## PERMUTATIONS \& COMBINATIONS

First let us see the definitions that we need to solve the questions.
Factorial: Factorial of any non-negative integer ' $n$ ' (denoted by $n!$ ) is equal to product of all the integers from 1 to n .
$\therefore \mathrm{n}!=1 \times 2 \times 3 \times$. $\qquad$ $\times n$
Try to memorize the factorial values of first 8 numbers.
$1!=1, \quad 2!=2, \quad 3!=6, \quad 4!=24, \quad 5!=120, \quad 6!=720, \quad 7!=5040, \quad 8!=40320$
Counting principle: If an operation can be performed in m-ways and a second operation can be performed in n-ways corresponding to each performance of the first operation, then the two operations in successions can be performed in $\mathrm{mn}(\mathrm{m} \times \mathrm{n})$ ways.

Permutations: An arrangement that can be formed by taking some or all the finite set of objects is called a permutation.

If the objects are arranged in a line then it is called a linear permutation and if the objects are arranged in the form of a circle then it is called a circular permutation.

The number of permutations that can be made by taking any r-objects at a time from a set of n -dissimilar objects ( $\mathrm{r} \leq \mathrm{n}$ ) is denoted by ${ }^{n} P_{r}$ or $\mathrm{n}(\mathrm{P}, \mathrm{r})$.

$$
{ }^{n} P_{r}=\frac{n!}{(n-r)!}=\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2) \ldots \ldots \ldots \ldots \ldots . .(\mathrm{n}-\mathrm{r}+1)
$$

The number of permutations that can be made by taking all the given 'n' dissimilar objects is equal to n !.

## Formulae:

1) The number of ways that ' $n$ ' distinct letters of a word can be arranged is equal to n!.
2) The number of arrangements that can be made by using the letters of a n-letter word in which set of 'a' letters are same and a set of 'b' letters are same is given by
$\frac{n!}{a!b!}$
3) The number of ways that the letters of a n-distinct letter word can be arranged such that a set of ' $k$ ' letters are always together is equal to $k!\times(n-k+1)$ !
4) The number of ways that the letters of a $n$-distinct letter word can be arranged such that a set of ' $k$ ' letters can never be together is equal to $n!-k!\times(n-k+1)$ !

Combination: A selection that can be formed by taking some or all of a finite set of things is called a combination.

The number of combinations of $n$ dissimilar things taken $r$ at a time is denoted by ${ }^{n} C_{r}$ or C(n, r).

$$
{ }^{n} C_{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)(n-2) \ldots \ldots \ldots \ldots \ldots . .(n-r+1)}{1.2 .3 \ldots \ldots \ldots \ldots . . . . . . . . . .}
$$

Also ${ }^{n} C_{0}=1 ; \quad{ }^{n} C_{n}=1$;
Note: (i) ${ }^{n} C_{r}+{ }^{n} C_{r-1}={ }^{(n+1)} C_{r}$
(ii) ${ }^{n} C_{r}={ }^{n} C_{n-r}$

1) In a group of n-members if each member offers a shake hand to the remaining members then the total number of handshakes $={ }^{n} C_{2}=\frac{n(n-1)}{1.2}=\frac{n(n-1)}{2}$
2) The number of diagonals in a regular polygon of ' $n$ ' sides is $\frac{n(n-3)}{2}$
3) From a group of $m$-men and $n$-women, if a committee of $r$-members ( $r \leq m+n$ ) is to be formed, then the number of ways it can be done is equal to ${ }^{(m+n)} C_{r}$.
4) The number of ways a group of r-boys (men) and s-girls (women) can be made out of $m$ boys (men) and $n$-girls (women) is equal to $\left({ }^{m} C_{r} \times{ }^{n} C_{s}\right)$
5) From a group of m-boys and n-girls the number of different ways that a committee of remembers can be formed so that the committee will have at least one girl is ${ }^{(m+n)} \mathrm{Cr}-{ }^{m} \mathrm{C}_{r}$

## PROBLEMS

1. In how many different ways can the letters of the word 'PRAISE' be arranged?
a) 720
b) 610
c) 360
d) 210
e) None of
these
ANSWER: a
The word 'PRAISE' has six different letters P, R, A, I, S and E i.e. $\mathrm{n}=6$

* The six different letters can be arranged in $6!=720$ ways

2. In how many different ways can the letters of the word 'STRESS' be arranged?
a) 360
b) 240
c) 720
d) 120
e) None of
these
ANSWER: d
The word 'STRESS' has a total of six letters (i.e. $n=6$ ) out of which a group of three letters are same (i.e. $a=3$ )
. The letters can be arranged in $\frac{n!}{a!}$ ways $=\frac{6!}{3!}=\frac{720}{6}=120$
3. In how many different ways can the letters of the word 'READERS' can be arranged?
a) 1260
b) 2520
c) 5040
d) 720
e) None of
these
ANSWER: a
The word 'READERS' has a total of seven letters (i.e. $n=7$ ) in which a group of two

R's are same (i.e. $\mathrm{a}=2$ ) and another group of two E's are same (i.e. $\mathrm{b}=2$ )

* The letters can be arranged in $\frac{n!}{a!b!}$ ways $=\frac{7!}{2!2!}=\frac{5040}{2 \times 2}=1260$

4. In how many different ways can the letters of the word 'PREVIOUS' be arranged in such a way that the vowels always come together?
a) 50400
b) 4840
c) 3260
d) 2880
e) 2420

ANSWER: d
Treating the group of four vowels (E, I O and U) as one distinct letter then total number of letters will be $8-4+1=5$ [i.e. P, R, V, S and (E, I, O, U)]

The five letters can be arranged in $5!=120$ ways
The vowels among themselves can be arranged in $4!=24$ ways
$\therefore$ The letters of the word 'PREVIOUS' can be arranged in $120 \times 24=2880$ ways such that vowels always come together.
5. In how many different ways can the letters of the word 'REASON' be arranged in such a way that all the vowels will never come together?
a) 720
b) 684
c) 576
d) 144
e) None of
these
ANSWER: c
The letters of the word 'REASON' can be arranged in 6 ! $=720$ ways
Number of ways that the letters can be arranged such that all vowels come together $=4!\times 3!=24 \times 6=144$

So number of arrangements in which all the vowels will not come together

$$
=720-144=576
$$

6. How many words can be formed out of the letters of the word 'ARTICLE' so that the vowels occupy the even places?
a) 144
b) 24
c) 6
d) 20
e) 60

ANSWER: a
The word 'ARTICLE' has Seven letters. So it contains seven places (3 even and 4 odd) to be filled with the letters

The three vowels can be placed in 3 even places in $3!=6$ ways
The four consonants can be placed in 4 odd places in $4!=24$ ways
$\therefore$ Seven letters can be placed in seven places in which vowels take only even positions $=6 \times 24=144$ ways
7. The letters of the word PROMISE are arranged so that no two of the vowels should come together. Find the total number of arrangements.
a) 49
b) 1440
c) 7
d) 1898
e) None of
these

## ANSWER: b

The consonants of the word 'PROMISE' (P, R, M and S) can be first arranged in $4!=24$ ways

The vowels of the word 'PROMISE' (O, I and E) then can be placed in between two consonants including the first and the last positions. Then five places will be available to place vowels.

The three vowels can be placed in 5 positions in $5 \mathrm{P}_{3}=5 \times 4 \times 3=60$ ways
$\therefore$ The required number of arrangements $=24 \times 60=1440$
8. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is
a) ${ }^{16} \mathrm{C}_{11}$
b) ${ }^{16} \mathrm{C}_{5}$
c) ${ }^{16} \mathrm{C}_{9}$
d) ${ }^{20} \mathrm{C} 9$
e) None of
these
ANSWER: c
If 4 members are excluded then number of available players $=22-4=18$. Out of 18 , two players must be included in the team. So number of players to be selected $=11-2=9$
: 9 players should be selected from $18-2=16$ players and it can be done in ${ }^{16} \mathrm{C}$ ${ }_{9}$ ways
9. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8 , he does not want to borrow Chemistry part II unless Chemistry part I is also borrowed. In how many ways can he choose the three books to be borrowed?
a) 56
b) 27
c) 26
d) 41
e) None of
these

## ANSWER: d

If he borrows Chemistry II then he will borrow Chemistry I. So the third book should be borrowed from remaining 6 books and this can be done in ${ }^{6} \mathrm{C}_{1}=6$ ways If he does not borrow Chemistry II then he can borrow 3 books from the
remaining 7 books and this can be done in $7 \mathrm{C}_{3}=\frac{7 \times 6 \times 5}{1 \times 2 \times 3}=35$ ways
$\therefore$ Total number of ways that he can borrow 3 books $=6+35=41$
Directions (Q. 10-11): Study the given information carefully and answer the questions that follow:

A committee of five members is to be formed out of 3 trainees, 4 professors and 6 research associates. In how many different ways can this be done if 10. The committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?
a) 12
b) 13
c) 24
d) 52
e) None of these

ANSWER: a
Five member team with 4 professors and 1 research associate can be selected in ${ }^{4} \mathrm{C}_{4} \times{ }^{6} \mathrm{C}_{1}=1 \times 6=6$ ways
Five member team with 3 trainees and 2 professors can be selected in ${ }^{3} \mathrm{C}_{3} \times{ }^{4} \mathrm{C}$ 2
$=1 \times 6=6$ ways
. Total number of ways of selecting the committee $=6+6=12$
11. The committee should have 2 trainees and 3 research associates?
a) 15
b) 45
c) 60
d) 9
e) None of these

## ANSWER: c

2 trainees and 3 research associates can be selected in ${ }^{3} \mathrm{C}_{2} \times{ }^{6} \mathrm{C}_{3}=3 \times 20=60$ ways

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Directions (Q. 12-13): Study the following information carefully to answer the questions that follow:

A committee of five members is to be formed out of 4 students, 3 teachers and 2 sports coaches. In how many ways can the committee be formed if 12. Any five people can be selected?
a) 126
b) 45
c) 120
d) 24
e) None of these

ANSWER: a
A committee of 5 out of $4+3+2=9$ members can be selected in $9 C_{5}={ }^{9} C_{4}$

$$
=\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4}=126 \text { ways }
$$

13. The committee should consist of 2 students, 2 teachers and 1 sports coach?
a) 25
b) 64
c) 9
d) 36
e) None of these

ANSWER: d
2 students, 2 teachers and 1 sports coach can be selected in $4 \mathrm{C}_{2} \times{ }^{3} \mathrm{C}_{2} \times{ }^{2} \mathrm{C}_{1}$ $=6 \times 3 \times 2=36$ ways

