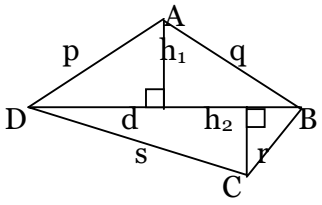
MENSURATION-II

Definitions and formulae:

Any closed two dimensional figure formed by four straight lines is called a quadrilateral.

The sum of the angles of a quadrilateral is equal to 360°.

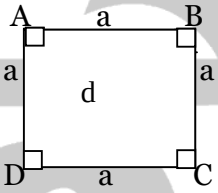


In the above quadrilateral Perimeter (P) = p + q + r + s

$$\text{Area (A)} = \frac{1}{2}\times d\times h_1 + \frac{1}{2}\times d\times h_2$$

$$= \frac{1}{2}\times d\times (h_1 + h_2)$$

SQUARE: Square is a quadrilateral in which all the four sides are equal and the angle between any two adjacent sides is 90°.



Perimeter (P) = 4a

$$\text{Area } A = \frac{1}{2}\times a\times a + \frac{1}{2}\times a\times a = a^2$$

$$d^2 = a^2 + a^2$$

$$\Rightarrow \quad d = \sqrt{2}\,a$$

$$\therefore A = a^2 = \frac{1}{2}d^2$$

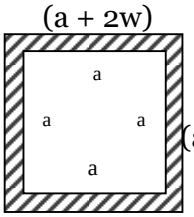
$$\text{Also } P^2 = 16A \quad \Rightarrow \quad A = \frac{P^2}{16}$$

Cost of fencing a square plot/field is equal to the product of perimeter and unit cost of fencing i.e. *Total Cost* = *Perimeter* × *Unit Cost* per metre

Similarly the cost of levelling a square plot/field is equal to the product of its area and unit cost.

∴ Cost of levelling = Area × Unit Cost per square metre

Area of Path: If a path of uniform width 'w' runs around all the four sides of a square of side 'a' then the area of the path (A_p) is given by



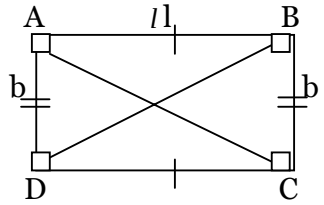
$$A_p = (a+2w)^2 - a^2$$

$$= 2w\,(2a + 2w)$$

$$= 4w\,(a + w)$$

RECTANGLE: A quadrilateral having two pairs of equal opposite sides and a right angle between any two adjacent sides is called a rectangle.

The longer dimension is called the **length (*l*)** and the shorter dimension is called the **breadth (b)**.



The line joining any two opposite sides is called the diagonal (d).

Perimeter (P) = 2 (l + b)

Area (A) = l × b = lb

$l^2 + b^2 = d^2$

$\Rightarrow$ d = $\sqrt{l^2 + b^2}$

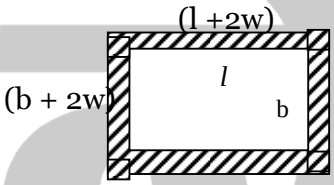
Also $(l+b)^2 = l^2 + b^2 + 2lb$

$\left(\frac{P}{2}\right)^2 = d^2 + 2A$

$\Rightarrow$ A = $\frac{1}{2}\left(\frac{P^2}{4} - d^2\right) = \frac{(P^2 - 4d^2)}{8}$

Area of Path:

If a path of uniform width (w) runs around a rectangular plot of length (l) and breadth (b) then the Area of Path (A_p) is given by

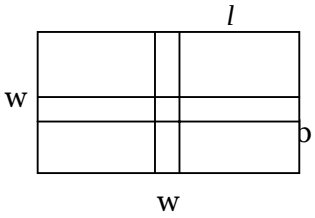


$A_p = (l+2w)(b+2w) - lb$

= 2w (l + b + 2w)

If the path runs inside the plot then its area (A_p) = 2w (l + b – 2w)

If two perpendicular paths of uniform width (w) run inside a rectangle, one parallel to length and the other parallel to the breadth, then its Area is given by



$A_p = l \times w + b \times w - w \times w$

= w (l + b – w)

If the length of a rectangle changes by l% and breadth by b% then the percentage change in Area (A) is given by

A = $l + b + \frac{lb}{100}$

Use (+) for increase and (-) for reduction.

PROBLEMS

1. If the length of a rectangle is increased in the ratio 6 : 7 and its breadth is diminished in the ratio 5 : 4 then its area will be diminished in the ratio

1) 17 : 16 2) 15 : 14 3) 9 : 8 4) 8 : 7 5) None of these

ANSWER: 2

The lengths of the rectangle be $6x$ and $7x$ and the breadths be 5y and 4y respectively.

Then its original area = $6x \times 5y = 30xy$

Its new area = $7x \times 4y = 28xy$

$\therefore$ Ratio of areas = $30xy : 28xy$ = 15 : 14

2. If each of the length and breadth of a rectangle is increased by 50%, by what percent does its area increase?

1) 125 2) 100 3) $55\frac{5}{9}$ 4) 50 5) None of these

ANSWER: 1

l and b be the length and breadth of the rectangle, then its area = $l \times b = lb$

If length and breadth are increased by 50% then their values are

$\left(\frac{100+50}{100}\right) \times l = \frac{15l}{10}$

= $1.5l$ and $\left(\frac{100+50}{100}\right) \times b = \frac{15b}{10} = 1.5b$ respectively. Then its new area = 1.5 × 1.5b =

$2.25lb$

Increase in area = $\left(\frac{2.25lb - lb}{lb}\right) \times 100 = \frac{1.25lb}{lb} \times 100$ = 125%

3. The perimeter of a rectangle and a square are 160 m each. The area of the rectangle is less than that of the square by 100 sq m. The length of the rectangle is

1) 30 m 2) 60 m 3) 40 m 4) 50 m 5) None of these

ANSWER: 4

Perimeter of square = 160

$\therefore$ Its side = $\frac{160}{4}$ =40

Area of square = $40^2 = 1600$

$\therefore$ Area of rectangle = 1600 – 100 = 1500

Perimeter = P = 2(l + b) = 160

$\therefore$ (l + b) = 80

$(l-b)^2 = (l+b)^2 - 4lb = 6400 - 4 \times 1500 = 400$

$l - b = \sqrt{400} = 20$

$\therefore$ $l = \frac{(l+b)(l-b)}{2} = \frac{80+20}{2} = 50$

4. The area of a square is 225 sq cm which is equal to the area of a rectangle. The length of the rectangle is 16 cm more than the breadth of the rectangle. What is the respective ratio between the side of the square and the breadth of the rectangle is

1) 3 : 5 2) 5 : 3 3) 5 : 4 4) 4 : 5 5) None of these

ANSWER: 2

Area of square = 225

$\therefore$ Its side = a = $\sqrt{225}$ = 15

If x be the breadth of the rectangle then its length will be $x+16$

$\therefore$ Area $= (x+16)x = 225$

$x^2 + 16x - 225 = 0$

$x^2 + 25x - 9x - 225 = 0$

$x(x+25) - 9(x+25) = 0$

$\Rightarrow x = 9$

$\therefore$ Required ratio = 15 : 9 = 5 : 3

5. The breadth of a rectangular is three-fourths of its length. If the area of the floor is 768 sq m then the difference between the length and breadth of the hall is:

1) 8 meters 2) 12 meters 3) 24 meters 4) 32 meters 5) None of these

ANSWER: 1

The length of the rectangle be $4x$ then its breadth $\frac{3}{4}(4x) = 4x$

Then its area $= 4x \times 3x = 12x^2 = 768$

$\therefore x^2 = \frac{768}{12} = 64$

$\therefore x = 8$

Difference between length and breadth == 8

6. The perimeter of a rectangle is 60 cm and its breadth is 12 cm. What is the area of the rectangle?

1) 261 cm^2 2) 263 cm^2 3) 213 cm^2 4) 216 cm^2 5) None of these

ANSWER: 4

If l and b be the length and breadth of the rectangle, then $2(l + b) = 60$

Then $2(l + b) = 60$

$\therefore l + b = 30$ Also b = 12

$\Rightarrow l = 30 - 12 = 18$

Area (A) $= l \times b = 18 \times 12 = 216$

7. The respective ratio of the length and breadth of a rectangular plot is 3 : 2. If the length of the plot is 40 meters more than its breadth, what is the perimeter of the rectangular plot?

1) 200 meter 2) 400 meter 3) 500 meter 4) Cannot be determined 5) None of these

ANSWER: 2

Length and breadth of the rectangle be $3x$ and $4x$

Given that $3x - 2x = 40$ $\therefore x = 40$

$\Rightarrow l = 3 \times 40 = 120$ b = $2 \times 40 = 80$

Perimeter (P) $= 2(l + b) = 2(120 + 80) = 400$ m

8. The area of a square is three times the area of a rectangle. The length of the rectangle is 5 cm more than $(5)^2$ which is three times its breadth. What is the perimeter of the square?

1) 240 cm 2) 60 cm 3) 120 cm 4) Cannot be determined 5) None of these

ANSWER: 3

$l = 5^2 + 5 = 30$ $\therefore$ b $= \frac{30}{3} = 10$

Area of rectangle = 30 × 10 = 300

$\therefore$ Area of square = 3 × 300 = 900

$\therefore$ Its side = $\sqrt{900}$ =30

Perimeter of the square = 4 × 30 = 120

9. The area of a square is four-third the area of a rectangle. If the area of the square is 1024 sq cm and the length of the rectangle is 64 cm, what is the difference between the breadth of the rectangle and the side of the square?

1) 18 cm 2) 24 cm 3) 15 cm 4) 20 cm 5) None of these

ANSWER: 4

Area of square = 1024

$\therefore$ Its side = a = $\sqrt{1024}$ = 32

Area o rectangle = $\frac{3}{4} \times 1024 = 3 \times 256 = 768$

Length of rectangle = 64

$\therefore$ Its breadth = $\frac{768}{64} = 12$

Required difference = 32 – 12 = 20

10. The area of a triangle is half the area of a square. The perimeter of the square is 224 cm. What is the area of the triangle?

1) 1856 cm^2 2) 1658 cm^2 3) 1558 cm^2 4) 1586 cm^2 5) None of these

ANSWER: 5

Perimeter of square = 224

$\therefore$ Its side = $\frac{224}{4} = 56$

$\therefore$ Square area = 56^2

$\therefore$ Area of triangle = $\frac{1}{2} \times 56 \times 56 = \frac{3136}{2} = 1568 cm^2$

11. The angles of a quadrilateral are in the ratio of 2 : 4 : 7 : 5. The smallest angle of the quadrilateral is equal to the smallest angle of a triangle. One of the angles of the triangle is twice the smallest angle of the triangle. What is the second largest angle of the triangle?

1) 80° 2) 60° 3) 120° 4) Cannot be determined 5) None of these

ANSWER: 2

Smallest angle of quadrilateral = $\left(\frac{360}{2+4+7+5}\right) \times 2 = 40$

$\therefore$ smallest angle of triangle = 40°

One of the remaining angles = 2×40 = 80

$\therefore$ Remaining angle = 180 – (40 + 80) = 60°

Second largest angle of triangle = 60°

12. If each side of a square is increased by 10%, its area will be increased by
1) 10% 2) 21% 3) 44% 4) 100% 5) None of these

ANSWER: 2

The original side of the square be 10.

Then its area = $10^2 = 100$

Side after increase = $\frac{(100+10)}{100}\times 10 = \frac{110}{100}\times 10 = 11$

$\therefore$ New area = $11^2 = 121$

$\therefore$ Increase in area = 21%