SIMPLE INTEREST and COMPOUND INTEREST

(For all Competitive Exams)

Theory:

Money borrowed by a borrower or the money lend by a lender is called the principal (P).

The time for which it is borrowed or lent is called time period (T). The extra money paid by the borrower to the lender is called the interest. Interest is defined as rate (R) per cent per annum (p.c.p.a.).

If the interest is paid only on the principal for every year then it is called simple interest (I).

Simple interest (I) on a sum of Rs P borrowed at R% p.a. for T-years is given by $I = \frac{PTR}{100}$

The total money paid by the borrower to the lender to clear the debt is called amount (A) and it is equal to P + I.

$$\therefore A = P + I = P + \frac{PTR}{100} = P\left(\frac{100 + TR}{100}\right)$$

$$\Rightarrow P = \left(\frac{100 \times A}{100 + TR}\right)$$

If the interest is calculated on the previous year's amount instead of principal except for the first year then it is called a compound interest.

If a sum of Rs P is lent out for n-years at R% p.a. compound interest then the amount received is given by $A = P\left(1 + \frac{R}{100}\right)^n$

$\therefore$ The compound interest earned (C.I) = A – P

$$\Rightarrow \text{C.I} = P\left(1 + \frac{R}{100}\right)^n - P = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

Some important formulae:

(i) If a sum of money becomes x-times of itself in T-years @ R% p.a. S.I. then (x -1)100 = T×R

(ii) If a sum of money becomes x_1-times of itself in a time of T_1-years @ R_1% p.a. S.I. it becomes x_2-times of itself in T_2-years @ R_2% p.a. S.I. then $\frac{x_1 - 1}{x_2 - 1} = \frac{T_1 R_1}{T_2 R_2}$

(iii) A sum (P) of money becomes an amount of A_1 in T_1 years at a certain rate and the same sum becomes an amount of A_2 in T_2 years ($T_2 > T_1$). Then P = $\left(\frac{A_1 T_2 - A_2 T_1}{T_2 - T_1}\right)$ and the rate

$$(R) = \frac{(A_1 - P)100}{PT_1}$$

(iv) If the rates of compound interest are $R_1, R_2, R_3 \ldots\ldots\ldots R_n$ for n-successive years then

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\times.....\times\left(1+\frac{R_n}{100}\right)$$

(v) If a sum of Rs P is lent out at R% p.a. compound interest for n-years then

$A = P\left(1+\frac{R}{200}\right)^{2n}$, if the interest is compounded half-yearly

$= P\left(1+\frac{R}{400}\right)^{4n}$, if the interest is compounded quarterly

(vi) The difference between compound interest and simple interest on a sum of Rs P for 2-years at R% p.a. is given by $d_2 = P\left(\frac{R}{100}\right)^2$

Also $d_2 = \frac{S.I \times R}{200}$, where S.I is simple interest for 2 years

(vii) The difference between compound interest and simple interest on a sum of Rs P for 3 years at R% p.a. is given by $d_3 = \frac{PR^2(R+300)}{10^6} = \frac{S.I \times R \times (R+300)}{3\times 10^4}$

where S.I. is simple interest for 3-years (viii) If a sum of money amounts to A_{n-1} and to A_n in (n – 1) and n years respectively then

$R = \left(\frac{A_n - A_{n-1}}{A_{n-1}}\right)\times 100$

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PROBLEMS

1. At what rate will a man get on simple interest of ` 1,071 on a principal of ` 2,550 in 3 years?

a) 12 p.c.p.a. b) 14 p.c.p.a. c) 16 p.c.p.a. d) 18 p.c.p.a. e) None of these

ANSWER: b

P = 2550, I = 1071, T = 3, R = ?

$I = \frac{PTR}{100}$

$1071 = \frac{2550\times 3\times R}{100} \Rightarrow R = \frac{1071\times 100}{2550\times 3} = 14$

2. The simple interest accrued in 5 years on a principal of ` 24,000 is one-tenth the principal. What is the rate of simple interest p.c.p.a. ?

a) 5 b) 4 c) 6 d) 2 e) None of these

ANSWER: d

P = 24,000 $I = \frac{P}{10} = 2{,}400$ T = 5 R = ?

$2400 = \frac{24000\times R\times 5}{100} \Rightarrow R = \frac{2400\times 100}{24000\times 5} = 2$

3. A lent ` 5000 to B for 2 years and ` 3000 to C for 4 years on simple interest at the same rate of interest and received ` 2200 in all from both as interest. The rate of interest per annum is

a) 7% b) 5% c) $7\frac{1}{8}\%$ d) 10% e) None of these

ANSWER: d

Let the rate of interest be x in each case then the total interest

$$\frac{5000\times 2\times x}{100}+\frac{3000\times 4\times x}{100}=2200$$

$$100x + 120x = 2200 \quad \Rightarrow \quad x=\frac{2200}{220}=10$$

4. Shamita took a loan at simple interest rate of 6 p.c.p.a. in the first year and it increased by 1.5 p.c.p.a. every year. If she pays ` 8,190 as interest at the end of 3 years, what was her loan amount

a) ` 36000 b) ` 35400 c) ` 36800 d) can't be determined e) None of these

ANSWER: e

P = ? $R_1=6$ $R_2=7.5$ $R_3=9$ I = 8190

But $I=\frac{P(R_1+R_2+R_3)}{100}$

$$8190=\frac{P(6+7.5+9)}{100}=\frac{22.5P}{100}$$

$$P=\frac{8190\times 100}{22.5}=364\times 100=36400$$

5. What would be the compound interest obtained on an amount of ` 6,000 at the rate of 7 p.c.p.a. for 2 years?

a) ` 767.5 b) ` 846.2 c) ` 769.4 d) ` 860.4 e) None of these

ANSWER: e

P = 6000 R = 7 n = 2 C.I = ?

$$\text{C.I}=P\left[\left(1+\frac{R}{100}\right)^n-1\right]$$

$$\text{C.I}=6000\left[\left(1+\frac{7}{100}\right)^2-1\right]=6000\left[\frac{107^2-100^2}{100^2}\right]$$

$$=6000\left[\frac{207\times 7}{100^2}\right]=6000\left[\frac{1449}{10000}\right]=869.4$$

6. What is the compound interest on ` 5,000 for 4 years if the rate of interest is 10% p.a. for the first 2 years and 20% p.a. for the next 2 years?

a) ` 2,320.50 b) ` 3,712 c) ` 3,745 d) ` 5,368 e) None of these

ANSWER:

P = 5000 $R_1=10$ $R_2=20$

$\therefore A = P\left(1+\frac{R_1}{100}\right)^2\left(1+\frac{R_2}{100}\right)^2$

$A = 5000\times\left(1+\frac{10}{100}\right)^2\times\left(1+\frac{20}{100}\right)^2$

$= 5000\times\frac{121}{100}\times\frac{144}{100} = 8712$

$\therefore$ C.I = 8712 – 5000 = 3712

7. A sum of money invested at compound interest amounts to ` 650 at the end of first year and ` 676 at the end of second year. The sum of money is:

a) ` 600 b) ` 540 c) ` 625 d) ` 560 e) None of these

ANSWER: c

$A_1 = 650$ $A_2 = 676$ R = ?

$R = \left(\frac{A_2 - A_1}{A_1}\right)\times 100$

$R = \frac{676-650}{650}\times 100 = \frac{26}{650}\times 100 = 4\%$

$A_1 = P\left(1+\frac{R}{100}\right)$

$650 = P\left(1+\frac{4}{100}\right) \Rightarrow P = \frac{650\times 100}{104} = 625$

8. The difference in simple and compound interest on a certain sum for 2 years at 5% per annum compounded annually is ` 75. Find the sum?

a) ` 40,000 b) ` 20,000 c) ` 50,000 d) ` 30,000 e) None of these

ANSWER: d

$d_2 = 75$ n = 2 R = 5

$d_2 = P\left(\frac{R}{100}\right)^2 \Rightarrow 75 = P\left(\frac{5}{100}\right)^2$

$P = 75\times 400 = 30{,}000$

9. Two customers borrowed the same amount of money, one at compound interest and at the other at simple interest. If after two years, the interest payable by one was ` 220 and the other ` 200, then what was the principle money lent to each of them?

a) ` 450 b) ` 500 c) ` 550 d) ` 575 e) None of these

ANSWER: b

$d_2 = 220 - 200 = 20$ S.I = 200 P = ?

$\therefore R = \frac{d_2\times 200}{S.I} = \frac{20\times 200}{200} = 20$

Also $d_2 = P\left(\frac{R}{100}\right)^2$

$$20 = P\left(\frac{20}{100}\right)^2 \Rightarrow \quad P = 20\times 25 = 500$$

10. If the compound interest on a certain sum for 2 years at 3% per annum is ` 101.50, then the simple interest on the same sum at the same rate and for the same time will be?

a) ` 90 b) ` 95.50 c) ` 100 d) ` 98.25 e) None of these

ANSWER: c

S.I = x R = 3

$$d_2 = \frac{S.I \times R}{200} = \frac{x\times 3}{200} = \frac{3x}{200}$$

$\therefore$ C.I = S.I + d_2

$$101.5 = x + \frac{3x}{200} = \frac{203x}{200}$$

$$\therefore \quad x = \frac{101.5\times 200}{203} = 100$$