PROBABILITY

PROBABILITY: If there are 'a' elementary events associated with a random experiment and 'b' of them are favorable to event 'E'. Then the probability of occurrence of event E is denoted by P (E) and is defined as $\frac{b}{a}$.

$$\therefore \quad P(E) = \frac{b}{a} \qquad \Rightarrow \ 0 \le P(E) \le 1$$

The probability of non-occurrence of event E denoted by $P(\bar{E})$ and is defined as $\frac{a-b}{a}$.

$$\Rightarrow P(\bar{E}) = \frac{a-b}{a} = 1 - \frac{b}{a} = 1 - P(E)$$

$$\Rightarrow P(E) + P(\bar{E}) = 1$$

If the random experiment is based on arrangement of objects then

$$P(E) = \frac{arrangements\ in\ favour}{Total\ number\ of\ arrangements}$$

Similarly if the random experiment is based on selection of objects then

$$P(E) = \frac{Selections\ in\ favour}{Total\ number\ of\ selections}$$

PROBLEMS

Directions (1-3): Study the given information carefully and answer the questions that follow:

A basket contains 4 red, 5 blue and 3 green marbles.

1. If three marbles are picked at random, what is the probability that either all are green or all are red?

a) $\frac{7}{44}$ b) $\frac{7}{12}$ c) $\frac{5}{12}$ d) $\frac{1}{44}$ e) None of these

ANSWER: d

Three marbles can be picked from 12 balls in ${}^{12}C_3$ ways

Number of picks where all three are green = ${}^{3}C_3$

Number of picks where all three are red = ${}^{4}C_3$

$\therefore$ Number of picks where all are green or red = ${}^{3}C_3 + {}^{4}C_3$

$$\Rightarrow P(E) = \frac{{}^{3}C_3 + {}^{4}C_3}{{}^{12}C_3} = \frac{1+4}{\frac{12\times11\times10}{1\times2\times3}} = \frac{5}{2\times11\times10} = \frac{1}{44}$$

2. If two marbles are picked at random, what is the probability that both are red?

a) $\frac{3}{7}$ b) $\frac{1}{2}$ c) $\frac{2}{11}$ d) $\frac{1}{6}$ e) None of these

ANSWER: e

Two marbles can be picked in ${}^{12}C_2 = \frac{12\times11}{1\times2} = 6\times11$ ways

Number of picks where both are red = ${}^{4}C_2 = \frac{4\times3}{1\times2} = 6$

$\Rightarrow$ Probability P(E) = $\frac{6}{6\times11} = \frac{1}{11}$

3. If three marbles are picked at random, what is the probability that at least one is blue?

a) $\frac{7}{12}$ b) $\frac{37}{44}$ c) $\frac{5}{12}$ d) $\frac{7}{44}$ e) None of these

ANSWER: b

Three marbles can be picked in ${}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 2\times11\times10$ ways

Number of picks where no marble is blue = ${}^{(4+3)}C_3 = {}^{7}C_3 = \frac{7\times6\times5}{1\times2\times3} = 7\times5$

$\Rightarrow P(E) = \frac{7\times5}{2\times11\times10} = \frac{7}{44}$

$\Rightarrow P(\overline{E}) = 1 - P(E) = 1 - \frac{7}{44} = \frac{37}{44}$

Directions (4-8): Study the following information carefully to answer the questions that follow:

A box contains 2 blue caps, 4 red caps, 5 green caps and 1 yellow cap.

4. If two caps are picked at random, what is the probability that both are blue?

a) $\frac{1}{6}$ b) $\frac{1}{10}$ c) $\frac{1}{12}$ d) $\frac{1}{45}$ e) None of these

ANSWER: e

Two caps can be picked in ${}^{12}C_2 = \frac{12\times11}{1\times2} = 66$ ways

Number of picks where both are blue = ${}^{2}C_2 = 1$

$\therefore$ Required Probability P(E) = $\frac{1}{66}$

5. If four caps are picked at random, what is the probability that none is green?

a) $\frac{7}{99}$ b) $\frac{5}{99}$ c) $\frac{7}{12}$ d) $\frac{5}{12}$ e) None of these

ANSWER: a

Four caps can be picked in ${}^{12}C_4 = \frac{12\times11\times10\times9}{1\times2\times3\times4} = 11\times5\times9$ ways

Number of picks where no cap is green = ${}^{(2+4+1)}C_4 = {}^{7}C_4 = {}^{7}C_3 = \frac{7\times6\times5}{1\times2\times3} = 7\times5$

$\therefore$ Required Probability P(E) = $\frac{7\times5}{11\times5\times9} = \frac{7}{99}$

6. If three caps are picked at random, what is the probability that two are red and one is green?

a) $\frac{9}{22}$ b) $\frac{6}{19}$ c) $\frac{1}{6}$ d) $\frac{3}{22}$ e) None of these

ANSWER: d

Three caps can be picked in ${}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 2\times11\times10$ ways

Number of picks with two red caps and one green cap = ${}^4C_2\times{}^5C_1 = 6\times5$

$\therefore$ Required Probability P(E) = $\frac{6\times5}{2\times11\times10} = \frac{3}{22}$

7. If one cap is picked at random, what is the probability that it is either blue or yellow?

a) $\frac{2}{9}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) $\frac{6}{11}$ e) None of these

ANSWER: b

One cap can be picked in ${}^{12}C_1 = 12$ ways

Number of picks with either blue or yellow cap = ${}^{(2+1)}C_1 = {}^3C_1 = 3$

$\therefore$ Required Probability P (E) = $\frac{3}{12} = \frac{1}{4}$

8. If two caps are picked at random, what is the probability that at least one is red?

a) $\frac{1}{3}$ b) $\frac{16}{21}$ c) $\frac{19}{33}$ d) $\frac{7}{19}$ e) None of these

ANSWER: c

Two caps can be picked in ${}^{12}C_2 = \frac{12\times11}{1\times2} = 66$ ways

Number of picks where none is red = ${}^{(2+5+1)}C_2 = {}^8C_2 = \frac{8\times7}{1\times2} = 28$

$\Rightarrow P(E) = \frac{28}{66} = \frac{14}{33}$

$\therefore P(\bar{E}) = 1 - P(E) = 1 - \frac{14}{33} = \frac{19}{33}$

Directions (9-13): Study the given information carefully to answer the questions that follow:

A basket contains 6 blue, 2 red, 4 green and 3 yellow marbles.

9. If 2 balls are picked at random, what is the probability that either both are green or both are yellow?

a) $\frac{2}{5}$ b) $\frac{3}{35}$ c) $\frac{1}{3}$ d) $\frac{3}{91}$ e) None of these

ANSWER: b

Two balls can be picked in ${}^{(6+2+4+3)}C_2 = {}^{15}C_2 = \frac{15\times14}{1\times2} = 105$ ways

Number of picks where both are green = ${}^4C_2 = \frac{4\times3}{1\times2} = 6$

Number of picks where both are yellow = ${}^3C_2 = 3$

∴ Number of picks where both are green or both are yellow = 6 + 3 = 9

∴ Required Probability P(E) = $\frac{9}{105} = \frac{3}{35}$

10. If 5 balls are picked at random, what is the probability that at least one is blue?

a) $\frac{137}{143}$ b) $\frac{9}{91}$ c) $\frac{18}{455}$ d) $\frac{2}{5}$ e) None of these

ANSWER: e

Five balls can be picked in ${}^{15}C_5 = \frac{15\times14\times13\times12\times11}{1\times2\times3\times4\times5} = 21\times13\times11$ ways

Number of picks with no blue ball = ${}^{(2+4+3)}C_5 = {}^9C_5 = {}^9C_4 = \frac{9\times8\times7\times6}{1\times2\times3\times4} = 9\times7\times2$

$\Rightarrow P(E) = \frac{9\times7\times2}{21\times13\times11} = \frac{42}{1001}$

∴ $P(\bar{E}) = 1 - \frac{42}{1001} = \frac{959}{1001}$

11. If 2 balls are picked at random, what is the probability that both are blue?

a) $\frac{1}{5}$ b) $\frac{8}{91}$ c) $\frac{2}{15}$ d) $\frac{7}{27}$ e) None of these

ANSWER: e

Two balls can be picked in ${}^{(6+2+4+3)}C_2 = {}^{15}C_2 = \frac{15\times14}{1\times2} = 15\times7$ ways

Two blue balls can be picked in ${}^6C_2 = \frac{6\times5}{1\times2} = 3\times5$

∴ Required Probability P(E) = $\frac{3\times5}{15\times7} = \frac{1}{7}$

12. If 4 balls are picked at random, what is the probability that 2 balls are red and 2 are green

a) $\frac{4}{15}$ b) $\frac{5}{27}$ c) $\frac{1}{3}$ d) $\frac{2}{455}$ e) None of these

ANSWER: d

Four balls can be picked in ${}^{15}C_4 = \frac{15\times14\times13\times12}{1\times2\times3\times4} = 15\times13\times7$ ways

2 red and 2 green balls can be picked in ${}^2C_2 \times {}^4C_2 = 1\times6$

∴ Required Probability P(E) = $\frac{1\times6}{15\times13\times7} = \frac{2}{455}$

13. If 3 balls are picked at random, what is the probability that none is yellow?

a) $\frac{3}{455}$ b) $\frac{1}{5}$ c) $\frac{44}{91}$ d) $\frac{4}{5}$ e) None of these

ANSWER: c

Three balls can be picked in ${}^{15}C_3 = \frac{15\times14\times13}{1\times2\times3} = 35\times13$ ways

Number of picks with no yellow ball = ${}^{(6+2+4)}C_3 = {}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 2\times11\times10$

$\therefore$ Required Probability P(E) = $\frac{2\times11\times10}{35\times13} = \frac{44}{91}$