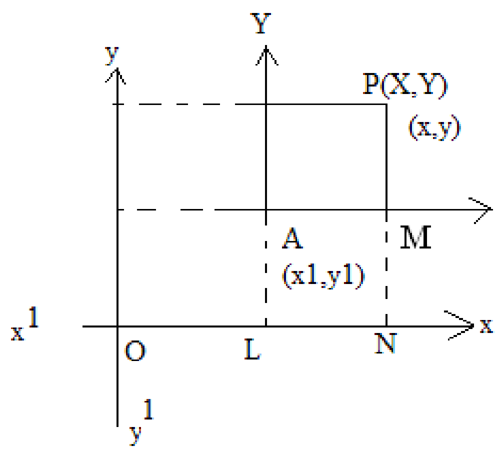
TRANSLATION OF AXES.

1. **Definition**: Without changing the direction of the axes, the transformation in which the origin is shifted to another position or point is called translation of axes.

2. **Theorem: To find the co-ordinates of a point (x, y) are translated by shifting the origin to a point (x_1, y_1)**

Proof :



Let xox^1 xox^1, yoy^1 be the original axes and $A(x_1, y_1)$ be a point to which the origin is shifted.

Let AX, AY be the new axes which are parallel to the original axes as in figure.

Let P be any point in the plane whose coordinates w.r.t old system are (x, y)

And w.r.t new axes are (X, Y) .

From figure, $A = (x_1, y_1)$ then $AL = y_1$, $OL = x_1$,

$P(x, y)$ then $x = ON = OL + LN = OL + AM = x_1 + X = X + x_1$

Hence $x = X + x_1$.

y=PN = PM +MN = X+ AL = X + y_1

therefore, $y = Y + y_1$

hence $x = X + x_1$, $y = Y + y_1$

3.Theorem: To find the point to which the origin is to be shifted by translation of axes to eliminate x, y terms(first degree terms) in the equation $ax^2+2xhy+by^2+2gx+2fy+c=0\left(h^2=ab\right)$

Proof : given equation is $ax^2+2xhy+by^2+2gx+2fy+c=0$

Let (x_1, y_1) be a point to which the origin is shifted by translation

Let (X, Y) be the new co-ordinates of the point (x, y) .

$\therefore$ the equations of the transformation are $x=X+x_1,\ y=Y+y_1$

Now the transformed equation is

$a(X+x_1)^2+2h(X+x_1)(Y+y_1)+b(Y+y_1)^2+2g(X+x_1)+2f[Y+y_1]+c=0$

$\Rightarrow a\left(X^2+2x_1X+x_1^2\right)+2h(XY+x_1Y+y_1X+x_1y_1)+b\left(Y^2+2y_1Y+y_1^2\right)+2g(X+x_1)+2f(Y+y_1)+c=0$

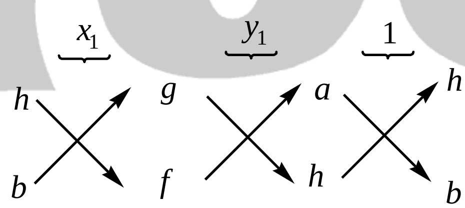
$\Rightarrow aX^2+2hXY+bY^2+2X(ax_1+hy_1+g)+2Y(hx_1+by_1+f)+\left(ax_1^2+2x_1+by_1^2+2gx_1+2fy_1+c\right)=0$

Since x, y terms (the first degree terms) are to be eliminated

$ax_1+hy_1+g=0$

$hx_1+by_1+f=0$

Solving these two equations by the method of cross pollination



$$\frac{x_1}{hf-bg}=\frac{y_1}{gh-af}=\frac{1}{ab-h^2} \quad \Rightarrow x_1=\frac{hf-bg}{ab-h^2},\ y_1=\frac{gh-af}{ab-h^2}$$

New origin is $\Rightarrow (x_1, y_1)=\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$

Note: (i) The point to which the origin has to be shifted to eliminate x, y terms by translation in the equation $ax^2+by^2+2gx+2fy+c=0$ is $\left(\frac{-g}{a}, \frac{-f}{b}\right)$

If b = a, then the new origin is $\left(\frac{-g}{a}, \frac{-f}{a}\right)$

(ii) The point to which the origin has to be shifted to eliminate x, y terms by translation of axes in the equation $a(x+x_1)^2+b(y+y_1)^2=c$ **is** $(-x_1, -y_1)$

iii) The point to which the origin has to be shifted to eliminate x, y terms by translation in the equation $2hxy+2gx+2fy+c=0$ is $\left(\frac{-f}{h}, \frac{-g}{h}\right)$

Theorem : To find the condition that the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ **to be in the form** $aX^2+2hXY+bY^2=0$ **when the axes are translated.** $(h^2 \neq ab)$

Proof : From theorem 3, we get

$$ax_1+hy_1+g=0 \quad \_\_\_\_(1)$$

$$hx_1+by_1+f=0 \quad \_\_\_\_(2)$$

Solving (1) and (2),

$$(x_1, y_1)=\left(\frac{hf-bg}{ab-h^2}, \frac{gf-af}{ab-h^2}\right)$$

Since the equation is to be in the form of $aX^2+2hXY+bY^2=0$,then for this we should have

$$ax_1^2+2hx_1y_1+by_1^2+2gx_1+2fy_1+c=0$$

$$\Rightarrow (ax_1+hy_1+g)x_1+(x_1+by_1+f)y_1+(gx_1+fy_1+c)=0$$

$\Rightarrow (0).x_1+(0), y_1+(gx_1+fy_1+c)=0$ from(1) and (2) $\Rightarrow gx_1+fy_1+c=0$ ____ (3)

Substituting x_1, y_1 in (3), we get

$$g\left(\frac{hf-bg}{ab-h^2}\right)+f\left(\frac{gh-af}{ab-h^2}\right)+c=0$$

$$\Rightarrow fgh-bg^2+fgh-af^2+abc+-ch^2=0$$

$$\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$$

PROBLEMS.

1. **When the origin is shifted to (4,-5) by the translation of axes, find the co-ordinates of the following points with reference to new axes.?**

Sol. i) (0,3)

New origin = (4,-5)= (h,k)

Old co-ordinates are (0,3) =(x,y)

$X = x - h = 0 - 4 = -4$

$Y = y - k = 3 + 5 = 8$

New co-ordinates are (-4,8)

ii) (-2,4) ANS. (-6,9)

iii) (4,-5)

ans. (0,0)

2. **The origin is shifted to (2,3) by the translation of axes. If the co-ordinates of a point P changes as follows, find the co-ordinates of Pin the original system.?**

i)(4,5) ii) (-4,3) iii) (0,0)

Sol. New origin = (2,3) =(h,k)

i) New co-ordinates are(4,5)

$x = 4, y = 5$

$x = x + h = 4 + 2 = 6$

$y = y + k = 5 + 3 = 8$

Old co-ordinates are (6,8)

ii) New co-ordinates are (-4,3)

ANS. (-2,6)

iii) New co-ordinates are (0,0)

ANS. (2,3)

3. Find the point to which the origin is to be shifted so that the point (3,0)may change to (2,-3)?

Sol. $(x, y) = (3, 0)$

$(x^1, y^1) = (2, -3)$

Let (h,k) be the new origin

$\therefore\ h = x - x^1 = 3 - 2 = 1$

$k = y - y^1 = 0 + 3 = 3$

$\therefore\ (h, k) = (1, 3)$

4. When the origin is shifted to (-1,2) by the translation of axes, find the transformed equations of the following.?

i) $x^2 + y^2 + 2x - 4y + 1 = 0$

ii) $2x^2 + y^2 - 4x + 4y = 0$

Sol. i)The given equation is $x^2 + y^2 + 2x - 4y + 1 = 0$ **---------(1)**

Origin is shifted to (-1,2) $\therefore$ h=-1, k=2

Equations of transformations are $x = X + h, y = Y + k$

i.e, $x = X - 1,\ y = Y + 2$

The new equation is

$(X-1)^2 + (Y+2)^2 + 2(X-1) - 4(Y+2) + 1 = 0$

$\Rightarrow X^2 + 1 - 2X + Y^2 + 4 + 4Y + 2X - 2 - 4Y - 8 + 1 = 0$

$\Rightarrow X^2 + Y^2 - 4 = 0$

ii) Old equation is $2x^2 + y^2 - 4x + 4y = 0$

Ans. $2X^2 + Y^2 - 8X + 8Y + 18 = 0$

5. The point to which the origin is shifted and the transformed equation are given below. Find the original equation.?

i) $(3,-4); x^2+y^2=4$ **ii)** $(-1,2); x^2+2y^2+16=0$

Sol. i) New origin = $(3,-4)=(h,k)$

$\therefore\ X=x-h,\quad Y=y-k$

= x -3 = y +4

The original equation of $(X)^2+(Y)^2=4$ is $(x-3)^2+(y+4)^2=4$

$x^2-6x+9+y^2+8y+16=4$

$\therefore x^2+y^2-6x+8y+21=0$

ii) New origin = $(h,k)=(-1,2)$

$X=x-h,\quad Y=y-k$

= x + 1 = y – 2

The original equation of $X^2+2Y^2+16=0$

$(x+1)^2+2(y-2)^2+16=0$

$x^2+2x+1+2y^2-8y+8+16=0 \Rightarrow x^2+2y^2+2x-8y+25=0$

6. **Find the point to which the origin is to be shifted so as to remove the first degree terms from the equation,** $4x^2+9y^2-8x+36y+4=0$

Sol. The given equation is

$4x^2+9y^2-8x+36y+4=0$ comparing with $ax^2+2hxy+by^2+2gx+2fy+c=0$

a = 4 h =0 g =-4 b = 9 f = 18

$-\frac{g}{a}=\frac{4}{4}=1,\ -\frac{f}{b}=-2$

Origin should be shifted to $\left(\frac{-g}{a},\frac{-f}{b}\right)$ = (1,-2)

SHORT ANSWER QUESTIONS

1. **When the origin is shifted to the point (2,3), the transformed equation of a curve is** $x^2+3xy-2y^2+17x-7y-11=0$. **Find the original equation of the curve?**

Sol. New origin =(2,3) = (h,k)

Equations of transformation are

$X = x+h,\ y = Y+k$

$X = x-h = x-2, Y = y-3$

Transformed equation is

$x^2+3xy-2y^2+17x-7y-11=0$

Original equation is

$(x-2)^2+3(x-2)(y-3)-2(y-3)^2+17(x-2)-7(y-3)-11=0$

$x^2+4x+4+3xy-9x-6y+18-2y^2+12y-18+17x-34-7y+21-11=0$

$\Rightarrow x^2+3xy-2y^2+4x-y-20=0$ is the required original equation.

3. **If the transformed equation of a curve is** $3x^2+xy-y^2-7x+y-7=0$ **when the origin is shifted to the point (1,2)by translation of axes, find the original equation of the curve?**

Sol. Same as above problem.

Ans. $3x^2+xy-y^2-15x+4y+13=0$.