

PREREQUISITES

(2-D GEOMETRY)

DISTANCE BETWEEN TWO POINTS

(i) The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$

$$AB \text{ (or } BA) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(ii) The distance from origin O to the point $A(x_1, y_1)$ is $OA = \sqrt{x_1^2 + y_1^2}$

(iii) The distance between two points $A(x_1, 0)$ and $B(x_2, 0)$ lying on the X - axis

$$\text{is } AB = \sqrt{(x_1 - x_2)^2 + (0 - 0)^2} = \sqrt{(x_1 - x_2)^2} = |x_1 - x_2|$$

(iv) The distance between two points $C(0, y_1)$ and $D(0, y_2)$ lying on the Y -axis is $CD = |y_1 - y_2|$

SECTION FORMULA

(i) The point P which divides the line segment joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ in the ratio $m : n$ internally is given by

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \quad (m+n \neq 0)$$

(ii) If P divides in the ratio $m:n$ externally then $P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right) \quad (m \neq n)$

Note: If the ratio $m : n$ is positive then P divides internally and if the ratio is negative P divides externally.

MID POINT

The mid point of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

POINTS OF TRISECTION

The points which divide the line segment \overline{AB} in the ratio $1 : 2$ and $2 : 1$ (internally) are called the points of trisection of \overline{AB} .

AREA OF A TRIANGLE

The area of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$\text{Area} = \frac{1}{2} |(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)|$$

$$\text{i.e., Area of ABC} = \frac{1}{2} \left| \sum (x_1 y_2 - x_2 y_1) \right|$$

Note:1. The area of the triangle formed by the points

(x_1, y_1) , (x_2, y_2) , (x_3, y_3) is the positive value of the determinant $\frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$.

2. The area of the triangle formed by the points (x_1, y_1) , (x_2, y_2) and the origin is $\frac{1}{2} |x_1 y_2 - x_2 y_1|$

AREA OF A QUADRILATERAL

The area of the quadrilateral formed by the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , (x_4, y_4) taken in that order is

$$\frac{1}{2} |x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + x_3 y_4 - x_4 y_3 + x_4 y_1 - x_1 y_4|$$

Note 1: The area of the quadrilateral formed by the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , (x_4, y_4) taken in order is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

CENTRES OF A TRIANGLE

Median : In a triangle, the line segment joining a vertex and the mid point of its opposite side is called a median of the triangle. The medians of a triangle are concurrent.

The point of concurrence of the medians of a triangle is called the centroid (or) centre of gravity of the triangle. It is denoted by G.

IN CENTRE OF A TRIANGLE

Internal bisector : The line which bisects the internal angle of a triangle is called an internal angle bisector of the triangle.

The point of concurrence of internal bisectors of the angles of a triangle is called the incentre of the triangle. It is denoted by I.

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

EXCENTRES OF A TRIANGLE:

The point of concurrence of internal bisector of angle A and external bisectors of angles B, C of ABC is called the ex-centre opposite to vertex A. It is denoted by I_1 . The excentres of ABC opposite to the vertices B, C are respectively denoted by I_2, I_3 .

$$I_1 = \text{Excentre opposite to A} = \left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c} \right)$$

$$I_2 = \text{Excentre opposite to B} = \left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c} \right)$$

$$I_3 = \text{Excentre opposite to C} = \left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c} \right)$$

ORTHO CENTRE OF A TRIANGLE

Altitude: The line passing through vertex and perpendicular to opposite side of a triangle is called an altitude of the triangle. Altitudes of a triangle are concurrent. The point of concurrence is called the ortho centre of the triangle. It is denoted by "O" or 'H'.

Circum centre of a Triangle:

Perpendicular bisector: The line passing through mid point of a side and perpendicular to the side is called the perpendicular bisector of the side.

The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrence is called **the** circum centre or the triangle. It is denoted by S.

EXERCISE

I. Find the distance between the following pairs of points. i) (4, 5), (5, 4) ii) (-3, 1), (3, 2) iii) $(a \cos a, a \sin a)$, $(a \cos b, a \sin b)$

Sol.

$$\text{i) Let } A = (4, 5), B = (5, 4) \Rightarrow AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow AB = \sqrt{(4-5)^2 + (5-4)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\text{ii) Let } A = (-3, 1), B = (3, 2) \Rightarrow AB = \sqrt{(3+3)^2 + (2-1)^2} = \sqrt{36+1} = \sqrt{37}$$

iii) Let $A = (a \cos a, a \sin a)$, $B = (a \cos b, a \sin b)$

$$AB = \sqrt{(a \cos \alpha - a \cos \beta)^2 + (a \sin \alpha - a \sin \beta)^2} = \sqrt{a^2 [2 - 2 \cos(\alpha - \beta)]} = \sqrt{2a^2 [1 - \cos(\alpha - \beta)]}$$

$$= \sqrt{4a^2 \sin^2 \left(\frac{\alpha - \beta}{2} \right)} = 2 \left| a \sin \left(\frac{\alpha - \beta}{2} \right) \right|$$

2. Find the value of 'a' if the distance between the points (a, 2), (3, 4) is $2\sqrt{2}$.

Sol. P (a, 2), Q (3, 4) are the given points.

$$PQ = 2\sqrt{2} \Rightarrow PQ^2 = 8$$

$$\Rightarrow (a - 3)^2 + (2 - 4)^2 = 8$$

$$\Rightarrow (a - 3)^2 = 8 - 4 = 4$$

$$\Rightarrow a - 3 = \pm 2 \Rightarrow a = 3 \pm 2 = 5 \text{ or } 1.$$

3. Find the point on the x-axis, which is equidistant from (7, 6) and (-3, 4).

Sol. Let A(7, 6), B(-3, 4)

Let P(x, 0) be the point on x-axis which is equidistant from A and B.

$$\text{Then } PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$$

$$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$$

$$\Rightarrow -14x + 85 = 6x + 25 \Rightarrow 20x = 60 \Rightarrow x = 3$$

The required point is P (3, 0).

4. Find the relation between x and y, if the point (x,y) is to be equidistant from (6,-1) and (2,3).

Sol. P(x, y), A (6, -1), B (2, 3) are the given points.

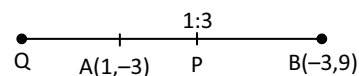
$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x - 6)^2 + (y + 1)^2 = (x - 2)^2 + (y - 3)^2$$

$$\Rightarrow x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$\Rightarrow 8y = 8x - 24 \Rightarrow y = x - 3$$

5. Find the points which divide the line segment joining A (1, -3) and B (-3, 9) in the ratio 1: 3 (i) internally and (ii) externally.



Sol. (i) Given points are A(1, -3) and B(-3, 9)

Let P be the point dividing AB internally in the ratio 1: 3.

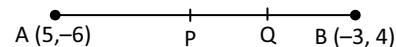
Then coordinates of P are

$$\left(\frac{1(-3) + 3 \cdot 1}{1+3}, \frac{1 \cdot 9 + 3(-3)}{1+3} \right) = \left(\frac{3-3}{4}, \frac{9-9}{4} \right) = (0, 0)$$

Let Q be the point dividing AB externally in the ratio 1 : 3. Then coordinates of Q are

$$\left(\frac{(1)(-3) - 3 \cdot 1}{1-3}, \frac{1 \cdot 9 - 3(-3)}{1-3} \right) = \left(\frac{-3-3}{-2}, \frac{9+9}{-2} \right) = (3, -9)$$

6. Find the points of trisection of the line segment joining (5, -6) and (-3, 4).



Sol. Given points are A (5, -6), B (-3, 4)

Let P and Q be the points of trisection of AB.

Let P divides AB in the ratio 1: 2 then Coordinates of P are

$$\left(\frac{1(-3) + 2(5)}{1+2}, \frac{1(4) + 2(-6)}{1+2} \right)$$

$$= \left(\frac{-3+10}{3}, \frac{4-12}{3} \right) = \left(\frac{7}{3}, \frac{-8}{3} \right)$$

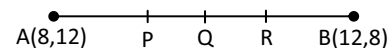
Let Q divides AB in the ratio 2 : 1, then Coordinates of Q are

$$\left(\frac{2(-3) + 1(5)}{2+1}, \frac{2(4) + 1(-6)}{2+1} \right)$$

$$= \left(\frac{-6+5}{3}, \frac{8-6}{3} \right) = \left(\frac{-1}{3}, \frac{2}{3} \right)$$

7. Find the points which divide the line segment joining (8, 12) and (12, 8) into four equal parts.

Sol. Given points A(8, 12), B(12, 8).



Let P, Q, R be the points dividing AB into four equal parts.

Let P divides AB in the ratio 1: 3, then

Coordinates of P are

$$\left(\frac{1(12) + 3(8)}{1+3}, \frac{1(8) + 3(12)}{1+3} \right) = (9, 11)$$

Q divides AB in the ratio 1: 1

Coordinates of Q are

$$\left(\frac{8+12}{1+1}, \frac{12+8}{2} \right) = \left(\frac{20}{2}, \frac{20}{2} \right) = (10, 10)$$

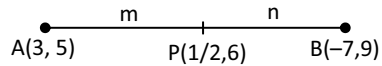
R divides AB in the ratio 3: 1, then Coordinates of R are:

$$\left(\frac{3(12) + 1(8)}{3+1}, \frac{3(8) + 1(12)}{3+1} \right) = (11, 9)$$

P (9, 11), Q (10, 10), R (11, 9) are the required points.

8. Find the ratio in which the point $(\frac{1}{2}, 6)$ divides the line segment joining $(3, 5)$ and $(-7, 9)$.

Sol. Let A (3, 5), B (-7, 9), P ($\frac{1}{2}$, 6). P divides AB in the ratio m: n.

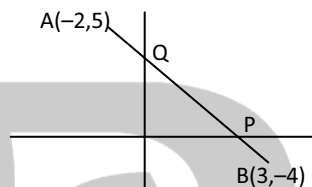


$$\frac{AP}{PB} = \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{\frac{1}{2} - 3}{-7 - \frac{1}{2}} = \frac{-\frac{5}{2}}{-\frac{15}{2}} = \frac{1}{3}$$

\therefore P divides AB in the ratio 1 : 3.

9. In what ratio do the coordinate axes divide the line segment joining $(-2, 5)$ and $(3, -4)$.

Sol. A(-2, 5), B(3, -4) are the given points. AB meets the X-axis in P and Y-axis in Q.

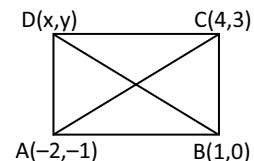


$$\begin{aligned} \text{Ratio in which X-axis divides AB} &= -y_1 : y_2 \text{ i.e., } = -5 : -4 = 5 : 4 \\ \text{Ratio in which Y-axis divides AB} &= -x_1 : x_2 \text{ i.e., } = 2 : 3 \end{aligned}$$

10. If $(-2, -1)$, $(1, 0)$ and $(4, 3)$ are three successive vertices of a parallelogram, find the fourth vertex.

Sol. Vertices of a parallelogram are A $(-2, -1)$, B $(1, 0)$, C $(4, 3)$

Let 4th vertex be D(x, y)



Diagonals AC and BD bisect each other.

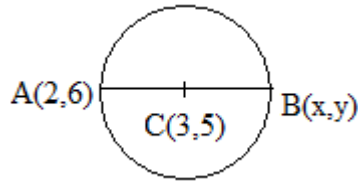
Midpoint of AC = Midpoint of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{-1+3}{2} \right) = \left(\frac{1+x}{2}, \frac{0+y}{2} \right) \Rightarrow \frac{x+1}{2} = \frac{2}{2} \Rightarrow x+1=2 \Rightarrow x=1$$

$$\Rightarrow \frac{y}{2} = \frac{2}{2} \Rightarrow y=2. \quad \text{Therefore, Coordinates of D are } (1, 2).$$

11. A (2, 6) is one of the extremities of a diameter of a circle with centre (3, 5), find the other point.

Sol. A (2, 6), B(x, y) are the ends of the diameter.



And C(3, 5) is the centre.

Now C is the mid point of AB. $\therefore \left(\frac{2+x}{2}, \frac{6+y}{2} \right) = (3, 5)$

$$\frac{2+x}{2} = 3 \Rightarrow 2+x = 6 \Rightarrow x = 4 \quad \text{and} \quad \frac{6+y}{2} = 5 \Rightarrow 6+y = 10 \Rightarrow y = 4$$

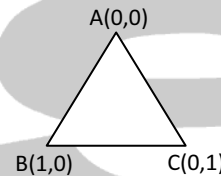
Coordinates of the other point B are (4, 4).

12. Find the area of the triangle formed by the following points.

i) (0, 0), (1, 0), (0, 1) ii) (5, 2), (-9, -3), (-3, -5)

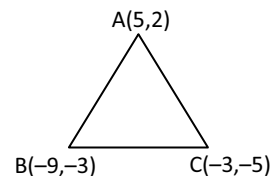
Sol. i) A(0, 0), B(1, 0), C(0, 1) are the vertices of the triangle.

$$\begin{aligned} \text{Area of } \triangle OAB &= \frac{1}{2} |x_1 y_2 - x_2 y_1| \\ &= \frac{1}{2} |1 \cdot 1 - 0 \cdot 0| = \frac{1}{2} \text{ sq. units} \end{aligned}$$



ii) A(5, 2), B(-9, -3), C(-3, -5) are the vertices of the triangle.

$$\begin{aligned} &= \frac{1}{2} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -9-5 & -3-2 \\ -3-5 & -5-2 \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} -14 & -5 \\ -8 & -7 \end{vmatrix} \\ &= \frac{1}{2} |98 - 40| = \frac{58}{2} = 29 \text{ sq. units} \end{aligned}$$



13. Show that the following points are collinear.

i) (0, -2), (-1, 1), (-2, 4) ii) (-1, 7), (3, -5), (4, -8)

Sol. i) A(0, -2), B(-1, 1), C(-2, 4) are the given points.

$$\text{Area of } \triangle ABC = \frac{1}{2} |0(1-4) - 1(4+2) - 2(-2-1)| = \frac{1}{2} |0 - 6 + 6| = 0$$

\therefore A, B, C are collinear.

ii) A(-1, 7), B(3, -5), C(4, -8)

$$\text{Area of } \triangle ABC = \frac{1}{2} |-1(-5+8) + 3(-8-7) + 4(7+5)| = \frac{1}{2} |-3-45+48| = 0$$

\therefore A, B, C are collinear.

14. Find the value of k, if the points (k, -1), (2, 1) and (4, 5) are collinear.

Sol. A(k, -1), B(2, 1), C(4, 5) are the given points.

A, B, C are collinear $\Rightarrow \Delta ABC = 0$

$$\frac{1}{2} |k(1-5) + 2(5+1) + 4(-1-1)| = 0$$

$$|-4k + 12 - 8| = 0 \Rightarrow |-4k + 4| = 0$$

$$\Rightarrow 4k = 4 \Rightarrow k = 1$$

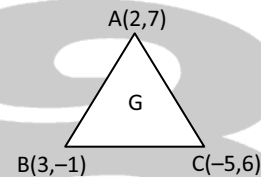
15. Find the centroid of the triangle whose vertices are (2, 7), (3, -1), (-5, 6).

Sol. A(2, 7), B(3, -1), C(-5, 6) are the vertices of the triangle.

Let G be the centroid of $\triangle ABC$.

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \left(\frac{2+3-5}{3}, \frac{7-1+6}{3} \right) = \left(0, \frac{12}{3} \right) = (0, 4)$$



16. A(4, 8), B(-2, 6) are two vertices of a triangle ABC. Find the coordinates of C if the centroid of the triangle is (2, 7).

Sol. Let C(x, y) be the third vertex. Given vertices are A(4, 8), B(-2, 6) and centroid of the triangle ABC is G(2, 7)

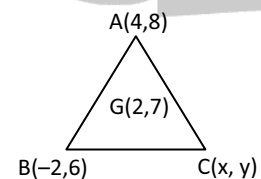
$$\Rightarrow G = \left(\frac{4-2+x}{3}, \frac{8+6+y}{3} \right)$$

$$\Rightarrow \left(\frac{x+2}{3}, \frac{y+14}{3} \right) = (2, 7)$$

$$\Rightarrow \frac{x+2}{3} = 2 \Rightarrow x+2 = 6 \Rightarrow x = 4 \text{ and}$$

$$\frac{y+14}{3} = 7 \Rightarrow y+14 = 21 \Rightarrow y = 7$$

Third vertex of C is (4, 7).



II.

1. Show that the following points taken in order from the figure mentioned against the points.

i) $(-2, 5), (3, -4), (7, 10)$: right angled isosceles triangle.

Sol. $A(-2, 5), B(3, -4), C(7, 10)$ are the given points.

$$AB^2 = (-2-3)^2 + (5+4)^2 = 25 + 81 = 106$$

$$BC^2 = (3-7)^2 + (-4-10)^2 = 16 + 196 = 212$$

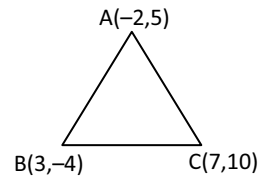
$$CA^2 = (7+2)^2 + (10-5)^2 = 81 + 25 = 106$$

$$AB^2 = CA^2 \Rightarrow AB = CA \quad \dots(1)$$

$$AB^2 + CA^2 = 106 + 106 = 212 = BC^2$$

$$AB^2 + AC^2 = BC^2 \quad \dots(2)$$

\therefore By (1) and (2), A, B, C are the vertices of a right angled isosceles triangle.



ii) $(1,3), (3,-1), (-5,-5)$: right angled triangle.

Sol. $A(1,3), B(3,-1), C(-5,-5)$ are the given points

$$AB^2 = (1-3)^2 + (3+1)^2 = 4 + 16 = 20$$

$$BC^2 = (3+5)^2 + (-1+5)^2 = 64 + 16 = 80$$

$$AC^2 = (1+5)^2 + (3+5)^2 = 36 + 64 = 100$$

From the above values

$$AB^2 + BC^2 = 20 + 80 = 100 = AC^2$$

\therefore ABC is a right angled triangle.

iii) $(2, 4), (2, 6), (2 + \sqrt{3}, 5)$: equilateral triangle.

Sol. $A(2, 4), B(2, 6), C(2 + \sqrt{3}, 5)$ are the given points.

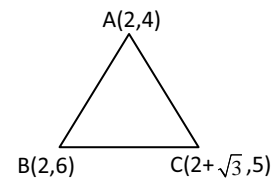
$$AB^2 = (2-2)^2 + (4-6)^2 = 0 + 4 = 4$$

$$BC^2 = (2-2-\sqrt{3})^2 + (6-5)^2 = 3 + 1 = 4$$

$$CA^2 = (2+\sqrt{3}-2)^2 + (5-4)^2 = 3 + 1 = 4$$

$$\therefore AB^2 = BC^2 = CA^2 \Rightarrow AB = BC = CA$$

\therefore ABC is an equilateral triangle.



iv) $(-3, 1), (-6, -7), (3, -9), (6, -1)$: parallelogram.

Sol. Let $A(-3, 1), B(-6, -7), C(3, -9), D(6, -1)$

$$\Rightarrow AB^2 = (-3+6)^2 + (1+7)^2 = 9 + 64 = 73$$

$$BC^2 = (-6-3)^2 + (-7+9)^2 = 81 + 4 = 85$$

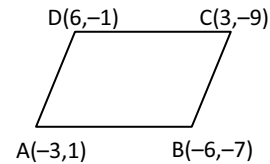
$$CD^2 = (3-6)^2 + (-9+1)^2 = 9 + 64 = 73$$

$$DA^2 = (6+3)^2 + (-1-1)^2 = 81 + 4 = 85$$

$$AB^2 = CD^2 \text{ and } BC^2 = DA^2$$

$$\therefore AB = CD \text{ and } BC = DA$$

\Rightarrow ABCD is a parallelogram.



v) $(8, 4), (5, 7), (-1, 1), (2, -2)$: rectangle.

Sol. Let $A(8, 4), B(5, 7), C(-1, 1), D(2, -2)$

$$AB^2 = (8-5)^2 + (4-7)^2 = 9 + 9 = 18$$

$$BC^2 = (5+1)^2 + (7-1)^2 = 36 + 36 = 72$$

$$CD^2 = (-1-2)^2 + (1+2)^2 = 9 + 9 = 18$$

$$DA^2 = (2-8)^2 + (-2-4)^2 = 36 + 36 = 72$$

$$AB^2 = CD^2 \Rightarrow AB = CD$$

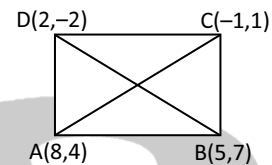
$$BC^2 = DA^2 \Rightarrow BC = AD$$

$$AC^2 = (8+1)^2 + (4-1)^2 = 81 + 9 = 90$$

$$BD^2 = (5-2)^2 + (7+2)^2 = 9 + 81 = 90$$

$$AC^2 = BD^2 \Rightarrow AC = BD$$

\therefore ABCD is a rectangle.



vi) $(3, -2), (7, 6), (-1, 2), (-5, -6)$: rhombus.

Sol. Let $A(3, -2), B(7, 6), C(-1, 2), D(-5, -6)$ $AB^2 = (3-7)^2 + (-2-6)^2 = 16 + 64 = 80$

$$BC^2 = (7+1)^2 + (6-2)^2 = 64 + 16 = 80$$

$$CD^2 = (-1+5)^2 + (2+6)^2 = 16 + 64 = 80$$

$$DA^2 = (-5-3)^2 + (-6+2)^2 = 64 + 16 = 80$$

$$AB = BC = CD = DA$$

$$AC^2 = (3+1)^2 + (-2-2)^2 = 16 + 16 = 32$$

$$BD^2 = (7+5)^2 + (6+6)^2 = 144 + 144 = 288$$

$$AC \neq BD$$

∴ ABCD is a rhombus.

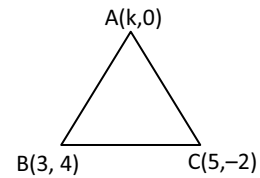
2. Find the value of k, if the area of the triangle formed by (k, 0), (3, 4) and (5, -2) is 10.

Sol. Let A(k, 0), B(3, 4) and C(5, -2)

$$\text{Area of } \triangle ABC = \frac{1}{2} |K(4+2) + 3(-2-0) + 5(0-4)| = 10$$

$$|6k - 6 - 20| = 20 \Rightarrow 6k - 26 = \pm 20$$

$$\Rightarrow 6k = 46 \text{ or } 6 \Rightarrow k = 1 \text{ or } \frac{46}{6} = \frac{23}{3}$$



3. Find the value of k if (k, 2 - 2k), (-k + 1, 2k), (-4 - k, 6 - 2k) are collinear.

Sol. A(k, 2 - 2k), B(-k + 1, 2k), C(-4 - k, 6 - 2k) are the given points.

A, B, C are collinear $\Delta ABC = 0$

$$\frac{1}{2} |k(2k - 6 + 2k) + (-k + 1)(6 - 2k - 2 + 2k) + (-4 - k)(2 - 2k - 2k)| = 0$$

$$\Rightarrow k(4k - 6) + 4(-k + 1) + (-4 - k)(2 - 4k) = 0$$

$$\Rightarrow 4k^2 - 6k - 4k + 4 - 8 + 16k - 2k + 4k^2 = 0$$

$$\Rightarrow 8k^2 + 4k - 4 = 0 \Rightarrow 2k^2 + k - 1 = 0$$

$$\Rightarrow 2k^2 + 2k - k - 1 = 0 \Rightarrow 2k(k + 1) - 1(k + 1) = 0 \Rightarrow (k + 1)(2k - 1) = 0 \Rightarrow k = -1 \text{ or } 1/2$$

III.

1. Find the in-centre of the triangle whose vertices are A(3, 2), B(7, 2), C(7, 5).

Sol. Vertices are A(3, 2), B(7, 2), C(7, 5)

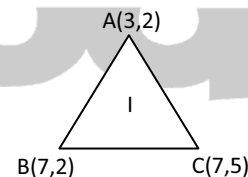
Sides of the triangle are

$$a = BC = \sqrt{(7-7)^2 + (2-5)^2} = \sqrt{0+9} = 3$$

$$b = CA = \sqrt{(7-3)^2 + (5-2)^2} = \sqrt{16+9} = 5$$

$$c = AB = \sqrt{(3-7)^2 + (2-2)^2} = \sqrt{16+0} = 4$$

$$\begin{aligned} \text{In centre } I &= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \\ &= \left(\frac{3 \cdot 3 + 5 \cdot 7 + 4 \cdot 7}{3+5+4}, \frac{3 \cdot 2 + 5 \cdot 2 + 4 \cdot 5}{3+5+4} \right) \\ &= \left(\frac{9+35+28}{12}, \frac{6+10+20}{12} \right) \\ &= \left(\frac{72}{12}, \frac{36}{12} \right) = (6, 3) \end{aligned}$$



PROBLEMS FOR PRACTICE

1. Find the area of the triangle formed by the points (1, 2), (3, -4) and (-2, 0).

Ans. 11

2. Find the area of the triangle formed by the points (9, -7), (2, 4) and (0, 0).

Ans. 25

3. Find the value of x, if the area of the triangle formed by (10, 2), (-3, -4) and (x, 1) is 5.

Ans. $19/2$ or $37/6$

4. Show that the triangle formed by the points (4, 4), (3, 5) and (-1, -1) is a right angled triangle.

5. The centroid of the triangle ABC is (2, 7). The points B and C lie on X, Y axes respectively and A = (4, 8). Find B and C.

Ans. B (2, 0), C (0, 13)

6. Find the in-centre of the triangle formed by the points A(7, 9), B(3, -7) and C(-3, 3).

ANS. $(13 - 8\sqrt{2}, 2\sqrt{2} - 1)$