MENSURATION-IV

Theory:

A solid is a figure bounded by one or more surface. Hence a solid has length, breadth and height. The plane surfaces that bind a solid are called its faces. The fundamental difference between a plane figure and a solid figure is that the plane figure lies in a plane and a solid figure lies in space.

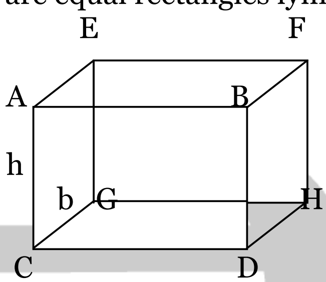
There are two types of three-dimensional figures

(1) The solid figure in which any of the cross section is the same throughout.
E.g. Cube, Cuboid, Cylinder etc.

(2) The solid figure in which none of the cross-sections is same throughout.
E.g. Cone, Sphere, Pyramid etc.

CUBOID:

A cuboid is bounded by 6 rectangular faces. The opposite faces of a rectangular solid are equal rectangles lying in parallel planes.



The areas of three different faces be A_1, A_2 and A_3 then $A_1 = $ lb $A_2 = $ bh $A_3 = $ lh

∴ Surface area = 2 ($A_1 + A_2 + A_3$) = 2 (lb + bh + lh)

Volume = Area of any face×corresponding height

V = lb×h = lbh

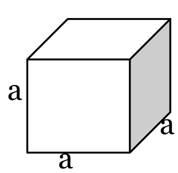
Diagonal (d) = $\sqrt{l^2 + b^2 + h^2}$

Diagonal is the biggest possible dimension of a cuboid.

Also $A_1 \times A_2 \times A_3 = (lb)(bh)(lh) = (lbh)^2 = V^2$

⇒ V = $\sqrt{A_1 A_2 A_3}$

CUBE:



A Cube is bounded by six square faces i.e. if the length ,breadth and height of a cuboid are all equal then it is called a cube.

If each side of the cube is of 'a' units,

then its surface area(S.A) =6a² and Its Volume(V) = a³

Diagonal of cube will be d = $a\sqrt{3}$

PROBLEMS

1. Each edge of a cube is decreased by 20%. The percentage of decrease in the surface area of the cube is

1) 44% 2) 36% 3) 20% 4) 60% 5) None of these

ANSWER: 2

Edge of the cube be 5 then its surface area = 6 × 5^2 = 150

After reduction new edge of the cube = $\left(\frac{100-20}{100}\right)\times 5 = \frac{4}{\cancel{5}}\times\cancel{5} = 4$

∴ New surface area of the cube = 6 × 4^2 = 96

∴ Surface area reduces by $\frac{150-96}{150}\times 100 = \frac{54}{150}\times 100 = 36\%$

Shortcut method:

(i) If each edge of a cube increased by $x\%$ then the surface area increases by

$$S = \left(2x + \frac{x^2}{100}\right)\%$$

(ii) If each edge of a cube decreased by $x\%$ then the surface area decreases by

$$S = \left(2x - \frac{x^2}{100}\right)\%$$

In the above problem x = 20%, then $S = \left(2\times 20 - \frac{20^2}{100}\right)\%$ = (40 – 4)% = 36%

2. A cuboid (3 cm × 4 cm × 5 cm) is cut into unit cubes. The ratio of the total surface area of all the unit cubes to that of the cuboid is

1) 180 : 3 2) 180 : 9 3) 180 : 36 4) 180 : 47 5) None of these

ANSWER: 4

The dimensions of a cuboid are 3 × 4 × 5

∴ Its surface area (S.A) = 2(3×4 + 4×5 + 3×5) = 94 cm^2

If the cuboid is cut into unit cubes, then the number of unit cubes so formed = 3 × 4 × 5

= 60

But surface area of each unit cube = 6 × 1^2 = 6

∴ Total surface area of unit cubes = 6 × 60 = 360

Required ratio = 360 : 94 = 180 : 47

3. If the diagonal of a cube is $10\sqrt{3}$ cm, then its surface area will be

1) 500 cm^2 2) 550 cm^2 3) 600 cm^2 4) 650 cm^2 5) None of these

ANSWER: 3

Diagonal (d) of a cube = $10\sqrt{3}$

∴ Its side a = $\frac{d}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}}$ = 10

Surface area (S.A) of cube = 6a^2 = 6 × 10^2 = 600 cm^2

4. If the volume of a cube is 216 cm^2, then the surface area of the cube will be

1) 214 cm^2 2) 216 cm^2 3) 218 cm^2 4) 220 cm^2 5) None of these

ANSWER:

Volume (V) of a cube = a^3 = 216

∴ a = $\sqrt[3]{216}$ = 6

Its surface area = $6a^2 = 6 \times 6^2 = 216$ cm²

5. If six cubes, each of 10 cm edge, are joined end to end, then the surface area of the resulting solid will be

1) 3600 cm^2 2) 3000 cm^2 3) 2600 cm^2 4) 2400 cm^2 5) None of these

ANSWER: 3

When six cubes are joined end to end, a cuboid will be formed whose length is 6×10 = 60 cm, breadth 10 cm and height 10 cm respectively i.e. l = 60, b = 10 & h = 10

∴ Surface area of cuboid = 2 (60×10 + 10×10 + 60×10)

= 2(600 + 100 + 600) = 2600 sq cm

6. If three cubes of copper, each with an edge of 6 cm, 8 cm and 10 cm respectively are melted to form a single cube, then the diagonal of the new cube will be

1) 18 cm 2) 19 cm 3) 19.5 cm 4) 20.8 cm 5) None of these

ANSWER: 4

If three cubes are melted to form a single larger cube then the volume of larger cube so formed will be equal to the sum of the volumes of the three cubes.

∴ Volume of the larger cube = $6^3 + 8^3 + 10^3$ = 216 + 512 + 1000 = 1728

∴ Side of larger cube = $\sqrt[3]{1728}$ = 12

∴ Diagonal of larger cube = $12\sqrt{3}$ = 12 × 1.732 = 20.8 cm

7. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is

1) 408 m³ 2) 360 m³ 3) 270 m³ 4) 208 m³ 5) None of these

ANSWER: 3

The cross-section of the swimming pool is a trapezium whose parallel sides are 1 m and 4 m and having a perpendicular distance of 9 m.

∴ Area of cross-section = $\left(\frac{1+4}{2}\right) \times 9$ = 22.5 sq m

∴ Volume of swimming pool = 22.5 × 12 = 270 cu.m

8. The length, breadth and height of a cuboid are in the ratio 1 : 2 : 3. The length, breadth and height of the cuboid are increased by 100%, 200% and 200% respectively. Then the increase in the volume of the cuboid is

1) 5 times 2) 6 times 3) 12 times 4) 17 times 5) None of these

ANSWER: 4

Length, breadth and height of cuboid be x, 2 x and 3 x respectively, then its volume = $x \times 2x \times 3x = 6x^3$

When length, breadth and height are increased by 100%, 200% and 300% respectively,

New length = $\left(\frac{100+100}{100}\right)\times x = 2x$

New breadth = $\left(\frac{100+200}{100}\right)\times 2x = 6x$

New height = $\left(\frac{100+300}{100}\right)\times 3x = 9x$

New volume = $2x\times 6x\times 9x = 108x^3$

∴ Increase in volume = $\left(\frac{108x^3 - 6x^3}{6x^3}\right) = \frac{102x^3}{6x^3}$ = 17 times

9. A cube of lead with edges measuring 6 cm each is melted and formed into 27 equal cubes. What will be the length of the edges of the new cubes?

1) 3 cm 2) 4 cm 3) 2 cm 4) 1 cm 5) None of these

ANSWER: 3

The edge of each smaller cube be 'a'.

Then total volume of 27 cubes = $27a^3$

But total volume of 27 cubes is equal to volume of cube of edge 6 cm

⇒ $27a^3 = 6^3 = 216$

$a^3 = \frac{216}{27} = 8$

∴ a = 2

10. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is

1) 120 cm^3 2) 64 cm^3 3) 48 cm^3 4) 24 cm^3 5) None of these

ANSWER: 3

The edges of cuboid are in the ratio of 1 : 2 : 3. So the edges can be assumed as x, $2x$ and $3x$

Surface area (S.A) = $2(x\times 2x + 2x\times 3x + x\times 3x) = 2(11x^2) = 22x^2$

∴ $22x^2 = 88$ ⇒ $x = 2$

∴ The dimensions of cuboid will be 2, 4 and 6. The volume of cuboid = 2×4×6 = 48 cm^3.

11. The areas of three adjacent faces of a cuboid are a,b and c. If the volume of the cuboid is V, then V^2 is equal to

1) abc 2) (ab + bc + ca) 3) $\frac{c}{ab}$ 4) (a + b + c) 5) None of these

ANSWER: 1

If the three adjacent dimensions are x, y and z, then $x\times y$ = a y×z = b $x\times z$ = c

∴ xy = a yz = b xz = c

(xy) (yz) (zx) = abc

$x^2y^2z^2 = abc$

But $x^2y^2z^2 = V^2$

$\therefore$ $V^2 = abc$

12. The sum of the length, breadth and depth a cuboid is 19 cm and its diagonal is $5\sqrt{5}$. Its surface area is

1) 361 cm^2 2) 125 cm^2 3) 236 cm^2 4) 144 cm^2 5) None of these

ANSWER: 3

If l, b and h are the three dimensions of cuboid then l + b + h = 19,

$\sqrt{l^2+b^2+h^2} = 5\sqrt{5}$

$\therefore$ $(l+b+h)^2 = l^2+b^2+h^2+2(lb+bh+hl)$

$\therefore$ $19^2 = \left(5\sqrt{5}\right)^2 + 2(lb + bh + hl)$

$\therefore$ $2(lb + bh + lh) = 361 - 125 = 236\ cm^2$

$\therefore$ Surface area = 236 cm^2

13. What is the time needed to empty a cubodial water reservoir 10 m long, 9 m wide and 3m deep at the rate of 45 L/m?

1) 100 h 2) 90 h 3) 80 h 4) 60 h 5) None of these

ANSWER: 1

Volume of water in reservoir = 10×9×3 = 270 cu. m

= 270×1000 lit = 270000 lit [$\because$ 1 cu. m = 1000 lit]

Time to empty tank = $\frac{270000}{45}$ = 6000 min = $\frac{6000}{60}$ = 100 h