MATHEMATICS PAPER IA- JUNE 2008.
ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY

TIME: 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A, B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Note: Attempt all questions. Each question carries 2 marks.

1. N is the set of natural numbers. Is the function f:N → N defined by f(x) = 2x+5 onto ? Explain the reason.
2. If $f=\{(1,2),(2,-3),(3,-1)\}$ then find $2+f$
3. The position vectors of A and B are **a** and **b** respectively. If C is a point on the line $\overleftrightarrow{AB}$ such that $\overline{AC}=5\overline{AB}$ then find the position vector of C.
4. Find the vector equation of the line joining the points 2**i** + **j** + 3**k** and **– 4i + 3j – k.**
5. Find the area of the parallelogram having 2 i -3j and 3i -3k as adjacent sides .
6. write the period of cos(6-5x).
7. If $\cos\theta=\frac{1}{4}$ and $270^0<\theta<360^0$ then find $\tan\frac{\theta}{2}$.
8. If $\tanh^2 x=\tan^2\theta$ then prove that cosh2x = sec 2θ.
9. In ΔABC , s=12 and A =90^0 then find r_1.
10. IF Z = x+iy is a point in the Argand plane such that $|Z-3+i|=4$ then find the locus of Z.

SECTION B

SHORT ANSWER TYPE QUESTIONS **5X4 =20**

Note : Answer any FIVE questions. Each question carries 4 marks.

11. Show that the points with position vectors -2i+3j+6k, 6i-2j+3k, 3i+6j-2k form an equilateral triangle.
12. By vector method prove that angle in a semi circle is a right angle.
13. If A+B =225^0 and none of A and B is an integral multiple of π then prove that $\left(\frac{\cot A}{1+\cot A}\right)\left(\frac{\cot B}{1+\cot B}\right)=\frac{1}{2}$.
14. Solve $\sqrt{2}(\sin x+\cos x)=\sqrt{3}$
15. solve $\mathrm{Tan}^{-1}\left(\frac{x+1}{x-1}\right)+\mathrm{Tan}^{-1}\left(\frac{x-1}{x}\right)=\pi+\mathrm{Tan}^{-1}(-7)$
16. In ΔABC show that $(b-c)^2\cos^2\frac{A}{2}+(b+c)^2\sin^2\frac{A}{2}=a^2$
17. Show that $32\cos^2\theta\sin^4\theta=\cos 6\theta-2\cos 4\theta-\cos 2\theta+2$

SECTION C

LONG ANSWER TYPE QUESTIONS **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. If $f : A \to B,\ g : B \to C$ be bijections. Then show that $gof : A \to C$ is a bijection.
19. Using mathematical induction, Show that $3.5^{2n+1} + 2^{3n+1}$ is divisible by 17 for all n € N.
20. **a,b,c** are non-zero vectors and **a** is perpendicular to both **b** and **c**. If |**a**| = 2, |**b**| = 3, |**c**| = 4 and
    (**b,c**) = , then find |[**abc**]|.
21. If $A+B+C=2S$, then prove that $\sin(S-A)\sin(S-B)+\sin S\sin(S-C)=\sin A\sin B$
22. In ΔABC , prove that $\frac{ab-r_1r_2}{r_3}=r$
23. A pillar is leaning towards east and a and b are the angles of elevation of the top of the pillar from two points due west of the pillar at distance a and b respectively. Show that the angle between the pillar and the horizontal is $\mathrm{Tan}^{-1}\left(\frac{b-a}{b\cot\alpha-a\cot\beta}\right)$.
24. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then show that
    (i) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$ (ii) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$