RATIO AND PROPORTION

If the ratio of two numbers is a : b then the two numbers can be assumed as ax and bx where x is the H.C.F (Highest Common Factor) of the two numbers.

In the above ratio a and b are known as the terms of the ratio.

Some Formulae:

(i) Two numbers are in the ratio of a : b. If both are increased/decreased by k then the ratio of the two numbers becomes c : d then the value x (H.C.F) is given by

$$x = \frac{|c-d| \times k}{|ad-bc|}$$

The two numbers will be ax and bx.

(ii) If the ratio of two numbers A and B is $N_1 : D_1$ and that of B and C is $N_2 : D_2$ then the ratio of three numbers i.e. A : B : C = $(N_1 \times N_2) : (D_1 \times N_2) : (D_1 \times D_2)$.

(iii) Three numbers are such that k_1 times the first number, k_2 times the second number and k_3 times the third number are all equal. then the three numbers are in the ratio of

$$\frac{1}{k_1} : \frac{1}{k_2} : \frac{1}{k_3}.$$

(iv) If two ratios a : b and c : d have equal values then a, b, c and d are said to be in proportion i.e. a : b = c : d. Here a and d are called the ends and b and c are called means of the proportion. In a proportion the product of extremes is equal to the product of the means.

$\Rightarrow$ a×d = b×c

(v) The number to be subtracted from each of a, b, c and d so that they become proportional

is $\frac{ad-bc}{(a+d)-(b+c)}$.

Similarly the least number to be added to each of a, b, c and d so that they became proportional is $\frac{ad-bc}{(b+c)-(a+d)}$

PROBLEMS

1. The average age of three boys is 25 years and their ages are in the proportion 3 : 5 : 7. The age of the youngest boy is:

1) 21 years 2) 18 years 3) 15 years 4) 9 years 5) None of these

ANSWER: 3

Total age of three boys = 25 × 3 = 75

$\therefore$ Their ages are in the ratio of 3 : 5 : 7 the age of youngest boy

$$= \left(\frac{75}{3+5+7}\right) \times 3 = \frac{75}{15} \times 3$$

= 15 years

2. A box contains 1-rupee, 50-paise and 25-paise coins in the ration 8 : 5 : 3. If the total amount of money in the box is ` 112.50, the number of 50-paise coins is

1) 80 2) 50 3) 30 4) 42 5) None of these

ANSWER: 2

The coins are in the ratio of 8 : 5 : 3

So their numbers can be assumed as $8x$, $5x$ and $3x$

$\therefore$ Total value of coins $= 8x \times 1 + 5x \times \frac{1}{2} + 3x \times \frac{1}{4}$

$$= 8x + \frac{5x}{2} + \frac{3x}{4} = \frac{32x + 10x + 3x}{4} = \frac{45x}{4}$$

$\Rightarrow \frac{45x}{4} = 112.5$

$\therefore x = \frac{112.5 \times 4}{45} = \frac{450}{45} = 10$

$\therefore$ Number of 50-paise coins = 5×10 = 50

3. Two natural numbers are in the ratio 3 : 5 and their product is 2160. The smaller of the numbers is

1) 36 2) 24 3) 18 4) 12 5) None of these

ANSWER: 1

Since the ratio of two numbers is 3 : 5, the two numbers can be assumed as $3x$ and $5x$

$\therefore$ Their product $= 3x \times 5x = 15x^2$

But $15x^2 = 2160$

$\Rightarrow x^2 = \frac{2160}{15} = 144$

$\therefore x = \sqrt{144} = 12$

Smaller number is 3×12 = 36

4. What must be added to each term of the ratio 7 : 11 so as to make it equal to 3 : 4

1) 8 2) 7.5 3) 6.5 4) 5 5) None of these

ANSWER: 4

The terms of the ratio 7 : 11 are 7 and 11 (not $7x$ and $11x$)

If 'k' be the number added to each term so that ratio becomes 3 : 4

then 7 + k : 11 + k = 3 : 4

$\Rightarrow$ 4(7 + k) = 3 (11 + k)

28 + 4k = 33 + 3k

$\therefore$ k = 33 – 28 = 5

5. If $W_1 : W_2 = 2 : 3$ and $W_1 : W_3 = 1 : 2$ then W2 : W3 is

1) 3 : 4 2) 4 : 3 3) 2 : 3 4) 4 : 5 5) None of these

ANSWER: 1

$W_1 : W_2 = 2 : 3 \Rightarrow W_2 : W_1 = 3 : 2$

$\therefore \frac{W_2}{W_1} = \frac{3}{2}$ also $\frac{W_1}{W_3} = \frac{1}{2}$

$\Rightarrow \frac{W_2}{\cancel{W_1}} \times \frac{\cancel{W_1}}{W_3} = \frac{3}{2} \times \frac{1}{2} \Rightarrow \frac{W_2}{W_3} = \frac{3}{4}$

$\therefore W_2 : W_3 = 3 : 4$

6. ` 2040 are divided among A, B and C such that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets. Then B's share is:

1) ` 180 2) ` 240 3) ` 360 4) ` 480 5) None of these

ANSWER: 3

$A + B + C = 2040$

$A = \frac{2}{3}B$ and $B = \frac{1}{4}C$

$\Rightarrow \frac{A}{B} = \frac{2}{3}$ and $\frac{B}{C} = \frac{1}{4}$

$\therefore A : B : C = (2 \times 1) : (3 \times 1) : (3 \times 4) = 2 : 3 : 12$

$\Rightarrow$ B's share $= \left(\frac{2040}{2+3+12}\right) \times 3 = \frac{2040}{17} \times 3 = 360$

7. ` 1870 are divided into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal. The third part is:

1) ` 510 2) ` 680 3) ` 850 4) ` 1020 5) None of these

ANSWER: 4

x, y and z be the three parts then $x + y + z = 1870$

Also $\frac{1}{2}x = \frac{1}{3}y = \frac{1}{6}z$

$\therefore x : y : z = 2 : 3 : 6$

$\Rightarrow z = \left(\frac{1870}{2+3+6}\right) \times 6 = \frac{1870}{11} \times 6 = 1020$

8. A sum of ` 1300 is divided among P, Q, R and S such that:

$\frac{P's\ share}{Q's\ share} = \frac{Q's\ share}{R's\ share} = \frac{R's\ share}{S's\ share} = \frac{2}{3}$

What is P's share?

1) ` 320 2) ` 240 3) ` 160 4) ` 140 5) None of these

ANSWER: 3

$P + Q + R + S = 1300$

$\frac{P}{Q} = \frac{2}{3}$ $\frac{Q}{R} = \frac{2}{3}$ $\frac{R}{S} = \frac{2}{3}$

$\Rightarrow$ P : Q : R : S = (2 × 2 × 2) : (3 × 2 × 2) : (3 × 3 × 2) : (3 × 3 × 3) = 8 : 12 : 18 : 27

$\therefore$ P's share $= \left(\frac{1300}{8+12+18+27}\right)\times 8 = \frac{1300}{65}\times 8 = 160$

9. The ages of X and Y are in the ratio of 3 : 1. Fifteen years hence, the ratio will be 2 : 1. Their present ages (in years) are:

1) 30, 10 2) 45, 15 3) 21, 7 4) 60, 20 5) None of these

ANSWER: 2

The ages of X and Y be $3x$ and x

Fifteen years hence their ages will be $(3x + 15)$ and $(x + 15)$

But $(3x + 15) : (x + 15) = 2 : 1$

$\Rightarrow$ $(3x + 15) \times 1 = (x + 15) \times 2$

$3x + 15 = 2x + 30$

$x = 15$

$\therefore$ The present ages are $3 \times 15 = 45$ and 15 respectively.

10. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper in C will be:

1) 5 : 9 2) 5 : 7 3) 7 : 5 4) 9 : 5 5) None of these

ANSWER: 3

Take the quantity of each of A and B as L.C.M of sum of the terms of the ratio.

$\therefore$ Quantity of each of A and B = L.C.M of [(7 + 2), (7 + 11)]

= L.C.M of (9, 18) = 18

$\therefore$ Gold in alloy A $= \left(\frac{18}{7+2}\right)\times 7 = 14$

Gold in alloy B $= \left(\frac{18}{7+11}\right)\times 7 = 7$

$\therefore$ In alloy C quantity of gold = 14 + 7 = 21

$\Rightarrow$ In alloy C quantity of copper = (2×18 – 21) = 15

$\therefore$ Gold to copper ratio = 21 : 15 = 7 : 5

11. A mixture contains milk and water in the ratio of 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the original mixture is:

1) 16 liters 2) 25 liters 3) 22.75 liters 4) 32.5 liters 5) None of these

ANSWER: 2

Milk and water in the mixture be $5x$ and x

When 5 litres water is added then milk and water will be $5x$ and $x + 5$

But $5x : x + 5 = 5 : 2$

$\Rightarrow$ $5x \times 2 : (x + 5)\times 5$

$10x = 5x + 25$

$\therefore$ $x = 5$

Milk in the original mixture = 5×5 = 25 litres

12. Two equal glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk. They are then filled with water and the contents mixed in a tumbler. The ratio of milk and water in a tumbler is:

1) 7 : 5 2) 7 : 17 3) 9 : 21 4) 11 : 23 5) None of these

ANSWER: 2

The glasses be of L.C.M of (3, 4) = 12 lit capacity

Then milk in first glass = $\frac{1}{3}(12)$ = 4 lit

Milk in second glass = $\frac{1}{4}(12)$ = 3 lit

$\therefore$ Water in first glass = 12 – 4 = 8 lit

Water in second glass = 12 – 3 = 9 lit

$\Rightarrow$ Milk in tumbler = 4 + 3 = 7 lit

Water in tumbler = 8 + 9 = 17 lit

Milk : Water = 7 : 17