11. Complex Numbers

Topics Covered

1. **Complex numbers as an ordered pair of real numbers fundamental operations**
2. **Representation of complex numbers on the form a+ib**
3. **Conjugate of a complex number, Modulus and ampletude of a complex numbers**
4. **Geometical representation of a complex numbers Argand plane**
5. **Demovier's theorem**
6. **Trigonometric expansions**

I. FUNDAMENTAL OPERATIONS

Very Short Answer Questions

1. **If** $z_1 = (2, -1)\ z_2 = (6, 3)$ **then** $z_1 - z_2$

Given $z_1 = 2 - i \qquad z_2 = 6 + 3i$

$z_1 - z_2 = -4 - 4i = (-4, -4)$

2. **If** $z_1 = (3, 5)$ *and* $z_2 = (2, 6)$ **find** z, z_2

Solution : -

Given $z_1 = 3 + 5i \qquad z_2 = 2 + 6i$

$z_1\ z_2 = (3 + 5i)(2 + 6i) = 6 + 28i + 30i^2 = 6 - 30 + 28i$

$z_1 . z_2 = -24 + 28i = (-24, 28)$

3. Write the additive inverse of the following

(i) Additive inverse of $(\sqrt{3}, 5)$ *is* $(-\sqrt{3}, -5)$

(ii) Additive Inverse of $(-6, 5) + (10, -4)$ *is* $(-4, -1)$

(iii) Additive inverse of $(2, 1)(-4, 6)$

Let $z_1 = 2 + i \quad z_2 = -4 + 6i$

$z_1 z_2 = (2 + l)(-4 + 6i) = -8 + 8i - 6i^2$

$= -2 + 8i$

Additive inverse is (2, -8)

4. **If** $z_1 = (6, -3)$ *and* $z_2 = (2, -1)$ **then find** z_1 / z_2

Solution : -

Given $z_1 = 6 + 3i \quad z_2 = 2 - i$

$$\frac{z_1}{z_2} = \frac{6+3i}{2-i} = \frac{(6+3i)(2+i)}{4-i^2} = \frac{8+12i+3i^2}{5}$$

$$1 + \frac{12}{5}i = \left(1, \frac{12}{5}\right)$$

5. **If** $z = \cos\theta + i\sin\theta$ **then find** $z - \frac{1}{z}$

Solution : - $z = \cos\theta + i\sin\theta \; \frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$

$$z - \frac{1}{z} = 2i\sin\theta$$

6. **Write the multiplicative inverse of the following complex numbers**

(i) $(3, 4)$ **(ii)** $(\sin\theta, \cos\theta)$ **(iii)** $(7, 24)$ **(iv** $(-2, 1)$

Solution : -

(i) **Let** $z = 3 + 4i$

Multiplicative inverse of z is $\frac{1}{3+4i} = \frac{3-4i}{(3+4i)(3-4i)}$

$$= \frac{3-4i}{25} = \left(\frac{3}{25}, \frac{-4}{25}\right)$$

(ii) **Let** $z = \sin\theta + i\cos\theta$

Multiplicative inverse of $z = \frac{1}{\sin\theta + i\cos\theta} = \frac{\sin\theta - i\cos\theta}{(\sin\theta - i\cos\theta)}$

$$= \frac{\sin\theta - i\cos\theta}{\sin^2\theta + \cos^2\theta} = \sin\theta - i\cos\theta$$

(iii) **Let** $z = 7 + 24i$

Multiplicative inverse of $z = \frac{1}{7+24i} = \frac{7-24i}{(7+24i)(7-24i)}$

$= \frac{7-24i}{625}$