MENSURATION

Some Important Definitions and formulae:
1. If any closed figure has three sides then it is called a *triangle.*
2. In a triangle the sum of three angles is 180°.
3. In a triangle the sum of the lengths of any two sides should be more than the third side.
4. Similarly the difference between any two sides of a triangle is less than the third side.
5. The side on which a triangle rests is called the base. The length of the perpendicular drawn on the base from opposite vertex is called the height of the triangle.
6. If the three sides of a triangle have three different lengths then it is called a *scalene triangle.*
7. If exactly two sides of a triangle are equal and the third side has different length then it is called an *isosceles triangle.*
8. If all the three sides of a triangle are equal then it is called an *equilateral triangle.*

SCALENE TRIANGLE
Perimeter (P) = a + b + c

Area (A) = $\sqrt{s(s-a)(s-b)(s-c)}$

where s = $P/2$

Area (A) = $\frac{1}{2} \times b \times h$

ISOSCELES TRIANGLE
Perimeter (P) = a + a + b = 2a+b

Area (A) = $\frac{b}{4}\sqrt{4a^2 - b^2}$

A = $\frac{1}{2} \times b \times h$

$\Rightarrow$ h = $\frac{1}{2}\sqrt{4a^2 - b^2}$

EQUILATERAL TRIANGLE
Perimeter (P) = a + a + a = 3a

Area (A) = $\frac{\sqrt{3}}{4}a^2$

A = $\frac{1}{2} \times a \times h$

$\Rightarrow$ h = $\frac{\sqrt{3}}{2}a$

RIGHT ANGLED TRIANGLE
Perimeter (P) = a + b + c
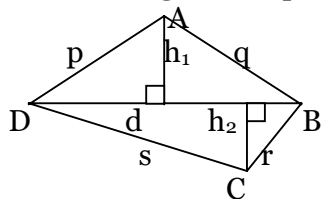
Area (A) = $\frac{1}{2} \times a \times b$

$\therefore$ $c^2 = a^2 + b^2$

$\Rightarrow$ $c = \sqrt{a^2 + b^2}$

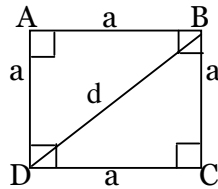Any closed two dimensional figure formed by four straight lines is called a quadrilateral.

The sum of the angles of a quadrilateral is equal to 360°.

In the above quadrilateral Perimeter (P) = p + q + r + s

$$\text{Area (A)} = \frac{1}{2} \times d \times h_1 + \frac{1}{2} \times d \times h_2 = \frac{1}{2} \times d \times (h_1 + h_2)$$

SQUARE: Square is a quadrilateral in which all the four sides are equal and the angle between any two adjacent sides is 90°.

Perimeter (P) = 4a

$$\text{Area } A = \frac{1}{2} \times a \times a + \frac{1}{2} \times a \times a = a^2$$

$$d^2 = a^2 + a^2$$
$$\Rightarrow d = \sqrt{2}\,a$$

Cost of fencing a square plot/field is equal to the product of perimeter and unit cost of fencing i.e. *Total Cost = Perimeter × Unit Cost* per meter. Similarly the cost of levelling a square plot/field is equal to the product of its area and unit cost.

∴ Cost of levelling = Area × Unit Cost per square meter

RECTANGLE: A quadrilateral having two pairs of equal opposite sides and a right angle between any two adjacent sides is called a rectangle.

The longer dimension is called the **length (l)** and the shorter dimension is called the **breadth (b)**.
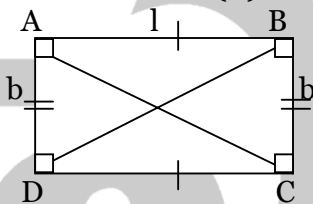
The line joining any two opposite sides is called the diagonal (d).
Perimeter (P) = 2 (l + b)
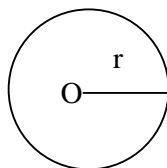Area (A) = l × b = lb
$$l^2 + b^2 = d^2$$
$$\Rightarrow d = \sqrt{l^2 + b^2}$$

CIRCLE:

A circle is a geometrical figure consisting of all those points in a plane which are at a given distance from a fixed point in the same plane. The fixed point is called the centre and the constant distance is known as the *radius*.

A circle with centre O and radius r is generally denoted by C (O, r)

CIRCUMFERENCE:

The perimeter of a circle is called its *Circumference*.

The ratio of the Circumference (C) of a circle and its diameter (D) is always constant, which is π.

$$\frac{C}{D} = \pi$$

$\Rightarrow$ C = πD = $2\pi r$

AREA OF A CIRCLE:

The Area (A) of a circle of radius (r) is given by A = πr^2

PROBLEMS

1. The largest and the smallest angles of a triangle are in the ratio of 3 : 1 respectively. The second largest angle of the triangle is equal to 44°. What is the value of 150 per cent of the largest angle of the triangle?

    1) 149   2) 129   3) 153   4) 173   5) None of these

ANSWER: 3

Sum of the angles of a triangle = 180°

Second largest angle = 44°

∴ Sum of the largest and the smallest angle = 180 − 144 = 136

But they are in the ratio of 3 : 1

∴ Largest angle = $\dfrac{136}{(3+1)} \times 3$ = 102

∴ Required answer = $\dfrac{150}{100} \times 102$ = 153

2. If the perimeter of a rectangular field is 80 meters and the breadth and length is in the ratio of 2 : 3 respectively. What is the area of the field?

    1) 360 sq meters   2) 430 sq meters   3) 384 sq meters 4) 160 sq meters 5) None of these

ANSWER: 3

Breadth (b), Length (l) of the rectangles be $2x$ and $3x$

Then its perimeter = 2($3x + 2x$) = $10x$

$\Rightarrow$ $10x$ = 80   ∴ $x = \dfrac{80}{10} = 8$

∴ b = 2×8 = 16 l = 3×8 = 24

∴ Area (A) = l×b = 24×16 = 384 sq m

3. The area of a square is 1024 sq cm. What is the respective ratio between the length and the breadth of a rectangle whose length is twice the side of the square and breadth is 12 cm less than the side of the square?

    1) 5 : 18   2) 16 : 7   3) 14 : 5   4) 32 : 5   5) None of these

ANSWER: 5

Side of square = $\sqrt{1024}$ = 32

∴ Length of rectangle = 2×32 = 64

Breadth of the rectangle = 32 − 12 = 20

∴ Required ratio = 64 : 20 = 16 : 5

4. The length of a rectangular field is 53 metres and the breadth is 28 metres. What will be the cost of putting a grass bed across the whole field if the cost of putting a grass bed per square bed per square metre is ` 27?

    1) ` 40,098    2) ` 40,048   3) ` 40,058    4) ` 40,088   5) None of these

    ANSWER: 5

    Length and breadth of the rectangle are 53 m and 28 m

    ∴ Its area = 53×28 = 1484 sq m

    Unit cost of putting grass bed = ` 27 per sq m

    ∴ Its total cost = 1484×27 = ` 40068

5. The breadth of a rectangular field is 25 meters. The total cost of putting a grass bed on this field was ` 12,375 at the rate of ` 15 per sq meter. What is the length of the rectangular field?

    1) 27 meters   2) 30 meters   3) 33 meters   4) 32 meters   5) None of these

    ANSWER: 3

    Total cost of putting grass bed = ` 12,375

    Unit cost = 15/sq m

    ∴ Area of the rectangular field = $\dfrac{12375}{15}$ = 825

    ∴ Length of the rectangular field = $\dfrac{Area}{Breadth}$ = $\dfrac{825}{25}$ = 33 m

6. The area of a square is four times the area of a rectangle. The length of the rectangle is 25 cm and its breadth is one cm less than one-fifths of its length. What is the perimeter of the square?

    1) 40 cm   2) 60 cm   3) 160 cm   4) Can't be determined   5) None of these

    ANSWER: 5

    Length of rectangle = 25

    Its breadth = $\dfrac{1}{5}$(25) − 1 = 4

    ∴ Area of rectangle = 25×4 = 100

    ∴ Area of the square = 100×4 = 400

    ⟹ Its side = $\sqrt{400}$ = 20

    Perimeter of the square = 4×20 = 80

7. The side of a square is twice the length of a rectangle of area 828 sq cm. Perimeter of the rectangle is 118 cm. What is the perimeter of the square?

    1) 184 cm   2) 288 cm   3) 144 cm   4) Can't be determined   5) None of these

    ANSWER: 2

    l and b be the length and breadth of the rectangle

    l×b = 828   2(l+b) = 118   ∴ l + b = 59

    (l − b)² = (l+b)² − 4lb = 59² − 4×828 = 3481 − 3312 = 169

    ∴ l − b = $\sqrt{169}$ = 13

$$\Rightarrow\ l = \frac{(l+b)+(l-b)}{2} = \frac{59+13}{2} = 36$$

∴ Side of square = 2×36 = 72

$\Rightarrow$ Its perimeter = 4×72 = 288

8.   The area of a circular plot is twice the area of a rectangular plot. If the area of the rectangular plot is 11088 sq meters, what is the circumference of the circular plot?

   1) 484 meters   2) 572 meters   3) 528 meters   4) 440 meters   5) None of these

ANSWER: 3

Area of circular plot = 2×11088 = 22176

If 'R' be the radius of the circle then $\frac{22}{7} \times R^2$ = 22176

$\Rightarrow\ R^2 = \frac{22176 \times 7}{22}$ = 1008×7 = 7×144×7

∴ R = $\sqrt{7 \times 144 \times 7}$ = 7×12 = 84

∴ Its circumference = $2 \times \frac{22}{7} \times 84$ = 528 m