(iii) Mod-amplitude form of complex number

Operations on Complex Numbers

VERY SHORT ANSWER QUESTIONS

1. Express the following complex numbers in Modulus amplitude from

(I) $1-i$ **(ii)** $1+i\sqrt{3}$ **(iii)** $-\sqrt{3}+i$ **(iv)** $-1-i\sqrt{3}$

Solution : -

Let $1-i=r\{\cos\theta+i\sin\theta\}$

$r\cos\theta=1\ r\sin\theta=-\ 1$

$(r\cos\theta)^2+(r\sin\theta)^2=2\Rightarrow r=\sqrt{2}$

$\cos\theta=\dfrac{1}{\sqrt{2}}\quad \sin\theta=-\dfrac{1}{\sqrt{2}}$

Principal value of $\theta=-\pi/4\quad\{\because -\pi\le\theta\le\pi\}$

$\therefore 1-i=\sqrt{2}\left\{\cos\left(-\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{-\pi}{4}\right)\right\}$

(ii) $1+i\sqrt{3}=r\{\cos\theta+i\sin\theta\}$

$\therefore r\cos\theta=1\quad r\sin\theta=\sqrt{3}$

$(r\cos\theta)^2+(r\sin\theta)^2=1+\left(\sqrt{3}\right)^2$

$r^2=4\Rightarrow r=2$

$\cos\theta=\dfrac{1}{2}\ \sin\theta=\dfrac{\sqrt{3}}{2}$

$\therefore$ Principal value of $\theta=\dfrac{\pi}{3}$

$\therefore 1+i\sqrt{3}=2\left\{\cos\dfrac{\pi}{3}+i\sin\pi/3\right\}$

(iii) $-\sqrt{3}+i=r\{\cos\theta+i\sin\theta\}$

$r\cos\theta=-\sqrt{3} \quad r\sin\theta=1$

$(r\cos\theta)^2\,(r\sin\theta)^2=3+1\Rightarrow r=2$

$\cos\theta=-\dfrac{\sqrt{3}}{2} \;\; \sin\theta=\dfrac{1}{2}$

$\therefore$ Principal value of $\theta=\dfrac{5\pi}{6}$

$\therefore-\sqrt{3}+i=2\left\{\cos\dfrac{5\pi}{6}+i\sin\dfrac{5\pi}{6}\right\}$

(iv) $-1-i\sqrt{3}=r\{\cos\theta+i\sin\theta\}$

$r\cos\theta=-1 \quad r\sin\theta=-\sqrt{3}$

$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+\left(-\sqrt{3}\right)^2\Rightarrow r^2=4\Rightarrow r=2$

$\cos\theta=-\dfrac{1}{2};\sin\theta=-\dfrac{\sqrt{3}}{2}$

Prove of $\theta=-2\pi/3$

$\therefore-1-i\sqrt{3}=2\left\{\cos\left(-\dfrac{2\pi}{3}\right)+i\sin\left(-\dfrac{2\pi}{3}\right)\right\}$

2. Simplify $-2i(3+i)(2+4i)(1+i)$ and obtain the modulus of that complex number

Solution : -

$-2i\{3+i\}(2+4i)(1+i)=-2i(3+i)\{2+2i+4i+4i^2\}$

$=-2i(3+i)(-2+6i)=-2i\{-6+18i-2i-6\}$

$=-2i\{-12+16i\}=24i-32i^2=32+24i$

$|32+24i|=\sqrt{(32)^2+(24)^2}=\sqrt{1024+576}=40$

3. **If** $z \neq 0$ **find** $Arg\ z + Arg\ \overline{z}$

If $Arg\ z = \theta$ then $Arg\overline{z} = -\theta$

$\therefore Argz + Arg\ \overline{z} = \theta - \theta = 0$

4. **If** $z_1 = -1$ *and* $z_2 = -i$ **then find** $Arg(z_1\ z_2)$

Solution: -

$$z_1 = -1 \Rightarrow Argz_1 = \pi \ \{\because -1 = \cos\pi + i\sin\pi\}$$

$$z_2 = -i \Rightarrow Arg\ z_2 = -\pi/2 \{\because -i = \cos -\pi/2 + 1\sin(-\pi/2)\}$$

$$Arg(z_1\ z_2) = Argz_1 + Arg\ z_2 = \pi - \pi/2 = \pi/2$$

5. **If** $z_1 = -1 : z_2 = i$ **then find** $Arg\left(\frac{z_1}{z_2}\right)$

Solution : -

$$Argz_1 = \pi \ \{\because -1 = \cos\pi + 1\sin\pi\}$$

$$Arg\ z_2 = \pi/2 \ \{\because i = \cos\pi/2 + i\sin\pi/2\}$$

$$Arg\left(\frac{z_1}{z_2}\right) = Argz_1 - Argz_2 = \pi - \pi/2 = \pi/2$$

SHORT ANSWER QUESTIONS

1. **Simplify and find the Modulus of the following complex no's**

(i) $\frac{(2+4i)(-1+2i)}{(-1-i)(3-i)}$ **(ii)** $\frac{(1+i)^3}{(2+i)(1+2i)}$

Solution: $\frac{(2+4i)(-1+2i)}{(-1-i)(3-i)} = \frac{-2+4i-4i-8}{-3+i-3i+i^2}$

$$= \frac{-10}{-4-2i} = \frac{5}{2+i} = \frac{5(2-i)}{(2+i)(2-i)} = \frac{\cancel{5}(2-i)}{\cancel{5}}$$

$$\left|\frac{(2+4i)(-1+2i)}{(-1-i)(3-i)}\right| = |2-i| = \sqrt{5}$$

(ii) $\dfrac{(1+i)^3}{(2+i)(1+2i)} = \dfrac{1+i^3+3i(1+i)}{2+5i+2i^2} = \dfrac{1-i+3i-3}{5i}$

$= \dfrac{-2+2i}{5i} \times \dfrac{5i}{5i} = \dfrac{-10i+10i^2}{25i^2}$

$\dfrac{-10-10i}{-25} = \dfrac{2+2i}{5}$

$\left|\dfrac{(1+i)^3}{(2+i)(1+2i)}\right| = \sqrt{\dfrac{4}{25}+\dfrac{4}{25}} = \dfrac{2\sqrt{2}}{5}$

2. If $(1-i)(2-i)(3-i)......(1-ni) = x = iy$ **then prove that**

$2.5.10..................(1+n^2) = x^2 + y^2$

Solution: -

$(1-i)(2-i)(3-i).......(1-ni) = x - iy$

Consider Modulus on both sides

$|(1-i)(2-i)(3-i).......(1-ni)| = \sqrt{x^2+y^2}$

$\sqrt{2}\ \ \sqrt{5}\ \ \sqrt{10}\ ....\sqrt{1+n^2} = \sqrt{x^2+y^2}$

Squaring on both sides

$2.5.10.......(1+n^2) = x^2 + y^2$

3. If the real part of $\dfrac{z+1}{z+i}$ **is 1 find the locus of z**

Solution: -

Let $z = x + iy$

$\text{Re}\left\{\dfrac{x+iy+1}{x+iy+1}\right\} = 1 \Rightarrow \text{Re}\dfrac{\{(x+1)+iy\}\{x-i(y+1)\}}{x^2+(y+1)^2} = 1$

Since real part of complex no is 1

$\dfrac{x(x+1) - i^2 y(y+1)}{x^2+(y+1)^2} = 1$

$$\cancel{x^2} + x + \cancel{y^2} + y = \cancel{x^2} + \cancel{y^2} + 2y + 1 \Rightarrow x - y = 1$$

4. **If** $z = x + iy \ \ and \ |z| = 1$ **find the locus of z**

Solution : -

$$|z| = 1 \Rightarrow |x + iy| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1$$

Squaring on both sides

$$x^2 + y^2 = 1$$

5. **If the amplitude of** $(z-1)$ **is** $\frac{\pi}{2}$ **then find the locus of z**

Solution : -

Let $z = x + iy \quad Amp(z-1) = \pi/2$

Amp $\{x + iy - 1\} = \pi/2 \Rightarrow Amp\{(x-1) + iy\} = \pi/2$

$$\tan^{-1}\left(\frac{y}{x-1}\right) = \pi/2 \Rightarrow \frac{y}{x-1} = \frac{\sin \pi/2}{\cos \pi/2}$$

$$\Rightarrow x - 1 = 0$$

6. **If the** Arg $\bar{z}_1$ **and** $Arg\ z_2$ **are** $\frac{\pi}{5}\ and\ \frac{\pi}{3}$ **respectively find** $(Arg\ z_1 + Arg\ z_2)$

Solution : -

$$Arg(\bar{z}_1) = \frac{\pi}{5} \Rightarrow Arg\ z_1 = -\pi/5$$

$$Arg\ z_2 = \pi/3 \quad Arg\ z_2 = \pi/3$$

$$\therefore Arg\ z_1 + Arg z_2 = -\pi/5 + \pi/3 = \frac{-3\pi + 5\pi}{15} = \frac{2\pi}{15}$$