

MATHEMATICS PAPER IA.- MARCH 2011.
ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY.

TIME : 3hrs

Max. Marks.75

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS.

10X2 =20

Note : Attempt all questions. Each question carries 2 marks.

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$ and $f:A \rightarrow B$ is surjection defined by $f(x)=\cos x$ then find B
2. If $f(x)=5^x$ then find $f^{-1}(x)$
3. If the position vectors of points A,B are $-2i+j-k, -4i+2j+2k,$ and $6i-3j-13k$ respectively and $AB=\lambda AC$ find λ
4. Find the vector equation line joining points $2i+j+3k$ and $-4i+3j-k$
5. If $4i+2\frac{p}{3}j+pk$ is parallel to the vector $i+2j+3k$ find p
6. Prove that $\cos 340 \cos 40 + \sin 200 \sin 140 = 1/2$
7. Show that $\frac{\cos 9 + \sin 9}{\cos 9 - \sin 9} = \cot 36$
8. If $\cosh x = 5/2$ find the value of i) $\cosh(2x)$ ii) $\sinh 2x$
9. If $a=26$ cm $b=30$ cm $\cos C=63/65$ then find c
10. Write $Z = -\sqrt{7} + i\sqrt{21}$ in polar form

SECTION B

SHORT ANSWER TYPE QUESTIONS.

5X4 =20

Note : Answer any FIVE questions. Each question carries 4 marks.

11. If a,b,c are non coplanar vectors then prove that the vectors $5a+6b+7c, 7a-8b+9c$ and $3a+20b+5c$ are coplanar
12. Find the volume of the parallelepiped with coterminous edges $2i-3j, i+j-k, 3i-k$
13. Prove that $\left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) = \frac{1}{8}$
14. solve $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$
15. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

16. If a, b, c are in A.P. then show that $\tan \frac{A}{2} \cos \frac{C}{2} = 1$
17. Show that $16 \sin^5 \theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin \theta$

SECTION C

LONG ANSWER TYPE QUESTIONS.

5X7 = 35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be bijection. Then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$
19. Prove By Mathematical Induction Show that

$$a + ar + ar^2 + \dots = \frac{a(r^n - 1)}{r - 1}$$

20. Find the equation Plane passing from points $A(2, 3, -1), B(4, 5, 2)$ and $C(3, 6, 5)$

21. Prove that $\sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2} = 1 + 4 \sin \left(\frac{\pi - A}{4} \right) \sin \left(\frac{\pi - B}{4} \right) \sin \left(\frac{\pi - C}{4} \right)$

22. If $a = (b - c) \sec \theta$ prove that $\tan \theta = \frac{2\sqrt{bc}}{b - c} \sin \frac{A}{2}$.

23. From the top of a tree on the bank of a lake, an Aeroplane in the sky makes an angle of elevation α and its image in the river makes an angle of depression β . If the height of the tree from the water surface is 'a' and that of the height of the aero plane is h, show that

$$h = \frac{a \sin(\alpha + \beta)}{\sin(\beta - \alpha)}$$

If α, β are roots of the equation $x^2 - 2x + 4 = 0$ then for any value n show that

24. $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$