

AVERAGE

Definition: Average (A) is the ratio between sum (S) of the quantities and the number (N) of quantities.

$$\therefore A = \frac{S}{N}$$
$$\Rightarrow S = A \times N$$

Formulae:

1. The average of n-consecutive even integers or n-consecutive odd integers is equal to the middle number if n is odd.

2. The average of n-consecutive even integers or n-consecutive odd integers is equal to the Average of middle two numbers if n is even.

Note: In the above case if the average is x the middle two numbers will be (x - 1) and (x + 1) respectively.

3. The average of a group of n-quantities is A. If one more quantity whose value is x is added to the group such that the average increases by 'i' then $x = A + i \times (n + 1)$

Note: In the above case if there is a reduction in the average then take 'i' as negative.

4. The average of a group of n-quantities is A. If a quantity whose value is y is replaced by another quantity whose value is x such that the average increases by 'i'.

Then $x = y + i \times n$

Note 1: If there is a reduction in average take 'i' as negative.

Note 2: The original average 'A' has no effect on the equation.

5. The average weight of a group of n-quantities is A but while taking the values one quantity 'p' is erroneously read as 'q'. Then the actual average = $A - \left(\frac{q - p}{n} \right)$

6. The average of first n-positive integers = $\frac{n + 1}{2}$

7. The average of first n-positive even integers = $n + 1$

8. The average of first n-positive odd integers = n

9. The average of squares of first n-numbers = $\frac{(n + 1)(2n + 1)}{6}$

PROBLEMS

1. Find the average of the following set of scores.

216, 463, 154, 605, 446, 336
1) 387 2) 370 3) 379 4) 380 5) None of these

ANSWER: 2

$$S = 216 + 463 + 154 + 605 + 446 + 336 = 2220$$

$$N = 6$$

$$\therefore A = \frac{S}{N} = \frac{2220}{6} = 370$$

2. The average of four consecutive even numbers A, B, C and D respectively is 55. What is the product of A and C?
1) 2652 2) 3248 3) 3024 4) 2808 5) None of these

ANSWER: 5

The average of A, B, C and D = Average of B and C
But B and C are consecutive even numbers. Their average will be equal to the odd number in between them (which is 55)

$$\therefore B = 55 - 1 = 54 \quad C = 55 + 1 = 56$$

$$\Rightarrow A = B - 2 = 54 - 2 = 52$$

$$\Rightarrow A \times C = 52 \times 56 = 2912$$

3. Average of four consecutive odd numbers is 106. What is the third number in ascending order?
1) 107 2) 111 3) 113 4) Cannot be determined 5) None of these

ANSWER: 1

A, B, C and D be the four consecutive odd numbers in ascending order.

Their average = Average of B and C = the even number between B and C = 106

$$\text{But } B = 106 - 1 = 105 \quad \text{and } C = 106 + 1 = 107$$

$$\therefore \text{The third number in ascending order} = C = 107$$

4. The average of odd numbers up to 100 is
1) 50.5 2) 50 3) 49.5 4) 49 5) None of these

ANSWER: 2

From 1 to 100 the number of odd numbers = 50

But the sum of first n-odd numbers = n^2

$$\therefore \text{Their average} = \frac{n^2}{n} = n$$

$$\therefore \text{The average of first 50 odd numbers} = 50$$

5. The sum of five consecutive numbers is 190. What is the sum of the largest and the smallest number?
1) 75 2) 77 3) 76 4) 73 5) None of these

ANSWER: 3

The sum of five consecutive numbers = 190

$$\therefore \text{Their average} = \frac{190}{5} = 38$$

In a set of consecutive numbers the average of the largest and the smallest numbers is equal to the average of all the numbers.

$$\Rightarrow \text{Average of the largest and the smallest} = 38$$

$$\therefore \text{Their sum} = 38 \times 2 = 76$$

6. The average of eight successive numbers is 6.5. The average of the smallest and the greatest numbers among them will be
1) 4 2) 6.5 3) 7.5 4) 9 5) None of these

ANSWER: 2

The average of the smallest and the largest of a set of n-consecutive numbers is equal to the average of given set of numbers.

∴ Average of the smallest and the largest = 6.5

7. The average of five numbers is 49. The average of the first and the second numbers is 48 and the average of the fourth and fifth numbers is 28. What is the third number?

- 1) 92 2) 91 3) 95 4) Cannot be determined 5) None of these

ANSWER: 5

A, B, C, D and E be the five numbers

$$A + B + C + D + E = 49 \times 5 = 245$$

$$\text{The sum of A and B} = A + B = 48 \times 2 = 96$$

$$\text{The sum of D and E} = D + E = 28 \times 2 = 56$$

$$\therefore C = (A + B + C + D + E) - (A + B + D + E) = 245 - (96 + 56) = 93$$

8. The average age of a man and his son is 48 years. The ratio of their ages is 2 : 1 respectively. What is the son's age?

- 1) 28 years 2) 35 years 3) 24 years 4) 32 years 5) None of these

ANSWER: 4

$$\text{Total age of man and son} = 48 \times 2 = 96$$

$$\text{Ratio of their ages} = 2 : 1$$

$$\therefore \text{Son's age} = \frac{96}{(2+1)} \times 1 = \frac{96}{3} \times 1 = 32$$

9. Average score of Rahul, Manish and Suresh is 63. Rahul's score is 15 less than Ajay and 10 more than Manish. If Ajay scored 30 marks more than the average score of Rahul, Manish and Suresh, what is the sum of Manish's and Suresh's score?

- 1) 120 2) 111 3) 117 4) Can't be determined 5) None of these

ANSWER: 2

$$\text{Total score of Rahul, Manish and Suresh} = 63 \times 3 = 189$$

$$\therefore \text{Ajay score is 30 more than the above average} \text{ Ajay's score} = 63 + 30 = 93$$

$$\text{Rahul's score is 15 less than Ajay's score, so Rahul's score} = 93 - 15 = 78$$

$$\therefore \text{Sum of Manish's and Suresh's score} = 189 - 78 = 111$$

10. The average marks of a student in seven subjects is 41. After reevaluation in one subject the marks were changed to 42 from 14 and in remaining subjects the marks remain unchanged. What are the new average marks?

- 1) 45 2) 44 3) 46 4) 47 5) None of these

ANSWER: 1

$$\text{After reevaluation, increase in total marks} = 42 - 14 = 28$$

$$\Rightarrow \text{Increase in the average} = \frac{28}{7} = 4$$

$$\therefore \text{New average} = 41 + 4 = 45$$

11. The average of 10 numbers is calculated as 15. It is discovered later on that while calculating the average one number, namely 36, was wrongly read as 26. The correct average is

- 1) 20 2) 18 3) 16 4) 14 5) None of these

ANSWER: 3

A number 36 was wrongly read as 26

∴ Reduction in total due to error = $36 - 26 = 10$

∴ Reduction due to error in average = $\frac{10}{10} = 1$

⇒ Actual average = $15 + 1 = 16$

12. The average age of 24 boys in a class is 11. When the teacher's age is included, the average increases by one. What is the age of the teacher?

- 1) 34 years 2) 42 years 3) 36 years 4) 48 years 5) None of these

ANSWER: 3

Total age of 24 boys = $24 \times 11 = 264$

The average age of 24 boys and teacher = $11 + 1 = 12$

∴ Total age of 24 boys and teacher = $25 \times 12 = 300$

⇒ Teacher's age = $300 - 264 = 36$

SHORTCUT METHOD:

$N = 24$ $A = 11$ $i = +1$ $x = ?$

∴ $x = A + i(N + 1) = 11 + 1 \times (24 + 1) = 11 + 25 = 36$

13. Average weight of 25 persons is increased by 1 kg when one man weighing 60 kg is replaced by a new person. Weight of new person is

- 1) 50 kg 2) 61 kg 3) 86 kg 4) 85 kg 5) None of these

ANSWER: 4

Applying the formula discussed in theory

$N = 25$ $i = +1$ $y = 60$ $x = ?$

∴ $x = y + i \times N = 60 + 1 \times 25 = 85$ kg

14. The average of the first 100 positive integers is

- 1) 100 2) 51 3) 50.5 4) 49.5 5) None of these

ANSWER: 3

The sum of first n-positive integers = $\frac{n(n+1)}{2}$

∴ Their average = $\frac{n(n+1)}{2n} = \frac{n+1}{2}$

⇒ The sum of first 100 positive integers = $\frac{100+1}{2} = \frac{101}{2} = 50.5$