m^2\omega^2}$   
 (3)  $\frac{3b\hbar^2}{2\pi m^2\omega^2}$  (4)  $\frac{15b\hbar^2}{4m^2\omega^2}$

42. A battery powers two circuits  $C_1$  and  $C_2$  as shown in the figure.



The total current  $I$  drawn from the battery is estimated by measuring the currents  $I_1$  and  $I_2$  through the individual circuits. If  $I_1$  and  $I_2$  are both 200 mA and if the errors in their measurement are 3 mA and 4 mA respectively, the error in the estimate of  $I$  is

- (1) 7.0 mA (2) 7.5 mA  
 (3) 5.0 mA (4) 10.5 mA

$I_0 = I_1 + I_2$   
 $\Delta I = \Delta I_1 + \Delta I_2$

$\Delta I = 3 + 4 \text{ mA}$   
 $\Delta I = 7 \text{ mA}$

$\frac{1}{2} m \omega^2 x^2$   
 $\frac{1}{2} h \omega$   
 $\omega = 2\omega$

43. Consider a Maxwellian distribution of the velocity of the molecules of an ideal gas. Let  $V_{mp}$  and  $V_{rms}$  denote the most probable velocity and the root mean square velocity, respectively. The magnitude of the ratio  $V_{mp}/V_{rms}$  is

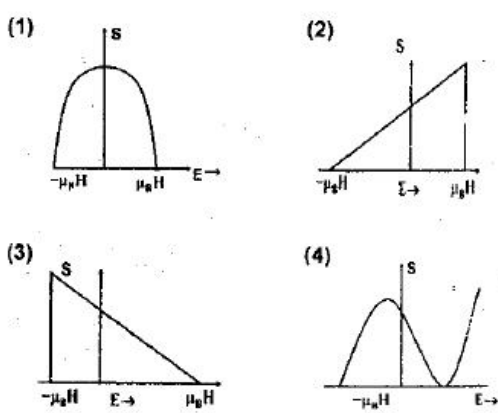
- (1) 1 (2) 2/3  
 (3)  $\sqrt{2/3}$  (4) 3/2

44. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temperature will

1. increase by a factor of 4  
 2. decrease by a factor of 4  
 3. increase by a factor of 8  
 4. decrease by a factor of 8

$E_F = kT_F$   
 $E_F \propto (n)^{2/3}$   
 $E_F \propto (8n)^{2/3}$   
 $E_F = 4T$

45. A system of  $N$  noninteracting spin- $\frac{1}{2}$  particles is placed in an external magnetic field  $H$ . The behaviour of the entropy of the system as a function of energy is given by

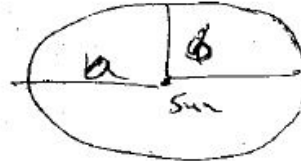


$\sin h(\frac{E}{2kT})$   
 $\frac{1}{\sin h(\frac{E}{2kT})}$

## PART C

46. A particle of mass  $m$  moves inside a bowl. If the surface of the bowl is given by the equation  $z = \frac{1}{2}a(x^2 + y^2)$ , where  $a$  is a constant, the Lagrangian of the particle is

1.  $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 - gar^2)$
2.  $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2]$
3.  $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2 - gar^2)$
4.  $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2]$



$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$   
 $V = -\frac{1}{2}mga^2r^2$   
 $L = T - V$

$-\frac{E}{g}$

$\frac{GMm}{r^2} = \frac{g}{4}$

47. An electron of energy 27 GeV collides with a proton of energy 820 GeV. The heaviest particle which can be produced in this collision has mass close to

- (1) 300 GeV
- (2) 821 GeV
- (3) 850 GeV
- (4) 1127 GeV

48. Let  $x_1(t)$  and  $x_2(t)$  be two linearly independent solutions of the differential

equation  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$ , and let  $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$ .

If  $w(0) = 1$ , then  $w(1)$  is given by

- (1) 1
- (2)  $e^2$
- (3)  $1/e$
- (4)  $1/e^2$

49. Assume that the free energy of a magnetic system has an expansion in the order parameter  $M$  of the form  $F(M, T) = a(T - T_c)M^2 + bM^4 + cM^6$ , with  $a, b$  and  $c > 0$ . As the temperature is lowered below  $T_c$ , the system undergoes a phase transition. The behaviour of the order parameter just below the transition, where  $(T - T_c)$  is very small, is best described by

- (1)  $M \propto (T_c - T)^{-1/2}$
- (2)  $M \propto (T_c - T)^{1/2}$
- (3)  $M \propto (T_c - T)$
- (4)  $M \propto (T_c - T)^3$

50. A planet of mass  $m$  moves in the inverse square central force field of the Sun of mass  $M$ . If the semi-major and semi-minor axes of the orbit are  $a$  and  $b$ , respectively, the total energy of the planet is:

- (1)  $-\frac{GMm}{a+b}$  (2)  $-GMm\left(\frac{1}{a} + \frac{1}{b}\right)$   
 (3)  $-\frac{GMm}{a}\left(\frac{1}{b} - \frac{1}{a}\right)$  (4)  $-GMm\frac{(a-b)}{(a+b)^2}$

$\sqrt{(p - ap)^2}$

51. Let  $|0\rangle$  and  $|1\rangle$  denote the normalized eigenstates corresponding to the ground and first excited states of a one dimensional harmonic oscillator. The uncertainty  $\Delta p$  in the state  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ , is

- (1)  $\Delta p = \sqrt{\hbar m \omega} / 2$  (2)  $\Delta p = \sqrt{\hbar m \omega} / 2$   
 (3)  $\Delta p = \sqrt{\hbar m \omega}$  (4)  $\Delta p = \sqrt{2\hbar m \omega}$

52. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at  $770 \text{ cm}^{-1}$ , the approximate positions of the Stokes and the anti-Stokes lines are

- (1) 481.5 nm and 520 nm (2) 481.5 nm and 500 nm  
 (3) 500 nm and 520 nm (4) 500 nm and 600 nm

53. The graph of the function

$$f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}$$

(where  $n=0,1,2,\dots$ ) is shown below.



Its Laplace transform  $\tilde{f}(s)$  is

- (1)  $\frac{1+e^{-s}}{s}$  (2)  $\frac{1-e^{-s}}{s}$   
 (3)  $\frac{1}{s(1+e^{-s})}$  (4)  $\frac{1}{s(1-e^{-s})}$

Handwritten calculations for the Laplace transform of the square wave function:

$$\int_0^\infty f(x)e^{-sx} dx = \int_0^1 e^{-sx} dx + \int_2^3 e^{-sx} dx + \int_4^5 e^{-sx} dx + \dots$$

$$= \frac{1-e^{-s}}{s} + \frac{e^{-2s}-e^{-3s}}{s} + \frac{e^{-4s}-e^{-5s}}{s} + \dots$$

$$= \frac{1}{s} \left[ (1-e^{-s}) + e^{-2s}(1-e^{-s}) + e^{-4s}(1-e^{-s}) + \dots \right]$$

$$= \frac{1}{s} \left[ (1-e^{-s}) \left( 1 + e^{-2s} + e^{-4s} + \dots \right) \right]$$

$$= \frac{1}{s} \left[ (1-e^{-s}) \frac{1}{1-e^{-2s}} \right] = \frac{1}{s} \left[ \frac{1-e^{-s}}{(1-e^{-s})(1+e^{-s})} \right] = \frac{1}{s(1+e^{-s})}$$

54. The energy levels of electrons of mass  $m$  and charge  $e$  confined in an area  $A$  in the  $xy$ -plane with a uniform magnetic field  $B$  applied in the  $z$ -direction are

given by  $E_n = \left(n + \frac{1}{2}\right) \frac{eB\hbar}{mc}$ ,  $n=0,1,2,\dots$ . The degeneracy of each level is

$\frac{eBA}{hc}$ . The lowest level is completely filled and the others are empty. The Fermi

energy  $\frac{\hbar^2 N}{2\pi m A}$ , where  $N$  is the total number of electrons, is

1. coincident with the  $n=0$  level
2. coincident with the  $n=1$  level
3. midway between the  $n=0$  and the  $n=1$  levels
4. midway between the  $n=1$  and the  $n=2$  levels

$$h + h_n =$$

$$h = 2 \times \pi k_F^2$$

$$h \times \frac{2\pi}{L} = 2 \pi k_F^2$$

55. An annulus of mass  $M$  made of a material of uniform density has inner and outer radii  $a$  and  $b$  respectively. Its principal moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is:

- (1)  $\frac{1}{2} M \frac{(b^4 + a^4)}{(b^2 - a^2)}$
- (2)  $\frac{1}{2} M \pi (b^2 - a^2)$
- (3)  $\frac{1}{2} M (b^2 - a^2)$
- (4)  $\frac{1}{2} M (b^2 + a^2)$

$$k_F = \left(\frac{2\pi}{A}\right)$$

$$E_F = \frac{\hbar^2 2\pi^2 n}{2\pi A}$$

56. The value of the real integral  $I = \int_{-3}^{+3} \frac{dx}{x^2 + 3x + 2}$  is

- (1)  $\frac{3\pi}{2}$
- (2)  $\ln\left(\frac{2}{5}\right)$
- (3)  $\infty$
- (4) 0

57. The minimum energy of an electron (the rest mass of which is 0.5 MeV) that can emit Cerenkov radiation while passing through water (of refractive index 1.5) is approximately

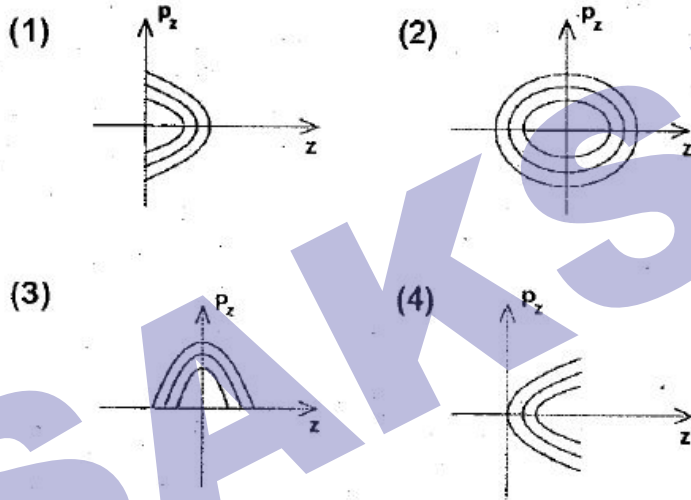
- (1) 1.0 MeV
- (2) 3.0 MeV
3. 0.6 MeV
- (4) 0.5 MeV

Penultimate Dange

58. A heater and a thermocouple are used to measure and control the temperature  $T$  of a sample at  $T_0 = 250^\circ\text{C}$ . A feedback circuit supplies power  $P$  to the heater according to the equation  $P = P_0 + G(T_0 - T) - D \frac{dT}{dt}$  with appropriately tuned values of the coefficients  $G$  and  $D$ . In order to maintain temperature stability in the presence of an external heat perturbation which causes small but rapid fluctuations of temperature, it is necessary to

- (1) decrease  $D$
- (2) increase  $D$
- (3) decrease  $G$
- (4) increase  $G$

59. The trajectory on the  $zp_z$ -plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at  $z=0$  is given by approximately by (neglect friction):



60. According to the shell model the spin and parity of the two nuclei  $^{125}_{51}\text{Sb}$  and  $^{89}_{38}\text{Sr}$  are, respectively,

- (1)  $(\frac{5}{2})^+$  and  $(\frac{5}{2})^+$
- (2)  $(\frac{5}{2})^+$  and  $(\frac{7}{2})^+$
- (3)  $(\frac{7}{2})^+$  and  $(\frac{5}{2})^+$
- (4)  $(\frac{7}{2})^+$  and  $(\frac{7}{2})^+$

deion  
 $\frac{89}{38}$   
 $\frac{125}{51}$   
 s p d s of p z s  
 d z s of p z s  
 d z s h

$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^2 (2s_{1/2})^2 (1d_{3/2})^2 (1f_{7/2})^8 (2p_{3/2})^4 (1f_{5/2})^4$   
 $2p_{1/2} (1g_{7/2})^2 (1g_{9/2})^2$   
 $3/2, 1/2$   
 $d_{5/2}, d_{3/2}, s_{1/2}$

61. The wave function of a particle at time  $t = 0$  is given by  $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$ , where  $|u_1\rangle$  and  $|u_2\rangle$  are the normalized eigenstates with eigenvalues  $E_1$  and  $E_2$  respectively, ( $E_2 > E_1$ ). The shortest time after which  $|\Psi(t)\rangle$  will become orthogonal to  $|\Psi(0)\rangle$  is

- (1)  $\frac{-\hbar\pi}{2(E_2 - E_1)}$                       (2)  $\frac{\hbar\pi}{E_2 - E_1}$   
 (3)  $\frac{\sqrt{2}\hbar\pi}{E_2 - E_1}$                       (4)  $\frac{2\hbar\pi}{E_2 - E_1}$

62. A gas of  $N$  non-interacting particles is in thermal equilibrium at temperature  $T$ . Each particle can be in any of the possible non-degenerate states of energy  $0$ ,  $2\varepsilon$  and  $4\varepsilon$ . The average energy per particle of the gas, when  $\beta\varepsilon \ll 1$ , is

- (1)  $2\varepsilon$                                       (2)  $3\varepsilon$   
 (3)  $2\varepsilon/3$                                 (4)  $\varepsilon$

63. The first few terms in the Taylor series expansion of the function  $f(x) = \sin x$  around  $x = \frac{\pi}{4}$  are

~~1.  $\frac{1}{\sqrt{2}} \left[ 1 + \left(x - \frac{\pi}{4}\right) + \frac{1}{2!} \left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$~~

~~2.  $\frac{1}{\sqrt{2}} \left[ 1 + \left(x - \frac{\pi}{4}\right) - \frac{1}{2!} \left(x - \frac{\pi}{4}\right)^2 - \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$~~

3.  $\left[ \left(x - \frac{\pi}{4}\right) - \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$

4.  $\frac{1}{\sqrt{2}} \left[ 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \dots \right]$

$f(x) = \sin x$   
 $f(\pi/4) = \frac{1}{\sqrt{2}}$

$f'(x) = \cos x$   
 $f'(\pi/4) = \frac{1}{\sqrt{2}}$

$f''(x) = -\sin x$   
 $f''(\pi/4) = -\frac{1}{\sqrt{2}}$

$f^{(n)}(a) = f(a) + (n-1)f'(a) + \frac{(n-2)f''(a)}{2!} + \dots$

$f^{(3)}(a) = \frac{(n-3)f'''(a)}{3!} + \dots$

$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$   
 $|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-iE_1 t/\hbar}|u_1\rangle + e^{-iE_2 t/\hbar}|u_2\rangle)$   
 $\langle\Psi(0)|\Psi(t)\rangle = \frac{1}{2}(e^{i(E_2 - E_1)t/\hbar} + e^{-i(E_2 - E_1)t/\hbar})$   
 $= \cos\left(\frac{(E_2 - E_1)t}{\hbar}\right)$   
 For orthogonality,  $\langle\Psi(0)|\Psi(t)\rangle = 0$   
 $\cos\left(\frac{(E_2 - E_1)t}{\hbar}\right) = 0$   
 $\frac{(E_2 - E_1)t}{\hbar} = \frac{\pi}{2}$   
 $t = \frac{\hbar\pi}{2(E_2 - E_1)}$

64. A one-dimensional chain consists of a set of  $N$  rods each of length  $a$ . When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is  $-\varepsilon$  when aligned parallel to the length of the chain and is  $+\varepsilon$  when perpendicular to it. When the chain is in thermal equilibrium at temperature  $T$ , its average length is

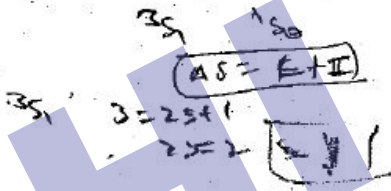
- (1)  $Na/2$  (2)  $Na$   
 (3)  $Na/(1+e^{-2\varepsilon/k_B T})$  (4)  $Na(1+e^{-2\varepsilon/k_B T})$

$z = 1 + e^{-2\varepsilon/k_B T}$

S(5H)

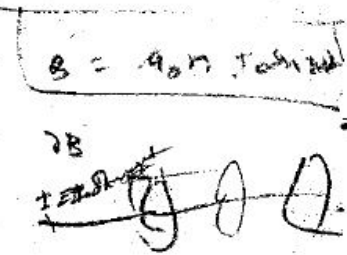
65. If the hyperfine interaction in an atom is given by  $H = a \vec{S}_e \cdot \vec{S}_p$ , where  $\vec{S}_e$  and  $\vec{S}_p$  denote the electron and proton spins, respectively, the splitting between the  $^3S_1$  and  $^1S_0$  state is

- (1)  $a\hbar^2/\sqrt{2}$  (2)  $a\hbar^2$   
 (3)  $a\hbar^2/2$  (4)  $2a\hbar^2$



66. Consider a solenoid of radius  $R$  with  $n$  turns per unit length, in which a time dependent current  $I = I_0 \sin \omega t$  (where  $\omega R/c \ll 1$ ) flows. The magnitude of the electric field at a perpendicular distance  $r < R$  from the axis of symmetry of the solenoid, is

- (1) 0 (2)  $\frac{1}{2r} \omega \mu_0 n I_0 R^2 \cos \omega t$   
 (3)  $\frac{1}{2} \omega \mu_0 n I_0 r \sin \omega t$  (4)  $\frac{1}{2} \omega \mu_0 n I_0 r \cos \omega t$



67. The difference in the Coulomb energy between the mirror nuclei  $^{49}_{24}\text{Cr}$  and  $^{49}_{25}\text{Mn}$  is 6.0 MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that  $e^2$  is approximately 1.0 MeV-fm, the radius of the  $^{49}_{25}\text{Mn}$  nucleus is

- (1)  $4.9 \times 10^{-13}$  m (2)  $4.9 \times 10^{-15}$  m  
 (3)  $5.1 \times 10^{-13}$  m (4)  $5.1 \times 10^{-15}$  m

$\Delta E = 6 \text{ MeV}$   
 $R = R_0 A^{1/3}$   
 $R = 1.2 \times 10^{-15} (49)^{1/3}$

Handwritten calculations for problem 67. It includes the formula for Coulomb energy  $E = \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$  and the derivation of the radius  $R = 1.2 \times 10^{-15} (49)^{1/3}$ . There are also some numerical calculations at the bottom right, such as  $12.25 / 3.5 = 3.5$  and  $3.5 \times 3.5 = 12.25$ .



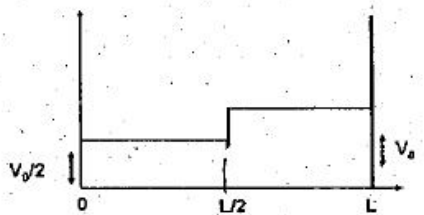
$I = \frac{3}{2} \cdot 3 = \frac{9}{2}$

$2 \times 3/2 = 3$   
 $1 \times 3/2 = 1.5$

68. The ratio of intensities of the  $D_1$  and  $D_2$  lines of sodium at high temperature is

- (1) 1:1                      (2) 2:3  
(3) 1:3                      (4) 1:2

69. A constant perturbation as shown in the figure below acts on a particle of mass  $m$  confined in an infinite potential well between 0 and  $L$ .



$\langle H_p \rangle = \int_0^L \psi_1^* \left( \frac{V_0}{2} \right) \psi_1 dx + \int_{L/2}^L \psi_1^* \left( \frac{V_0}{2} \right) \psi_1 dx$

$\frac{V_0}{2} \int_0^L \left( \frac{2}{L} \sin \frac{2\pi x}{L} \right)^2 dx$

$\frac{V_0}{2} \int_0^{L/2} \left( \frac{2}{L} \sin \frac{2\pi x}{L} \right)^2 dx + \frac{V_0}{2} \int_{L/2}^L \left( \frac{2}{L} \sin \frac{2\pi x}{L} \right)^2 dx$

The first-order correction to the ground state energy of the particle is

- (1)  $\frac{V_0}{2}$                       (2)  $\frac{3V_0}{4}$   
(3)  $\frac{V_0}{4}$                       (4)  $\frac{3V_0}{2}$

70. An atom of mass  $M$  can be excited to a state of mass  $M + \Delta$  by photon capture. The frequency of a photon which can cause this transition is

- (1)  $\Delta c^2 / 2h$                       (2)  $\Delta c^2 / h$   
(3)  $\Delta^2 c^2 / 2Mh$                       (4)  $\Delta(\Delta + 2M)c^2 / 2Mh$

$\frac{d}{dt} (Mv) = \text{const}$

71. The excitations of a three-dimensional solid are bosonic in nature with their frequency  $\omega$  and wave-number  $k$  are related by  $\omega \propto k^2$  in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

- (1)  $T^{1/2}$                       (2)  $T$   
(3)  $T^{3/2}$                       (4)  $T^3$

$C_V \propto T^3$

$\epsilon = NkT$

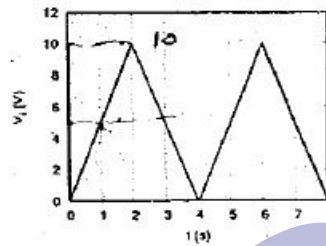
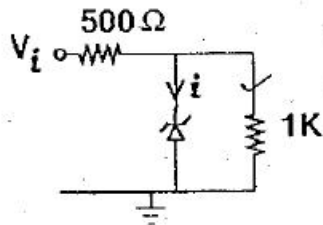
$E_V \propto T^3$

$\Delta c^2 = \frac{h^2 \omega^2}{2m}$   
 $\omega = \frac{\Delta c^2}{h}$

72. Gas molecules of mass  $m$  are confined in a cylinder of radius  $R$  and height  $L$  (with  $R \gg L$ ) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that  $mgL \gg k_b T$ ) is given by

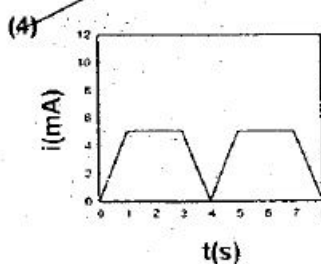
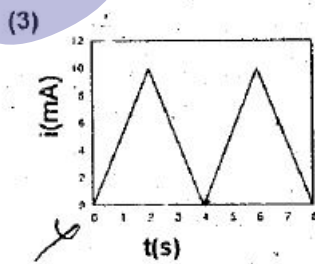
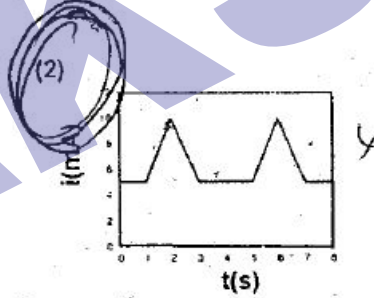
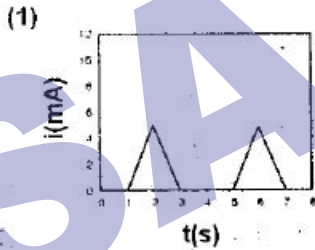
- (1)  $Nk_b T/2$                       (2)  $3Nk_b T/2$   
 (3)  $2Nk_b T$                       (4)  $5Nk_b T/2$

73. The figure below shows a voltage regulator utilizing a Zener diode of breakdown voltage 5 V and a positive triangular wave input of amplitude 10 V.



$V_b = 5V$   
 $V_m = 10V$   
 $V_{Z} = 5V$   
 $V = \frac{1 \times 100}{150} = \frac{2}{3}$

For  $V_i > 5 V$ , the Zener regulates the output voltage by channeling the excess current through itself. Which of the following waveforms shows the current  $i$  passing through the Zener diode?



$V = \frac{10 \times 1000}{1500}$   
 $V = \frac{1000}{150} = \frac{20}{3}$

$\frac{2k\Omega}{2m\Omega}$

$\frac{1}{2m\Omega}$        $\frac{3}{2m\Omega}$

$\Delta D = \sqrt{\langle D^2 \rangle - \langle D \rangle^2}$

Δ

74. A constant electric current  $I$  in an infinitely long straight wire is suddenly switched on at  $t=0$ . The vector potential at a perpendicular distance  $r$  from the wire is given by  $\vec{A} = \frac{\mu_0 I}{2\pi} \ln \left[ \frac{1}{r} (ct + \sqrt{c^2 t^2 - r^2}) \right]$ . The electric field at a distance  $r (< ct)$  is

- (1) 0  
 (2)  $\frac{\mu_0 I}{2\pi t \sqrt{2}} (\hat{i} - \hat{j})$   
 (3)  $\frac{c\mu_0 I}{2\pi \sqrt{c^2 t^2 - r^2}} \frac{1}{\sqrt{2}} (\hat{i} + \hat{j})$   
 (4)  $-\frac{c\mu_0 I}{2\pi \sqrt{c^2 t^2 - r^2}} \hat{k}$

75. Monochromatic light of wavelength 660 nm and intensity 100 mW/cm<sup>2</sup> falls on a solar cell of area 30 cm<sup>2</sup>. The conversion efficiency of the solar cell is 10%. If each converted photon results in an electron-hole pair, what is the maximum circuit current supplied by the solar cell? (Take  $h = 6.6 \times 10^{-34}$  J-s,  $c = 3 \times 10^8$  m/s and  $e = 1.6 \times 10^{-19}$  C).

- (1) 160 mA  
 (2) 320 mA  
 (3) 1600 mA  
 (4) 3200 mA

$$I = \frac{V}{R}$$

SAKSHI

$$\frac{V_0}{2} \left( 1 - \cos \frac{n\pi x}{L} \right) dx$$

$$\frac{V_0}{2} \left[ x - \frac{L}{n\pi} \sin \frac{n\pi x}{L} \right]_{L/2}^L$$

$$\frac{V_0}{2} \left[ L - \frac{L}{2} \right] = \frac{L}{2} \frac{V_0}{4}$$

$$\frac{V_0}{4} \int_0^{L/2} 2 \sin^2 \frac{n\pi x}{L} dx = \frac{V_0}{4} \left[ 1 - \cos \frac{n\pi x}{L} \right]_{L/2}^L$$

$$\frac{V_0}{4} \left[ x - \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_{L/2}^L$$

$$\frac{V_0 L}{8} + \frac{V_0}{4} = \frac{3V_0 L}{8}$$