Geometrical Representation of Complex Numbers

Very Short Answer Questions

1. Show that the triangle formed with the points in the argand diagram represented by $(2+2i), -2-2i -2\sqrt{3}+2\sqrt{3}\,i$ is equilateral triangle.

Solution: -

Let $A(2,2)\,B(-2,-2)\,C\left(-2\sqrt{3},2\sqrt{3}\right)$ be the points represented by the given complex numbers $2+2i, -2-2i -2\sqrt{3}+2\sqrt{3}\,i$ respectively

$$AB=\sqrt{(2+2)^2+(2+2)^2}=\sqrt{32}$$

$$BC=\sqrt{\left(-2\sqrt{3}+2\right)^2+\left(2\sqrt{3}+2\right)^2}=\sqrt{32}$$

$$CA=\sqrt{\left(-2\sqrt{3}-2\right)^2+\left(2\sqrt{3}-2\right)^2}=\sqrt{32}$$

$$AB=BC=CA$$

$\therefore$ Triangle ABC is equilateral

2. Find the equation of the perpendicular bisector of the lines segment joining the points $7+7i, 7-7i$ in the argand diagram

Solution: -

Let A(7, 7) B (7, - 7) be the ends of the line segment represented by the complex numbers $(7+7i)$ *and* $(7-7i)$ represents

Mid point of $AB=(7,0)$

Slope of $AB=\dfrac{-7-7}{7-7}$

$\therefore$ Slope of perpendicular bisector = 0

Equal of perpendicular bisector is $(y-0)=0(x-7)$

$y=0$

3. **If** $z = x + iy$ **and if the point P in the argand plane represents** z_1 **find the locus of z satisfying the equation**

(i) $|z - 2 - 3i| = 5$ **(ii)** $2|z - 2| = |z - 1|$ **(iii)** $\text{Im}\left(z^2\right) = 4$

Solution (i) $|z - 2 - 3i| = 5$

$$|z - 2 - 3i| = 5 \Rightarrow \sqrt{(x-2)^2 + (y-3)^2} = 5$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

(ii) $z|z - 2| = |z - 1|$

$$2|(x-2) + iy| = |(x-1) + iy| \Rightarrow 2\sqrt{(x-2)^2 + y^2} = (x-1)^2 + y^2$$

$$4\left\{(x-2)^2 + y^2\right\} = (x-1)^2 + y^2 \Rightarrow 4x^2 + 4y^2 - 16x + 16 = x^2 + y^2 - 2x + 1$$

$$\therefore 3x^2 + 3y^2 - 14x + 15 = 0$$

(iii) $\text{Im}\left(z^2\right) = 4$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy$$

$\text{Im}\left(z^2\right) = 2xy$ {$\text{Im}\left(z^2\right)$ means Imaginary part of z^2}

$$im\left(z^2\right) = 4 \Rightarrow 2x\,y = 4 \Rightarrow xy = 2$$

SHORT ANSWER QUESTIONS

1. **If** $\frac{z_3 - z_1}{z_2 - z_1}$ **is a real number then show that the points represented by the complex number** z_1, z_2, z_3 **are collinear**

Solution : -

Let $\frac{z_3 - z_1}{z_2 - z_1} = k$ where K is a real number

$$z_3 - z_1 = kz_2 - kz_1$$

$$kz_1 - z_1 = kz_2 - z_3$$

$$z_1 = \frac{kz_2 - z_2}{k-1}$$

i.e, z_1 divides the line joining of z_2 *and* z_3

Externally in the ratio $K:1$

Hence z_1, z_2, z_3 are collinear

2. **Show that the four points in the argand plane represented by the complex numbers** $z+i, 4+3i, 2+5i, 3i$ **are the vertices of a square**

Solution: -

Let $A(2,1)\, B(4,3)\, C(2,5)\, D(0,3)$ be the given vertices

$AB = \sqrt{(4-2)^2 + (3-1)^2} = \sqrt{8}$ $\quad$ $BC = \sqrt{(2-4)^2 + (5-3)^2} = \sqrt{8}$

$CD = \sqrt{(0-2)^2 + (3-5)^2} = \sqrt{8}$ $\quad$ $AD = \sqrt{(0-2)^2 + (3-1)^2} = \sqrt{8}$

$AC = \sqrt{(4-2)^2 + (5-1)^2} = 4$ $\quad$ $BD = \sqrt{(0-4)^2 + (3-3)^2} = 4$

Here $AB = BC = CD = AD$ *and* $AC = BD$

Hence ABCD is a square

3. **Show that the points represented by the complex number** $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ **are the vertices of a rhombus**

Solution ; -

Let $A(-2,7)\, B\left(-\frac{3}{2}, \frac{1}{2}\right) C(4,-3)\, D\left(\frac{7}{2}, \frac{7}{2}\right)$ be the given vertices

$AB = \sqrt{\left(-\frac{3}{2}+1\right)^2 + \left(\frac{1}{2}-7\right)^2} = \sqrt{\frac{170}{4}}$ $\quad$ $CD = \sqrt{\left(\frac{7}{2}-4\right)^2 + \left(\frac{7}{2}+3\right)^2} = \sqrt{\frac{170}{4}}$

$$BC = \sqrt{\left(4+\frac{3}{2}\right)^2 + \left(-3-\frac{1}{2}\right)^2} = \sqrt{\frac{170}{4}}$$

$$AD = \sqrt{\left(\frac{7}{2}+2\right)^2 + \left(\frac{7}{2}-7\right)^2} = \sqrt{\frac{170}{4}}$$

$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{136}$

$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{136}$

$AB=BC=CD=AD$ and $AC\neq BD$

Hence ABCD is a rhombus

4. Show that the points in the argand diagram represented by the complex numbers z_1, z_2, z_3 are collinear if and only if there exists three real numbers p, q, r not all zero satisfying $pz_1+qz_2+rz_3=0$ *and* $p+q+r=0$

Solution : -

Given $pz_1+qz_2+r\,z_3=0$ *and* $p+q+r=0$

$\therefore r=-p-q$

$\therefore pz_1+qz_2+(-p-q)z_3=0 \quad \{\because r=-p-q\}$

$pz_1+qz_2=(p+q)z_3$

$z_3=\dfrac{qz_2+pz_1}{q+p}$

z_3 divides the line joining of z_1 *and* z_2 in the ratio q : p

$\therefore z_1, z_2, z_3$ are collinear

Given z_1, z_2, z_3 are collinear the

$\therefore$ Let z_2 divide line joining of z_1 & z_3 in the ratio K : l

$\therefore z_2=\dfrac{k\,z_3+l\,z_1}{k+l}$

$(z_1+(-k-l)\,z_2+k\,z_3=0)$

This is of the form $pz_1+qz_2+r\,z_3=0$

Where $p=l \;\; q=-k-l\,;\, r=k$

$p+q+r=0$

Hence Proved

5. **The points P, Q denote the complex numbers $z_1\ z_2$ in the argand diagram. O is the origin if $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ then show that $\lfloor POQ = 90^0$**

Solution : -

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\therefore P = (x_2\ y_1)\ \ Q = (x_2\ y_2)$

Slope of $OP = \frac{y_1}{x_1}$ slope of $OQ = \frac{y_2}{x_2}$

Given $z_1\bar{z}_2 + z_1\ \bar{z}_2 = 0$

$(x_1 + iy_1)(x_2 - iy_2) + (x_1 - iy_1)(x_2 + iy_2) = 0$

$x_1x_2 - i\ \cancel{x_1}\ y_2 + i\ \cancel{x_2}\ y_1 + y_1y_2 + x_1x_2 + i\ \cancel{x_1}\ y_2 - i\ \cancel{x_2}\ y_1 + y_1y_2 = 0$

$x_1x_2 + y_1y_2 = 0 \Rightarrow = -\ x_1x_2 = y_1y_2$

$\therefore \frac{y_1}{x_1} \times \frac{y_2}{x_2} = -1$

Slope of $OP \times$ slope of $OQ = -1$

$\therefore \lfloor POQ = 90^0$