MENSURATION-V

RIGHT PRISM:

A prism is a solid whose base is a rectangular polygon and whose faces are rectangular.

The vertical distance between the midpoints of the bases of a prism is called height of the prism.

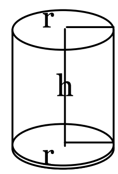
The sum of areas of all rectangular faces is called Lateral Surface Area (L.S.A) or Slant Surface Area (S.S.A).

Lateral Surface Area = Lateral Surface Area + 2$\times$Base Area

Volume of a prism = Area of base$\times$height.

CYLINDER:

Cylinder is a prism whose base is a circle. So the lateral surface will be a curved one. So Lateral Surface Area is called Curved Surface Area



If ‘r’ be the radius of the cylinder and ‘h’ be its height, then

Curved Surface Area (C.S.A) = Circumference of Base$\times$Height

$$= 2\pi r \times h = 2\pi r h$$

∴ Total Surface Area = C.S.A + 2$\times$Base Area

$$= 2\pi r h + 2\pi r^2$$

$$= 2\pi r (r + h)$$

Volume = Base Area$\times$Height

$$= \pi r^2 \times h = \pi r^2 h$$

Important Tip: While calculating the percentage changes in Lateral Surface Area, Total Surface Area and Volume of a prism the constant term π and any numeral in the formula can be neglected.

For e.g. C.S.A of a cylinder can be taken as rh instead of $2\pi rh$, T.S.A can be taken as $r(r+h)$ instead of $2\pi r(r+h)$ and Volume can be taken as $r^2 h$ instead of $\pi r^2 h$.

PYRAMID:

A pyramid is a solid whose base is a plane rectilinear figure and whose side-faces are triangles having a common vertex outside the plane of the base. The length of the perpendic- ular from the vertex to the base is called height, the straight line joining the vertex to the center of the base is called axis and the line segment joining the vertex to the mid-point of any one of the sides of the base.

A pyramid is said to be right pyramid if the perpendicular dropped from vertex meets the base at the central point.

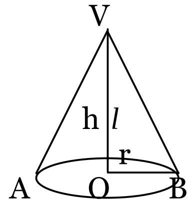
Vol. of right pyramid = $\frac{1}{3}\times$(Area of base$\times$height)

Lateral Surface Area = $\frac{1}{2}$(Perimeter of base$\times$slant height)

Total Surface Area = Lateral Surface area + Base Area

RIGHT CIRCULAR CONE:

Right circular cone is a right pyramid whose base is a circle.



The point v-is called vertex and fixed line vo is called height (h) and the lengths VA and VB are called slant heights (l).

Also $h^2 + r^2 = l^2$

$\Rightarrow \ l = \sqrt{r^2 + h^2}$

$\therefore$ Volume (V) = $\frac{1}{3} \times$ Base Area $\times$ Height

$= \frac{1}{3}(\pi r^2 \times h) = \frac{1}{3}\pi r^2 h$

Curved Surface Area (C.S.A) = $\frac{1}{2}$(Perimeter $\times$ Slant Length)

$= \frac{1}{2}(2\pi r \times l) = \pi rl$

Total Surface Area (T.S.A) = C.S.A + Base Area

$= \pi rl + \pi r^2 = \pi r(r + l)$

SPHERE:

The set of all points in space which are at equidistant from a fixed point is called a sphere.

The fixed point is called the centre of the sphere and the constant distance is called its radius.

The volume of a sphere of radius 'r' is given by V = $\frac{4}{3}\pi r^3$.

The surface area of the sphere is given by S = $4\pi r^2$.

PROBLEMS

1. On increasing the radius of a cylinder by 6 units, the volume increases by x cubic units. On increasing the altitude of the cylinder by 6units, the volume also increases by x cubic units. If the original altitude is 2 units, what is the original radius?

1) 2 units 2) 4 units 3) 6 units 4) 8 units 5) None of these

ANSWER: 3

If r and h are the radius and height of cylinder, then its volume = $\pi r^2 h$

If r increases by 6 then increase in volume = $\pi(r + 6)^2 h - \pi r^2 h = \pi h[(r + 6)^2 - r^2]$

$= \pi h(2r + 6)(6)$

If h increases by 6, then the increase in volume = $\pi r^2(h + 6) - \pi r^2 h = \pi r^2(\cancel{h} + 6 - \cancel{h})$

$= \pi r^2 6$

$\therefore \ \cancel{\pi} h(2r + 6)\cancel{6} = \cancel{\pi} r^2 \cancel{6}$

But h = 2 $\therefore$ 2(2r + 6) = r^2

$r^2 - 4r - 12 = 0$
$(r - 6)(r + 2) = 0$
$\therefore\ r = 6$

2. There are 5 cones and 5 cylinders each of base radius r and height r. What is the number of spheres of radius r that can be molded out of these (assuming each body to be solid)?

1) 3 2) 2 3) 5 4) 4 5) None of these

ANSWER: 3

Total volume of 5 cones and 5 cylinders = $\left(\frac{1}{3}\pi r^2 r + \pi r^2 \times r\right)5$

$$= \frac{4}{3}\pi r^3 \times 5 = \frac{20}{3}\pi r^3$$

If 'n' spheres each of radius 'r' are formed by moulding 5 cones and 5 cylinders, the total

volume of spheres = total volume of cones and cylinders

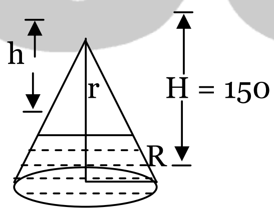
$\therefore\ n \times \frac{4}{3}\cancel{\pi r^3} = \frac{20}{3}\cancel{\pi r^3}$

$\therefore\ n = \frac{20}{3} \times \frac{3}{4} = 5$

3. A conical vessel is lying on a table with its base downwards. The capacity of the vessel is 500 liter and its vertical height is 150 cm. If 244 litre of water is put in the vessel, then what is the height of the water level in the conical vessel above the table?

1) 25 cm 2) 30 cm 3) 35 cm 4) 40 cm 5) None of these

ANSWER: 2



If 'r' and 'R' be the radii of cylinders at the top of water level and the base of cone and h and H are the height of water level from apex and height of cone then $\frac{h}{H} = \frac{r}{R}$

Volume of empty part of cone = 500 – 244 = 256

$$\therefore\ \frac{\text{Volume of upper part}}{\text{Total volume}} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{r^2 h}{R^2 H} = \left(\frac{h}{H}\right)^2 \times \frac{h}{H} = \frac{h^3}{H^3}$$

$$\frac{256}{500} = \left(\frac{h}{H}\right)^3$$

$$\therefore\ \frac{h}{H} = \sqrt[3]{\frac{256}{500}} = \frac{4}{5}$$

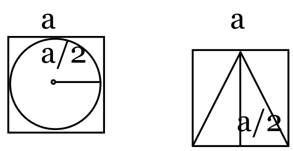
$$\therefore\ h = \frac{4}{5} \times H = \frac{4}{5} \times 150 = 120$$

Water level above table = 150 – 120 = 30

4. There are two identical cubes. Out of one cube, a sphere of maximum volume (V_s) is cut off. Out of the second cube, a cone of maximum volume (V_c) is cut such that its base lies on one of the faces of the cube. Which one of the following is correct?

1) $V_s = V_c$ 2) $V_s = 2V_c$ 3) $2V_s = 3V_c$ 4) $3V_s = 4V_c$ 5) None of these

ANSWER: 2



If the side of each cube be ‘a’ then radius of maximum sphere that can be cut = a/2

$\therefore$ Volume of sphere (V_s) = $\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3}\times\pi\times\frac{a^3}{8} = \frac{\pi a^3}{6}$

Similarly the radius of maximum cone (r) = a/2 and its height = a.

$\therefore$ Volume of maximum sphere (V_c) = $\frac{1}{3}\pi\left(\frac{a}{2}\right)^2 a = \frac{1}{3}\pi\left(\frac{a^2}{4}\right)a = \frac{1}{12}\pi a^3$

$\because \frac{\pi a^3}{12} = \frac{1}{2}\left(\frac{\pi a^3}{6}\right)$

$V_c = \frac{1}{2}V_s$

$V_s = 2V_c$

5. If the diameter of a wire is decreased by 10%, by how much percent (approximately) will the length be increased to keep the volume constant?

1) 5% 2) 17% 3) 20% 4) 23% 5) 21%

ANSWER: 2

Radius and length be 10 cm each. Then volume of wire = $\pi(10^2)(10) = 1000\pi$

If radius is increased by 10% then new radius = $\frac{110}{100}\times 10 = 11$

$\therefore$ If volume is to remain same, then new length should be $\frac{1000\pi}{11^2\pi} = \frac{1000}{121} = 8.26$

$\cong$ 8.3

$\therefore$ Length is decreased by $\left(\frac{10-8.3}{10}\right)\times 100 = 17\%$

6. A right circular cylinder and a right circular cone have equal bases and equal volumes. But the lateral surface area of the right circular cone is 15/8 times the lateral surface area of the right circular cylinder. What is the ratio of radius to height of the cylinder?

1) 3 : 4 2) 9 : 4 3) 15 : 8 4) 8 : 15 5) None of these

ANSWER: 2

If r be the radius of cone and cylinder and h_1 and h_2 be the heights of cylinder and cone.

Given $\pi r^2 h_1 = \frac{1}{3}\pi r^2 h_2$

$\Rightarrow h_2 = 3h_1$

Also $\pi r\sqrt{r^2 + h_2^2} = (2\pi r h_1)\frac{15}{8}$

$\therefore r^2 + h_2^2 = (4h_1^2)\times\frac{225}{64}$

$\therefore r^2 + 9h_1^2 = 4h_1^2 \times \frac{225}{64} = \frac{225h_1^2}{16}$

$r^2 = \frac{225h_1^2}{16} - 9h_1^2 = \frac{81h_1^2}{16}$

$\frac{r^2}{h_1^2} = \frac{81}{16}$

$\therefore$ r : h_1 = 9 : 4