MATHEMATICS

31. If $\vec{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$

then the value of

$\left(2\vec{a}-\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is:

1) 5 2) 3 3) –5 4) –3

Ans: 3

Sol: $\left(2\bar{a}-\bar{b}\right)\left\{\left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right\}$

$=-5\left|\bar{a}\right|^2\left|\bar{b}\right|^2=-5$

$\because \left|\bar{a}\right|=1\quad \left|\bar{b}\right|=1\quad \bar{a}.\bar{b}=0$

32. Coefficient of x^7 in the exp-ansion of $(1-x-x^2+x^3)^6$ is?

1) –144 2) 132 3) 144 4)–132

Sol: $(1-x-x^2+x^3)^6=(1-x)^6(1-x^2)^6$

coefficient of x^7 in

$(1-6c_1x+6c_2x^2-6c_3x^3+6c_4x^4-6c_5x^5+6c_6x^6)$

$(1-6c_1x^2+6c_2x^4-6c_3x^6+6c_4x^8-6c_5x^{10}+6c_6x^{12})$

$=(-6c_1\times-6c_3)+(-6c_3\times-6c_2)+(-6c_5\times-6c_1)$

$=120-300+36$

$=-144$

Ans : 1

33. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line

Re z = 1, then it is necessary that:

1) |β| = 1 2) β∈ (1, ∞)

3) β∈ (0, 1) 4) β∈ (–1, 0)

Ans: 2

Sol: $z^2+\alpha z+\beta=0$

$z=x+iy$

$x=1\Rightarrow z=1+iy\Rightarrow(1+iy)^2+\alpha(1+iy)+\beta=0$

$\Rightarrow(1-y^2+\alpha+\beta)+i(2y+\alpha y)=0$

$\Rightarrow 2y+ay=0$

$\Rightarrow\alpha=-2$

$1-y^2+\alpha+\beta=0$

Put α = –2

$1+y^2=\beta\geq1$

$\Rightarrow\beta\in[1,\infty)$

34. Consider the following state-ments

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :

1) ~Q↔~P∧R 2) ~(P∧~R)↔Q

3) ~P∧(Q↔~R)

4) ~(Q↔(P∧~R))

Ans: 4

Sol: ~{P∧~R⇔Q}

= ~{Q⇔P∧~R}

35. $\frac{d^2x}{dy^2}$ equals :

1) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

$-\left(\frac{2d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

3) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ 4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

Ans: 2

Sol: $\left(\frac{d^2x}{dy^2}\right)=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$

$-\frac{1}{\left(\frac{dy}{dx}\right)^2}\times\frac{d}{dy}\left(\frac{dy}{dx}\right)$

$=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\times\frac{d^2y}{dx^2}\times\frac{dx}{dy}$

$=-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

36. **Statement-1 :**

The point A (1, 0, 7) is the mirror images of the point B(1, 6, 3) in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2 : The line segment joining A(1, 0, 7) and B(1, 6, 3)

1) Statement-1 is true, Statement-2 is false

2) Statement-1 is false, Statement-2 is true

3) Statement-1 is true, Stateme-nt-2 is true; Statement-2 is a correct explanation for State-ment-1

4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for State-ment-1

Ans: 4

37. If C and D are two events such that C ⊂ D and

1) P(C|D)<P(C)

2) $P(C|D)=\frac{P(D)}{P(C)}$

3) P(C|D) = P(C)

4) P(C|D)≥P(C)

Ans: 4

38. Consider 5 independent Berno-uli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to 31/32, then p lies in the interval :

1) $\left[0,\frac{1}{2}\right]$ 2) $\left[\frac{11}{12},1\right]$

3) $\left(\frac{1}{2},\frac{3}{4}\right]$ 4) $\left(\frac{3}{4},\frac{11}{12}\right]$

Ans: 1

Sol: Pb of at least one failure = 1–Pb of no failure

$=1-5_{c_5}\left(p^n\right)\left(q^0\right)=\frac{31}{32}$

$1-P^5=\frac{31}{35}$

$\Rightarrow p=1/2$

39. The value of $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$ is:

1) π/2 log2 2) log2

3) π log2 4) π/8 log2

Ans: 3

Sol: $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$

put x = tanθ

$\Rightarrow\int_0^{\pi/4}\frac{8\log(1+\tan\theta)}{\sin^2\theta}\times\sin^2\theta\,d\theta=\frac{8\pi\log2}{8}=\pi\log2$

40. For $x\in\left(0,\frac{5\pi}{2}\right)$, define $f(x)=\int_0^x\sqrt{t}$ sint dt then f has :

1) local minimum at π and local maximum at 2π

2) local maximum at π and local minimum at 2π

3) local maximum at π and 2π

4) local minimum at π and 2π

Ans: 2

Sol: $x\in\left(0,\frac{5\pi}{2}\right)$

$f(x)=\int_0^x\sqrt{t}\sin t\,dt$

$f'(x)=\sqrt{x}\sin x=0\Rightarrow x=0,\pi,2\pi$

$f''(x)<0;\ f''(2\pi)>0$

⇒ f(x) has maximum at x = π minimum at x = 2π

41. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying :

$\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. The vector $\vec{d}$ is equal to :

1) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ 2) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

3) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ 4) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Ans: 2

Sol: $\bar{a}\not\perp\bar{b}\quad\bar{a}.\bar{b}=0\ \Rightarrow\bar{a}\perp\bar{d}$

$\bar{b}\times\bar{c}=\bar{b}\times\bar{d}$ take cross product with $\bar{a}$

$\bar{a}\times\left(\bar{b}\times\bar{c}\right)=\bar{a}\times\left(\bar{b}\times\bar{d}\right)$

$\Rightarrow\bar{d}=\bar{c}-\frac{\left(\bar{a}.\bar{c}\right)}{\left(\bar{a}.\bar{b}\right)}\bar{b}$

42. Let R be the set of real numbers.

Statement-1: A = {(x, y)∈R×R: y–x is an integer} is an equivalence relation on R.

Statement-2: B={(x, y)∈ R× R: x=αy for some rational number α} is an equivalence relation of R.

1) Statement-1 is true, State-ment-2 is false.

2) Statement-1 is false, Statement-2 is true

3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

4) Statement-1 is true, State-ment-2 is true; Statement-2 is not a correct

explanation for Statement-1

Ans: 1

Sol: St(1) is Reflexive, symm-etric & Transitive

$\Rightarrow$ It is equivalence relation on R

St(2) is Reflexive, symmetric do not transitive

$\therefore$ not equivalence relation on R

43. Let A and B be two symmetric matrices of order 3.

Statement-1: A(BA) and (AB)A are symmetric matrices.

Statement-2: AB is symmetric matrix multiplication of A with B is commutative.

1) Statement-1 is true, Statement-2 is false.
2) Statement-1 is false, Statement-2 is true
3) Statement-1 is true, State-ment-2 is true; Statement-2 is a correct explanation for Statement-1
4) Statement-1 is true, Statem-ent-2 is true; Statement-2 is not a correct explanation for Statement-1

Ans: 4

Sol: Given $A^T = A$

$B^T = B$

$(A(BA))^T =$

$(BA)^T A^T = (A^T B^T) A^T$

$= (AB)A = A(BA)$

$((AB)A)^T = A^T(AB)^T =$

$A^T(B^T A^T)$

$= A(BA) = (AB)A$

$\Rightarrow$ ST(1) is true

$(AB)^T = B^T A^T = BA$

= AB if commutative

$\Rightarrow$ ST (2) is true

44. The two circles $x^2+y^2= ax$, $x^2+y^2 =c^2$ $(c>0)$ touch each other if :

1) $a = 2c$ 2) $|a| = 2c$
3) $2|a| = c$ 4) $|a| = c$

Ans: 4

Sol: $x^2 + y^2 - ax = 0$ $x^2 + y^2 = c^2$

$c_1 = \left(\frac{a}{2}, 0\right)$ $c_2 = (0,0)$

$r_1 = a/2$ $r_2 = c$

$c_1c_2 = |r_1 - r_2| \Rightarrow \frac{a}{2} = \left(\frac{a}{2} - c\right)$ $\Rightarrow |a| = c$

45. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

1) equals $-\sqrt{2}$ 2) equals $1/\sqrt{2}$
3) does not exist 4) equals $\sqrt{2}$

Ans: 3

Sol: $\underset{x\to 2}{\text{Lt}} \frac{\sqrt{1-\cos 2(x-2)}}{x-2}$

$= \underset{x\to 2}{\text{Lt}} \frac{\sqrt{2}|\sin(x-2)|}{x-2}$

LHL = $-\sqrt{2}$ RHL = $+\sqrt{2}$

does not exist

46. If $A = \sin^2 x + \cos^4 x$, then for all real x:

1) $1 \le A \le 2$ 2) $\frac{3}{4} \le A \le \frac{13}{16}$
3) $\frac{3}{4} \le A \le 1$ 4) $\frac{13}{16} \le A \le 1$

Ans: 3

Sol: $A = \sin^2 x + \cos^4 x$

$= 1 - \cos^2 x \sin^2 x = 1 - \frac{\sin^2 2x}{4}$

$\because 0 \le \sin^2 x \le 1 \Rightarrow 3/4 \le A \le 1$

47. The lines L_1: $y-x=0$ and L_2: $2x+y=0$ intersect the line L_3: $y+2=0$ at P and Q respe-ctively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement-2: In any triangle into two similar triangles.

1) Statement-1 is true, Statement-2 is false.
2) Statement-1 is false, Statement-2 is true
3) Statement-1 is true, Statem-ent-2 is true; Statement-2 is a correct explanation for State-ment-1
4) Statement-1 is true, State-ment-2 is true; Statement-2 is not a correct explanation for Statement-1

Ans: 1

Sol: $p(-2,-2)Q(1,-2)R\left(\frac{2\sqrt{2}-2\sqrt{5}}{2\sqrt{2}+\sqrt{5}}, -2\right)$

$|PR| : |RQ| = 2\sqrt{2} : \sqrt{5}$

Statement 1 is true

Statement 2 is false

48. The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is :

1) $(-\infty, 0)$ 2) $(-\infty, \infty)-\{0\}$
3) $(-\infty, \infty)$ 4) $(0, \infty)$

Sol: $|x|-x>0$ $\Rightarrow x<0$ **Ans : 1**

49. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x+2y+3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals :

1) 2/5 2) 5/3 3) 2/3 4) 3/2

Sol: $\frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}} = \frac{3}{\sqrt{14}}$

$\Rightarrow \lambda = 2/3$ **Ans : 3**

50. The shortest distance between line $y-x = 1$ and curve $x = y^2$ is:

1) $8/3\sqrt{2}$ 2) $4/\sqrt{3}$ 3) $\sqrt{3}/4$ 4) $3\sqrt{2}/8$

Sol: $p(t^2, t)\lambda = \frac{|t^2-t+1|}{\sqrt{2}} = \frac{t^2-t+1}{\sqrt{2}}$

$\lambda_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$ **Ans : 4**

51. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :

1) 20 months 2) 21 months
3) 18 months 4) 19 months

Sol: $600 + \frac{n-3}{2}[480 + (n-4)40] = 11040$

$\Rightarrow n = 21$ **Ans : 2**

52. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals :

1) 4 2) 5 3) 2 4) 3

Sol: $MD = \frac{\Sigma|xi - M|}{n}$ $M = \frac{51a}{2}$

$|a| = 4$, $n = 50$ **Ans : 1**

53. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A+B\omega$. Then (A, B) equals :

1) (1,0) 2) (–1,1) 3) (0,1) 4) (1,1)

Sol: $(1 + \omega)^7 = A + b\omega$

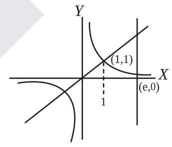
$\Rightarrow 1 + \omega = A + b\omega$

$\Rightarrow$ A, B = (1, 1) **Ans : 4**

54. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is :

1) 3/2 square units 2) 5/2 square units
3) 1/2 square units 4) 1 square units

Sol:



$A = \frac{1}{2} + \int_1^e \frac{1}{x} dx = 3/2$

Ans : 1

55. The number of values of k for which the linear equations

$4x + ky + 2z = 0$, $kx + 4y + z = 0$

$2x + 2y + z = 0$

posses a non-zero solution is :

1) 1 2) zero 3) 3 4) 2

Sol: $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0 \Rightarrow k = 2, 4$

Ans : 4

56. The values of p and q for which the function

$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x<0 \\ q, & x=0 \text{ is continuous} \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x<0 \end{cases}$

for all x in R, are:

1) $p = -\frac{3}{2}, q = \frac{1}{2}$ 2) $p = \frac{1}{2}, q = \frac{3}{2}$
3) $p = \frac{1}{2}, q = -\frac{3}{2}$ 4) $p = \frac{5}{2}, q = \frac{1}{2}$

Sol: $\underset{x\to 0^+}{\text{Lt}} \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} = 1/2$

$\underset{x\to 0^-}{\text{Lt}} \frac{\sin(p+1)x + \sin x}{x} = p+2$

$f(0) = q \Rightarrow q = p + 2 = 1/2$

$(p,q) = \left(\frac{-3}{2}, \frac{1}{2}\right)$ **Ans : 1**

57. **Statement-1**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$

Statement-2

The number of ways of choosing any 3 places from 9 different places is $^{9}C_3$

1) Statement-1 is true, Statement-2 is false
2) Statement-1 is false, Statement-2 is true
3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

Sol: ST(1): $^{n-1}C_{r-1} = {}^{10-1}C_{4-1} = {}^{9}C_3$

ST(2): $^{n}C_r = {}^{9}C_3$

Ans : 4

58. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is:

1) $3x^2 + 5y^2 - 15 = 0$
2) $5x^2 + 3y^2 - 32 = 0$
3) $3x^2 + 5y^2 - 32 = 0$
4) $5x^2 + 3y^2 - 48 = 0$

Sol: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Passes through(–3,1)

$\Rightarrow \frac{9}{a^2}+\frac{1}{b^2}=1(1)$

$(1)b^2 = a^2\left(1-e^2\right)=\frac{3a^2}{5}-(2)$

$(2)\text{in}(1)\Rightarrow a^2=\frac{32}{3};b^2=\frac{32}{5}\Rightarrow 3x^2+5y^2-32=0$

Ans : 3

59. Let I be the purchase value of an equipment and d V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-1)$, where k>0 is a constant and t is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

1) $I-\frac{k(T-t)^2}{2}$ 2) e^{-kT}

3) $T^2-\frac{I}{k}$ 4) $I-\frac{KT^2}{2}$

Sol: $\frac{d(v(t))}{dt}=-k(T-t);\ k>0$

Integrating

$v(t)=-kTt+\frac{kt^2}{2}+c$

t = 0, v(0) = I $\Rightarrow$ c = I

$v(t)=-kTt+\frac{k}{2}t^2+I$

t = T $\Rightarrow$ v(t) v(T)

$v(T)=I-\frac{1}{2}kT^2$

Ans : 4

60. If $\frac{dy}{dx}=y+3>0$ and y(0) = 2,

then y(In2) is equal to :

1) 13 2) 2 3) 7 4) 5

Sol: $\int\frac{dy}{y+3}=\int dx\Rightarrow \text{In}(y+3)=x+\text{In}5$

$\Rightarrow$ y + 3 = 10, y = 7 **Ans : 3**