### LENGTH OF THE PERPENDICULAR FROM A POINT TO A STRAIGHT LINE AND DISTANCE BETWEEN TWO PAPALLEL LINES

#### THEOREM

The perpendicular distance from a point  $P(x_1, y_1)$  to the line ax + by + c = 0 is

$$\frac{\left|ax_{1}+by_{1}+c\right|}{\sqrt{a^{2}+b^{2}}}$$

**Proof:** 

Let the axes be translated to the point  $P(x_1, y_1)$ .

Let (X,Y) be the new coordinates of (x, y). Then  $x = X + x_1$ ,  $y = Y + y_1$  The transformed equation of the given line is

$$a(X + x_1) + b(Y + y_1) + c = 0$$
  
$$\Rightarrow aX + bY + (ax_1 + by_1 + c) = 0$$

The perpendicular distance from the new origin P to the line is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ 

(from normal form) The perpendicular distance from a point

P(x<sub>1</sub>, y<sub>1</sub>) to the line ax+ by + c = 0 is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ .

### DISTANCE BETWEEN PARALLEL LINES THEOREM

The distance between the two parallel lines  $ax + by + c_1 = 0$  and  $ax + by + c_2 = 0$  is

$$\frac{\left|c_{1}-c_{2}\right|}{\sqrt{a^{2}+b^{2}}}$$

**Proof:** 

Given lines are  $ax + by + c_1 = 0 \quad --- \quad (1)$ 

$$ax + by + c_2 = 0$$
 --- (2)

Let  $P(x_1, y_1)$  be a point on the line (2).





Distance between the parallel lines = Perpendicular distance from P to line (1)

$$=\frac{|ax_1+by_1+c_1|}{\sqrt{a^2+b^2}}=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$$

# FOOT OF THE PERPENDICULAR THEOREM

If (h, k) is the foot of the perpendicular from  $(x_1, y_1)$  to the line ax + by + c = 0



$$= \frac{ah+bk-ax_1-by_1}{a^2+b^2} = \frac{-ax_1-by_1-c}{a^2+b^2}$$
$$= \frac{-(ax_1+by_1+c)}{a^2+b^2}$$
Hence  $\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$ 

### **IMAGE OF A POINT**

### THEOREM

If (h, k) is the image of (x<sub>1</sub>, y<sub>1</sub>) w.r.t the line ax + by + c = 0 (a  $\neq 0, b \neq 0$ ), then  $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(x_1 + by_1 + c)}{a^2 + b^2}$ .

Proof: 
$$(h, k)$$
  $(x_1, y_1)$   $(x_1, y_1)$   $(h, k)$ 

Let  $A(x_1, y_1)$ , B(h, k)

Mid point of is 
$$P = \left(\frac{x_1 + h}{2}, \frac{y_1 + k}{2}\right)$$

Since B is the image of A, therefore mid point P lies on ax + by + c = 0.

$$\Rightarrow a\left(\frac{x_1+h}{2}\right) + b\left(\frac{y_1+k}{2}\right) + c = 0$$
  
$$\Rightarrow ax_1 + by_1 + ah + bk + 2c = 0$$
  
$$\Rightarrow ah + bk = -ax_1 + by_1 - 2c.$$
  
Slope of  $\overline{AB}$  is  $\frac{k-y_1}{h-x_1}$ 

And Slope of given line is  $-\frac{a}{b}$ 

 $\overline{AB}$  is perpendicular to the given line

$$\Rightarrow \left(\frac{k - y_1}{h - x_1}\right) \left(-\frac{a}{b}\right) = -1$$
$$\Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$$

By the law of multipliers in ratio and proportion

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{a(h - x_1) + b(k - y_1)}{a^2 + b^2}$$
$$= \frac{ah + bk - ax_1 - by_1}{a^2 + b^2}$$
$$= \frac{-ax_1 - by_1 - 2c - ax_1 - by_1}{a^2 + b^2}$$
$$= \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$
Hence  $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(x_1 + by_1 + c)}{a^2 + b^2}$ 

Note 1: The image of  $(x_1, y_1)$  w.r.t the line x = y is  $(y_1, x_1)$ 

**Note 2:** The image of  $(x_1, y_1)$  w.r.t the line x + y = 0 is  $(-y_1, -x_1)$ 

### THEOREM

If the four straight lines ax + by + p = 0, ax + by + q = 0, cx + dy + r = 0 and cx + dy + s = 0 form a parallelogram. Then the area of the parallelogram so formed is

 $\frac{(p-q)(r-s)}{bc-ad}$ 

### **Proof:**

Let  $L_1 = ax + by + p = 0$   $L_2 = ax + by + q = 0$   $L_3 = cx + dy + r = 0$  $L_4 = cx + dy + s = 0$ 

Clearly

 $L_1 \parallel L_2$  and  $L_3 \parallel L_4$ . So  $L_1$  and  $L_3$  are nonparallel. Let be the angle between  $L_1$  and  $L_3$ .

Let 
$$d_1 = \text{distance between } L_1 \text{ and } L_{2=} \frac{|p-q|}{\sqrt{a^2 + b^2}}$$
  
Let  $d_2 = \text{distance between } L_3 \text{ and } L_4 = \frac{|r-s|}{\sqrt{c^2 + d^2}}$   
Now  $\cos \theta = \frac{|ac+bd|}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$  and  $\sin \theta = \sqrt{\frac{(a^2 + b^2)(c^2 + d^2) - (ac+bd)^2}{(a^2 + b^2)(c^2 + d^2)}}$   
 $= \frac{|bc-ad|}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$ 

Now area of the parallelogram is  $\frac{d_1d_2}{\sin\theta} = \left|\frac{(p-q)(r-s)}{bc-ad}\right|$ 

### ANGLE BETWEEN TWO LINES THEOREM

If  $\theta$  is an angle between the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$  then

$$\cos\theta = \pm \frac{a_1 a_2 + b_1 b_2}{\sqrt{a_1^2 + b_2^3} \sqrt{a_2^2 + b_2^2}}$$

**Proof:** The lines passing through the origin and parallel to the given lines are



Let  $\theta_1, \theta_2$  be the inclinations of (1) and (2) respectively  $(\theta_1 > \theta_2)$ 

Now  $\theta$  is an angle between (1) and (2)

$$\theta = \theta_1 - \theta_2$$

 $P(-b_1,a_1)$  satisfies eq(1), the point lies on (1)

Similarly,  $Q(-b_2, a_2)$  lies on (2)

Let L, M be the projection of P,Q respectively on the x - axis.

$$\therefore \cos \theta_{1} = \frac{OL}{OP} = \frac{-b_{1}}{\sqrt{a_{1}^{2} + b_{1}^{2}}}, \sin \theta_{1} = \frac{PL}{OP} = \frac{a_{1}}{\sqrt{a_{1}^{2} + b_{1}^{2}}}$$
$$\therefore \cos \theta_{2} = \frac{OM}{OQ} = \frac{-b_{2}}{\sqrt{a_{2}^{2} + b_{2}^{2}}}, \sin \theta_{2} = \frac{OM}{OQ} = \frac{a_{2}}{\sqrt{a_{2}^{2} + b_{2}^{2}}}$$
$$\theta = \theta_{1} - \theta_{2}$$

$$\cos = \cos (\theta_1 - \theta_2)$$
  
=  $\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2$   
=  $\frac{(-b_1)}{\sqrt{a_1^2 + b_1^2}} \frac{(-b_2)}{\sqrt{a_1^2 + b_1^2}} + \frac{a_1}{\sqrt{a_1^2 + b_1^2}} \frac{a_2}{\sqrt{a_1^2 + b_1^2}}$   
=  $\frac{a_1a_1 + b_1b_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$ 

Note 1: If is the acute angle between the lines then

$$\cos \theta = \frac{|a_1a_1 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$



**Note 3:** If is an angle between two lines are not a right angle then the angle between the lines means the acute angle between the lines.

Note 4: If  $\theta$  is an angle between the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ 

then 
$$\tan \theta = \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2}$$

Note 5: If is the acute angle between the lines

 $a_{1}x + b_{1}y + c_{1} = 0, \ a_{2}x + b_{2}y + c_{2} = 0 \text{ then}$  $\tan \theta = \left| \frac{a_{1}b_{2} - a_{2}b_{1}}{a_{1}a_{2} + b_{1}b_{2}} \right| = \left| \frac{a_{1} / b_{1} - a_{2} / b_{2}}{(a_{1}a_{2}) / (b_{1}b_{2}) + 1} \right|$  $= \left| \frac{(-a_{1} / b_{1}) - (-a_{2} / b_{2})}{1 + (-a_{1} / b_{1})(-a_{2} / b_{2})} \right| \text{ where } m_{1}, \ m_{2} \text{ are the slopes of the lines.}$ 

### THEOREM

The equation of the line parallel to ax + by + c = 0 and passing through  $(x_1, y_1)$  is  $a(x - x_1) + b(y - y_1) = 0.$ 

### **Proof:**

Slope of the given line is -a/b.  $\Rightarrow$  Slope of the required line is -a/b.(lines are parallel)

Equation of the required line is

$$y - y_1 = -\frac{a}{b} (x - x_1)$$
  
b(y - y\_1) = -a(x - x\_1)  
a(x - x\_1) + b(y - y\_1) = 0.

Note 1: The equation of a line parallel to ax + by + c = 0 may be taken as ax + by + k = 0.

Note 2: The equation of a line parallel to ax + by + c = 0 and passing through the origin is ax + by = 0.

### THEOREM

# The equation of the line perpendicular to ax + by + c = 0 and passing through $(x_1, y_1)$ is $b(x - x_1) - a(y - y_1) = 0$ .

### **Proof:**

Slope of the given line is  $-a/b \Rightarrow$  Slope of the required line is b/a.

(since product of slopes = -1)

Equation of the required line is  $y - y_1 = \frac{b}{a} (x - x_1)$ 

 $a(y - y_1) = b(x - x_1)$  $b(x - x_1) - a(y - y_1) = 0.$ 

Note 1: The equation of a line perpendicular to ax + by + c = 0 may be taken as

bx - ay + k = 0

Note 2: The equation of a line perpendicular to ax + by + c = 0 and passing through the origin is bx - ay = 0.

### EXERCISE -3 (d)

I. Find the angle between the following straight lines.

1. 
$$y = 4 - 2x, y = 3x + 7$$

**Sol:** given lines are

 $y = 4 - 2x \implies 2 x + y - 4 = 0$  and 3x - y + 7 = 0Let  $\theta$  be the angle between the lines, then

$$\cos \theta = \frac{|a_1 a_2 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}}$$
$$= \frac{|2.3 + 1(-1)|}{\sqrt{4 + 1} \sqrt{9 + 1}} = \frac{5}{\sqrt{5} \sqrt{10}} = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \frac{\pi}{4}$$

2. 
$$3x + 5y = 7, 2x - y + 4 = 0$$
  
Sol.  $ans: \theta = cos^{-1} \left( \frac{1}{\sqrt{170}} \right)$ 

3. 
$$\mathbf{y} = -\sqrt{3}\mathbf{x} + 5, \mathbf{y} = \frac{1}{\sqrt{3}}\mathbf{x} - \frac{2}{\sqrt{3}}$$

**Sol.** slope of  $1^{\text{st}}$  line is  $m_1 = -\sqrt{3}$ 

Slope of 2<sup>nd</sup> line is 
$$m_2 = \frac{1}{\sqrt{3}}$$
  
 $m_1 m_2 = (-\sqrt{3}) \frac{1}{\sqrt{3}} = -1.$ 

The lines are perpendicular, hence angle between the lines is  $\theta = \frac{\pi}{2}$ 

# 4. ax + by = a + b, a(x - y) + b(x + y) = 2b

**Sol.** given lines ax + by = a + b, (a + b) x + (-a + b) y = 2b

let 
$$\theta$$
 be the angle between the lines, then  

$$\cos \theta = \frac{|a(a+b)+b(-a+b)|}{\sqrt{a^2+b^2} + \sqrt{(a+b)^2 + (-a+b)^2}}$$

$$= \frac{|a^{2} + ab - ab + b^{2}|}{\sqrt{a^{2} + b^{2}}\sqrt{2(a^{2} + b^{2})}} = \frac{a^{2} + b^{2}}{\sqrt{2}(a^{2} + b^{2})} = \frac{1}{\sqrt{2}}$$

 $\therefore \theta = \frac{\pi}{4}$ 

Find the length of the perpendicular drawn form the point given against the following straight lines.

5. 
$$5x - 2y + 4 = 0$$
, (-2, -3)

Sol. Length of the perpendicular =  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|5(-2) - 2(-3) + 4|}{\sqrt{25 + 4}} = \frac{|-10 + 10|}{\sqrt{29}} = 0$ 

- 6. 3x 4y + 10 = 0, (3, 4)
- **Sol.** Length of the perpendicular  $=\frac{|3.3-4.4+10|}{\sqrt{9+16}} = \frac{3}{5}$

7. 
$$x - 3y - 4 = 0$$
, (0, 0)  
Sol Ans:  $\frac{4}{3}$ 

**Sol.** Ans;  $\frac{1}{\sqrt{10}}$ 

Find the distance between the following parallel lines.

### 8. 3x - 4y = 12, 3x - 4y = 7

**Sol.** Given lines are 3x - 4y = 12, 3x - 4y = 7

Distance between parallel lines 
$$=\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} = \frac{|-12 + 7|}{\sqrt{9 + 16}} = \frac{5}{5} = 1$$

9. 5x - 3y - 4 = 0, 10x - 6y - 9 = 0

Sol. Equations of the lines can be taken as 
$$10x - 6y - 8 = 0$$
,  $10x - 6y - 9 = 0$   
Distance between parallel lines  $=\frac{|-8+9|}{\sqrt{100+36}} = \frac{1}{2\sqrt{34}}$ 

- 10. Find the equation of the straight line parallel to the line 2x + 3y + 7 = 0 and passing through the point (5, 4).
- Sol. Given line is 2x + 3y + 7 = 0Equation of the parallel to 2x + 3y + 7 = 0 is 2x + 3y = k. This line is passing through P (5, 4)  $\Rightarrow 10 + 12 = k \Rightarrow k = 22$ Equation of the required line is 2x + 3y - 22 = 0
- 11. Find the equation of the straight line perpendicular to the line 5x 3y + 1 = 0 and passing through the point (4, -3).

Sol. Equation of the given line is 5x - 3y + 1 = 0Equation of the perpendicular to 5x - 3y + 1 = 0 is 3x + 5y + k = 0This line is passing through P (4, -3)  $\Rightarrow 12 - 15 + k = 0 \Rightarrow k = 3$ Equation of the required line is 3x + 5y + 3 = 0

- 12. Find the value of k, if the straight lines 6x 10y + 3 = 0 and kx 5y + 8 = 0 are parallel.
- Sol. Given lines are 6x 10y + 3 = 0 and kx 5y + 8 = 0. Since the lines are parallel  $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow -30 = -10$  k  $\Rightarrow$  k = 3
- 13. Fund the value of P, if the straight lines 3x + 7y 1 = 0 and 7x py + 3 = 0 are mutually perpendicular.

**Sol.** Given lines are 3x + 7y - 1 = 0, 7x - py + 3 = 0Since the lines are perpendicular

 $\Rightarrow$  a<sub>1</sub>a<sub>2</sub> + b<sub>1</sub>b<sub>2</sub> = 0  $\Rightarrow$  3.7 + (-p) = 0  $\Rightarrow$  7p = 21  $\Rightarrow$  p = 3

- 14. Find the value of k, the straight lines y 3kx + 4 = 0 and (2k 1) x (8k 1) y = 6 are perpendicular.
- **Sol.** Given lines are -3kx + y + 4 = 0

(2k - 1)x - (8k - 1)y - 6 = 0These lines are perpendicular  $\Rightarrow a_1a_2 + b_1b_2 = 0$  $\Rightarrow -3k(2k - 1) - 1(8k - 1) = 0 \Rightarrow -6k^2 + 3k - 8k + 1 = 0$  $6k^2 + 5k - 1 = 0 \Rightarrow (k + 1)(6k - 1) = 0$ k = -1 or 1/6

15. (-4, 5) is a vertex of a square and one of its diagonal is 7x - y + 8 = 0. Find the equation of the other diagonal.



D (-4, 5) is a point on this line 
$$\Rightarrow$$
 -4 + 35 + k = 0  $\Rightarrow$  k = 4 - 35 = - 31

Equation of BD is x + 7y - 31 = 0

### II.

### 1. Find the equation of the straight lines passing through (1, 3) and

### i)parallel to ii) perpendicular to the line passing through the points (3, -5) and (-6, 1).

- Sol. Given points A (3, -5), B (-6, 1) Slope of AB =  $\frac{-5-1}{3+6} = \frac{-6}{9} = \frac{-2}{3}$
- i) slope of the line parallel to AB is  $\frac{-2}{3}$  equation of the line parallel to AB and passing through (1, 3) is  $y-3 = \frac{-2}{3}(x-1) \Rightarrow 3y-9 = -2x+2 \Rightarrow 2x+3y-11 = 0$
- ii) slope of the line perpendicular to AB is 3/2.

Equation of the line passing through (1, 3) and having slope 3/2 is  $y-3 = \frac{-2}{3}(x-1)$  $\Rightarrow 2x+3y-5=0.$ 

2. The line  $\frac{x}{a} - \frac{y}{b} = 1$  meets the X – axis at P. Find the equation of the line perpendicular to this line at P.

Sol. given line is 
$$\frac{x}{a} - \frac{y}{b} = 1 - \dots - (1)$$
  
On x- axis y= 0  $\Rightarrow \frac{x}{a} - \frac{0}{b} = 1 \Rightarrow x = a$   
Point P=(a,0)  
Equation of the line perpendicular to (1) is  $\frac{x}{b} + \frac{y}{a} = k$   
This line is passing through P (a,0)  $\Rightarrow \frac{a}{b} + 0 = k \Rightarrow k = a/b$   
Equation of the line is  $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$ .  
3. Find the equation of the line perpendicular to the line 3x + 4y + 6

3. Find the equation of the line perpendicular to the line 3x + 4y + 6 = 0 and making intercept -4 on X – axis .

**Sol.** Given line is 3x + 4y + 6 = 0.

Equation of the perpendicular to 3x + 4y + 6 = 0 is 4x - 3y = k

$$\Rightarrow \frac{4x}{k} - \frac{3y}{k} = 1 \Rightarrow \frac{x}{\left\lfloor \frac{k}{4} \right\rfloor} + \frac{y}{\left\lfloor \frac{-k}{3} \right\rfloor} = 1$$

x - int ercept =  $\frac{k}{4} = -4 \Rightarrow k = -16$ 

Equation of the required line is  $4x - 3y = -16 \implies 4x - 3y + 16 = 0$ .

- 4. A (-1, 1), B (5,3) are opposite vertices of a square in the XY plane. Find the equation of the other diagonal (not passing through A, B) of a square.
- **Sol.** A(-1,1), B (5, 3) are opposite vertices of the square.

Slope of AB = 
$$\frac{1-3}{-1-5} = \frac{-2}{-6} = \frac{1}{3}$$



Diagonal CD is passing through O (2,2), Equation of CD is y - 2 = -3 (x - 2)=  $-3x + 6 \Rightarrow 3x + y - 8 = 0$ .

# 5. Find the foot of the perpendicular drawn from (4,1) upon the straight line 3x - 4y + 12=0.

**Sol.** Equation of the line is 3x - 4y + 12 = 0

If (h,k) is the foot of the perpendicular from  $(x_1, y_1)$  on the line ax + by + c = 0, then

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$$

$$\frac{h-4}{3} = \frac{k-1}{-4} = -\frac{(12-4+12)}{9+16}$$

$$\frac{h-4}{3} = \frac{k-1}{-4} = -\frac{20}{25} = -\frac{4}{5}$$

$$h-4 = -\frac{12}{5} \Longrightarrow h = 4 - \frac{12}{5} = \frac{20-12}{5} = \frac{8}{5}$$

$$k-1 = \frac{16}{5} \Longrightarrow k = 1 + \frac{16}{5} = \frac{5+16}{5} = \frac{21}{5}$$

$$\therefore \text{ Foot of the perpendicular} = \left(\frac{8}{5}, \frac{21}{5}\right)$$

6. Find the foot of the perpendicular drawn from (3,0) upon the straight line 5x + 12y - 41 = 0.

**Sol.** Ans:  $\left(\frac{49}{13}, \frac{24}{13}\right)$ 

- 7. x 2y 5 = 0 is the perpendicular bisector of the line segment joining the points A,B. If A = (-1,-3), find the co-ordinates of B.
- Sol. If PQ is the perpendicular bisector of AB, then B is the image of A in the line PQ. Equation of the line is x - 3y - 5 = 0Given point A = (-1,-3). Here B is the image of A w.r.t x - 3y - 5 = 0. Let B (h,k) be the image of A (-1,-3), then

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$
$$\frac{h+1}{1} = \frac{k+3}{-3} = \frac{-2(-1+9-5)}{1+9}$$
$$\frac{h+1}{1} = \frac{y_2+3}{-3} = -\frac{3}{5}$$
$$\frac{h+1}{1} = \frac{-3}{5} \Rightarrow h+1 = -\frac{3}{5}$$
$$h = -1 - \frac{3}{5} = -\frac{8}{5} \Rightarrow \frac{y_2+3}{-3} = \frac{-3}{5}$$
$$k+3 = \frac{9}{5} \Rightarrow k = \frac{9}{5} - 3 = -\frac{6}{5}$$

Co-ordinates of B are 
$$\left(\frac{-8}{5}, \frac{-6}{5}\right)$$

- 8. Find the image of the point straight line 3x + 4y 1 = 0. Sol. Ans:  $\left(-\frac{7}{5}, -\frac{6}{5}\right)$
- 9. Show that the distance of the point (6,-2) from the line 4x + 3y = 12 is half of the distance of the point (3,4) from the line 4x 3y = 12.
- **10.** Find the locus of foot of the perpendicular from the origin to a variable straight line which always passes through a fixed point (a,b).



Sol.

Let P(x,y) be the foot of the perpendicular from O(0,0) to the line.

Slope of OP = 
$$\frac{y}{x}$$
  
Line is passing through A(a,b).  
Slope of AP =  $\frac{y-a}{x-b}$   
Line AP is perpendicular to OP,  $\Rightarrow$  product of slopes = -1  
 $\Rightarrow \frac{y}{x} \cdot \frac{y-b}{x-a} = -1 \Rightarrow y^2 - by = -(x^2 - ax)$   
 $\Rightarrow x^2 + y^2 - ax - by = 0$ 

III.

1. Show that the lines x - 7y - 22 = 0, 3x + 4y + 9 = 0 and 7x + y - 54 = 0 form a right angled isosceles triangle.

Sol. Given lines 
$$x - 7y - 22 = 0$$
 ------(1)  
 $3x + 4y + 9 = 0$  ------(2)  
 $7x + y - 54 = 0$  ------(3)  
C  
(1) (3)  
A (2) B  
Let 'A' be the angle between (1),(2)  
 $\cos A = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}} = \frac{|3 - 28|}{\sqrt{1 + 49}\sqrt{9 + 16}} = \frac{25}{5\sqrt{2.5}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$   
 $\Rightarrow A = 45^{\circ}$ 

Let B be the angle between (2) ,(3) then  $\cos B = \frac{21+4}{\sqrt{9+16}\sqrt{49+1}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$ 

 $\Rightarrow$  B = 45°

Let 'C' be the angle between (3), (1)

$$\cos C = \frac{7-7}{\sqrt{1+49}\sqrt{49+1}} = 0 \implies C = 90^{\circ}$$

Since  $|\underline{A}| = |\underline{B}| = 45^{\circ} \text{and} |\underline{C}| = 90^{\circ}$ 

: Given lines form a right angled isosceles triangle.

- 2. Find the equation of the straight lines passing through the point (-3,2) and making an angle of  $45^{\circ}$  with the straight line 3x y + 4 = 0.
- **Sol.** Given point P (-3,2)

Given line 3x - y + 4 = 0 -----(1)

Slope 
$$m_1 = -\frac{a}{b} = 3$$
  
 $45^{\circ}$   $45^{\circ}$   
 $3x - y + 4 = 0$ 

Let m be the slope of the required line.

Then 
$$\tan 45^\circ = \frac{m-3}{1+3m}$$
  

$$\Rightarrow \left| \frac{m-3}{1+3m} \right| = 1 \Rightarrow \frac{m-3}{1+3m} = 1$$

$$\Rightarrow m-3 = 1+3m \Rightarrow 2m = -4 \text{ or } m = -2$$

$$\Rightarrow \frac{m-3}{1+3m} = - \Rightarrow m-3 = -1-3m$$

$$\Rightarrow 4m = 2 \Rightarrow m = \frac{1}{2}$$

**case** (1) m = -2 and point (-3,2) Equation of the line is

$$y - 2 = -2(x + 3) = -2x - 6 \implies 2x + y + 4 = 0$$

**case (2)**  $m = \frac{1}{2}$ , point (-3,2)

Equation of the line is

$$y-2 = \frac{1}{2}(x+3) \Longrightarrow 2y-4 = x+3 \qquad \qquad \Rightarrow x-2y+7 = 0$$

- 3. Find the angle of the triangle whose sides are x + y 4 = 0, 2x + y - 6 = 0, 5x + 3y - 15 = 0.
- **Sol.** Ans:  $\cos^{-1} = \left(\frac{4}{\sqrt{17}}\right), \quad \cos^{-1}\left(\frac{13}{\sqrt{170}}\right), \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$
- 4. Prove that the foot of the perpendiculars from the origin on the lines x + y = 4, x + 5y = 26 and 15x 27y = 424 are collinear.
- 5. Find the equation of the lines passing through the point of intersection of the lines 3x + 2y + 4 = 0, 2x + 5y = 1 and whose distance form (2, -1) is 2.
- Sol. Equation of the lines passing through the point of intersection of the line

$$\begin{split} L_{1} &= 3x + 2y + 4 = 0, L_{2} = 2x + 5y - 1 = 0 \text{ is} \\ L_{1} + \lambda L_{1} = 0 \\ &\Rightarrow (3 + 2\lambda)x + (2 + 5\lambda)y + (4 - \lambda) = 0 - \dots - (1) \\ \text{Given distance from } (2, -1) \text{ to } (1) &= 2 \\ &\frac{|3 + 2\lambda)2 + (2 + 5\lambda)(-1) + (4 - \lambda)|}{\sqrt{(3 + 2\lambda)^{2} + (2 + 5\lambda)^{2}}} = 2 \\ &\Rightarrow \frac{|-2\lambda + 8|}{\sqrt{(3 + 2\lambda)^{2} + (2 + 5\lambda)^{2}}} = 2 \\ &\Rightarrow (-\lambda + 4)^{2} = 9 + 4\lambda^{2} + 12\lambda + 4 + 25\lambda^{2} + 20\lambda \\ &\Rightarrow 28\lambda^{2} + 40\lambda - 3 = 0 \\ &\Rightarrow 28\lambda^{2} - 2\lambda + 42\lambda - 3 = 0 \\ &\Rightarrow (2\lambda + 3)(14\lambda - 1) = 0 \Rightarrow \lambda = \frac{1}{14}, \lambda = -\frac{3}{2} \\ \text{From (1)} \\ \text{If } \lambda = \frac{1}{14}, \text{ then equation of the line is } 4x + 3y + 5 = 0 \\ \text{If } \lambda = -\frac{3}{2}, \text{ then equation of the line is } y - 1 = 0. \end{split}$$

6. Each side of a square is of length 4 units. The center of the square is (3, 7) and one of its diagonals is parallel to y = x. Find the Co-ordinates of its vertices.

**Sol.** Let ABCD be the square. Side AB = 4

Point of intersection of the diagonals is the center P(3, 7)



From P drawn PM  $\perp$  AB. Then M is midpoint of AB  $\therefore$  AM = MB = PM = 2

Since a diagonal is parallel to y = x, its sides are parallel to the co-ordinate axes.

M(3, 5)

 $\Rightarrow$ A (3-2, 5), B(3+2, 5), C(3+5, 7+2),D(3-2,7+2)

 $\Rightarrow$ A (1, 5), B(5, 5), C(5, 9),D(1,9)

- 7. If ab > 0, find the area of the rhombus enclosed by the four straight lines  $ax \pm by \pm c = 0$ .
- Sol. let the Equation of AB be ax + by + c = 0 ---(1) Equation of CD be ax + by - c = 0 ---(2)



Equation of BC be ax - by + c = 0 ---(3) Equation of AD be ax - by - c = 0 ---(4) Solving (1) and (3),  $B = \left(-\frac{c}{a}, 0\right)$ Solving (1) and (4),  $A = \left(0, -\frac{c}{b}\right)$ Solving (2) and (3),  $C = \left(0, \frac{c}{b}\right)$ 

Solving (2) and (4), 
$$D = \left(\frac{c}{a}, 0\right)$$

Area of rhombus ABCD =  $\frac{1}{2} |\sum x_1(y_2 - y_4)|$ 

$$= \frac{1}{2|0(0-0) - \frac{c}{a}\left(\frac{c}{b} + \frac{c}{b}\right) + 0(0-0) + \frac{c}{a}\left(\frac{-c}{b} - \frac{c}{b}\right)|}$$
$$= \frac{1}{2} \cdot \frac{4c^2}{|ab|} = \frac{2c^2}{|ab|} \text{ sq. units}$$

- 8. Find the area of the parallelogram whose sides are 3x + 4y + 5 = 0, 3x + 4y 2 = 0, 2x + 3y + 1 = 0 and 2x + 3y - 7 = 0
- Sol. Given sides are

3x + 4y + 5 = 0	(1)
3x + 4y - 2 = 0	(2)
2x + 3y + 1 = 0	(3)
2x + 3y - 7 = 0	(4)

Area of parallelogram formed by (1), (2), (3), (4)

$$= \left| \frac{(c_1 - c_2)(d_1 - d_2)}{a_1 b_2 - a_2 b_1} \right| = \left| \frac{(5+2)(1+7)}{3(3) - 2(4)} \right|$$
$$= \left| \frac{7 \times 8}{9-8} \right| = \frac{56}{1} = 56 \text{ sq. units}$$

