

MATHEMATICS PAPER IA.- MAY 2011.
ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY.

TIME : 3hrs

Max. Marks.75

Note: This question paper consists of three sections A, B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS.

10X2 =20

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f: A \rightarrow B$ is surjection defined by $f(x) = \cos x$ then find B
2. Find the domain of the function $f(x) = \log(x^2 - 4x + 3)$
3. If $a = 2i + 5j + k$ and $b = 4i + mj + nk$ are collinear vectors then find the value of m and n
4. If $a = i + 2j - 3k$ and $b = 3i - j + 2k$ then S.T. $(a+b)$, $(a-b)$ are mutually perpendicular
5. Find vector equation of the plane passing through points $(1, -2, 5)$, $(0, -5, -1)$ and $(-3, 5, 0)$
6. If $\cos\theta + \sin\theta = \sqrt{2} \cos\theta$ prove that $\cos\theta - \sin\theta = \sqrt{2} \sin\theta$
7. Find the value of $\cos 42^\circ + \cos 78^\circ + \cos 162^\circ$
8. If $\cosh x = 5/2$ then find i) $\cosh(2x)$ ii) $\sinh 2x$
9. In $\triangle ABC$ express $\sum r_1 \cot(A/2)$ in terms of "s"
10. If $Z_1 = -1$, $Z_2 = i$ then find the value of $\arg(Z_1/Z_2)$

SECTION B

SHORT ANSWER TYPE QUESTIONS.

5X4 =20

Note : Answer any FIVE questions. Each question carries 4 marks.

11. If ABCDEF is a regular hexagon with centre G, then prove that

$$AB + AC + AD + AE + AF = 3AD = 6AG$$

12. If $a = i - 2j - 3k$, $b = 2i + j - k$ and $c = i + 3j - 2k$ verify $a \times (b \times c) \neq (a \times b) \times c$

13. If $A + B = 45^\circ$ then S.T. $(1 + \tan A)(1 + \tan B) = 2$ hence deduce that

$$\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$$

14. Solve $3\tan^4 \alpha - 10\tan^2 \alpha + 3 = 0$

15. Prove that $\tan^{-1} \frac{1}{8} + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} = \frac{\pi}{4}$

16. If $a = (b + c) \cos \theta$ prove that $\sin \theta = \frac{2\sqrt{bc}}{b+c} \cos \frac{A}{2}$.

17 . prove that $\frac{\sin 6\theta}{\sin \theta} = 32 \cos^5 \theta - 35 \cos^3 \theta + 6 \cos \theta$

SECTION C

LONG ANSWER TYPE QUESTIONS.

5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

18 . Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be bijection. Then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

19 .Show that $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots$ up to n terms $= \frac{n}{3n+1}$ for all $n \in \mathbb{N}$

20 .Prove that by vector method the angle between two diagonals of a cube is $\cos^{-1}(1/3)$

21. In triangle ABC, prove that $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

22 . If $a=13, b=14, c=15$, prove that $R = \frac{65}{8}, r = 4, r_1 = \frac{21}{2}, r_2 = 12, r_3 = 14$.

23 . From the top of a tree on the bank of a lake, an Aeroplane in the sky makes an angle of elevation α and its image in the river makes an angle of depression β . if the height of the tree from the water surface is 'a' and that of the height of the aero plane is h, show that $h = \frac{a \sin(\alpha + \beta)}{\sin(\beta - \alpha)}$.

24 . Show that $(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n \left(\frac{\theta}{2} \right) \cos \left(\frac{n\theta}{2} \right)$.