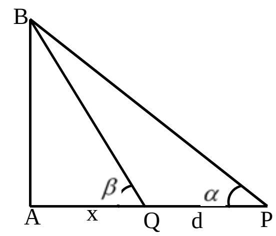
10. HEIGHTS AND DISTANCES

LONG ANSWER QUESTIONS

1. From a point on the level ground the angle of elevation of the top of the hill is α. After walking through a distance 'd' meters towards the foot of the hill the angles of elevation of the top is found to be β. Find the height of the hill



Solution:

AB be the hill of height h

P be the first point of observation

Q be the second point of observation

The angle of elevation of β

From P and Q are α *and* β respectively

Given PQ = d let AQ = x

In Δ^{le} BAQ $\tan\beta = \frac{h}{x} \Rightarrow x = h\cot\beta$

In Δ^{le} BAP $\tan\alpha = \frac{h}{d+x} \Rightarrow d + x = h\cot\alpha$

$$d + h\cot\beta = h\cot\alpha$$

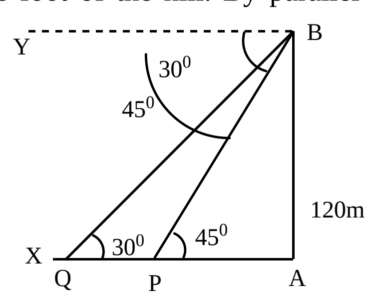
$$d = h\{\cot\alpha - \cot\beta\}$$

$$h = \frac{d}{\cot\alpha - \cot\beta}$$

2. From the cliff of a mountain 120 meters above the sea level the angles of depression of two boats on the same side of the mountain are 45^0 and 30^0 respectively. Find the distance between the both boats {assume the both the boats lie along the line from foot of the hill on the same sides of the hill ?

Solution:

Let AB be the hill Ax be the horizontal through the foot of the hill. By parallel to horizontal line passing through the observing.



Exp: Let P, Q be the two boats

Given $\angle y\ BQ = 30^0$

From the fig $\angle y\ BP = \angle BPA = 45^0$

$\underline{|BQ} = \lfloor BQA = 30^0$

Let PQ = x

In triangle PQAB

$$\tan 45^0 = \frac{AB}{PA} \Rightarrow 1 = \frac{120}{PA}$$

$$\Rightarrow PA = 120$$

In triangle BAQ

$$\tan 30^0 = \frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{120}{x + 120^0}$$

$$x = 120\left(\sqrt{3} - 1\right)$$

3. The angles of elevation of the top of a mountain from the top and bottom of a pillar AB of height 5 meters are 30^0 *and* 60^0 respectively. Find the height of the mountain?

Solution:

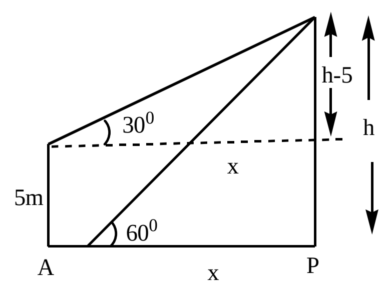
Let PQ be the mountain of height ‘h’

AB is the pillar of height 5m

Angles of elevation of Q from A and B are respectively $60^0 \textit{and } 30^0$

Let $AP = BC = x$

Since $AB = 5\ \ CQ = h - 5$



In triangle QPA $\tan 60^0 = \frac{h}{x} \rightarrow (1)$

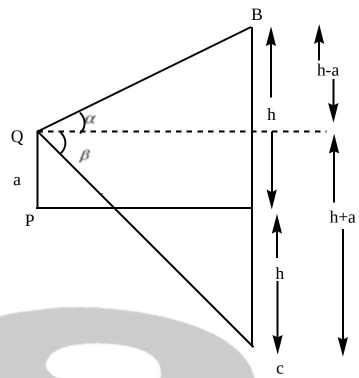
In triangle QCB $\tan 30^0 = \frac{h-5}{x} \rightarrow (2)$

(1) ÷ (2) we have $\frac{\tan 60^0}{\tan 30^0} = \frac{\frac{h}{x}}{\frac{h-5}{x}} \Rightarrow \frac{\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{h}{h-5}$

$$3=\frac{h}{h-5}\Rightarrow 3h-15=h\Rightarrow 2h=15\Rightarrow h=\frac{15}{2}$$

$\therefore$ Height of mountain $=7.5m$

4. From the top of a tree on the bank of a lake an aeroplane in the sky makes an angle of elevation α and its image in the river makes an angle of depression β. If the height of the tree from the water surface in a and the height of the aeroplane in h show that $h=\dfrac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$



Solution :

Let 'β' the position of aeroplane C b the its image

PQ be the tree of height 'a' draw QR parallel the AP

Angle of elevation of B from Q is α

Angle of depression of C from Q is β

BR = h – a $CR = h+Q$ Let $QR = PQ = x$

In triangle QRB $\tan\alpha=\dfrac{h-a}{x}\rightarrow$ (1)

In triangle QRC $\tan\beta=\dfrac{h+a}{x}\rightarrow$ (2)

(1) $\div$(2) we have $\dfrac{\tan\alpha}{\tan\beta}=\dfrac{h-a}{h+a}$

$$\therefore\frac{h+a}{h-a}=\frac{\tan\beta}{\tan\alpha}$$

Using compounds and dividends we have

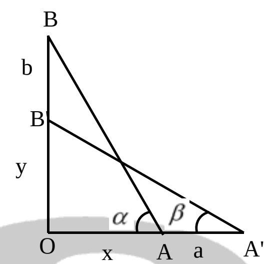
$$\frac{h+\not{a}-h-\not{a}}{\not{h}+a-\not{h}+a}=\frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}\Rightarrow\frac{h}{a}=\frac{\dfrac{\sin\beta}{\cos\beta}+\dfrac{\sin\alpha}{\cos\alpha}}{\dfrac{\sin\beta}{\cos\beta}-\dfrac{\sin\alpha}{\cos\alpha}}$$

$$\frac{h}{a}=\frac{\sin\beta\cos\alpha+\cos\beta\sin\alpha}{\sin\beta\cos\alpha-\cos\beta\sin\alpha}\Rightarrow h=\frac{a\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

5. **one end of the ladder is in contact with a wall and the other end is in contact with level ground making an angle α when the foot of the ladder is moved through a distance 'a' cms away from the wall, the end in contact with the wall slides through 'b' cms along the wall and the angle made by the ladder with the level is now β . Show that** $a = b\tan\left(\frac{\alpha+\beta}{2}\right)$

Solution : -

Let l be the length of the ladder OA = x and OB' = y where AB is the initial position of ladder and A'B' is the final position of ladder



Given AA = a   BB' = b

AB = A'B' = l

In triangle B OA

$$\sin\alpha = \frac{y+b}{l} \Rightarrow y+b = l\sin\alpha \rightarrow (1)$$

$$\cos\alpha = \frac{x}{2} \Rightarrow x = l\cos\alpha \rightarrow (2)$$

In triangle B'OA'

$$\sin\beta = \frac{y}{l} \Rightarrow y = l\sin\beta \rightarrow (3)$$

$$\cos\beta = \frac{a+x}{l} \Rightarrow a+x = l\cos\beta \rightarrow (4)$$

(1) - (3) we have

$$b = l(\sin\alpha - \sin\beta) \rightarrow (5)$$

(4) – (2) we have

$$a = l(\cos\beta - \cos\alpha) \rightarrow (6)$$

(5) ÷ (6) we have

$$\frac{b}{a} = \frac{l(\sin\alpha - \sin\beta)}{l(\cos\beta - \cos\alpha)}$$

$$\frac{b}{a} = \frac{l\left(\cancel{2\cos}\frac{\alpha+\beta}{2}\sin\cancel{\frac{\alpha-\beta}{2}}\right)}{\left(\cancel{2\sin}\left(\frac{\alpha+\beta}{2}\right)\sin\left(\cancel{\frac{\alpha-\beta}{2}}\right)\right)}$$

$$\frac{b}{a} = \cot\left(\frac{\alpha+\beta}{2}\right)$$

$$a = b\tan\left(\frac{\alpha+\beta}{2}\right)$$

6. The angles of depression of two mile stones along a road when viewed from an areoplane are α, β. Show that the aeroplane is flying at a height of $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$ from the road level. Taking the two mile stones to be on opposite side?

Solution: -

Let A be the position of the aeroplane P, Q be two mile stones

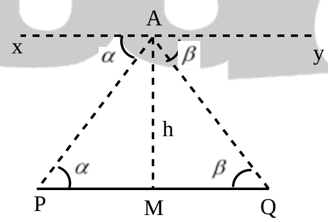
Given PQ = 1 mile

Draw XAY parallel to PQ through the observes eye

Given $\angle \times AP = \angle APQ = \alpha \qquad \angle YAQ = \beta$

$\therefore \quad \angle XAP = \angle APQ = \alpha \qquad \angle YAQ = \angle PQA = \beta$

In triangle PMA



$$\tan\alpha = \frac{h}{PM} \Rightarrow PM = h\cot\alpha$$

In triangle $\tan\beta = \frac{h}{QM} \Rightarrow QM = h\cot\beta$

Given PQ = 1

$$PM + QM = 1 \Rightarrow h\cot\alpha + h\cot\beta = l$$

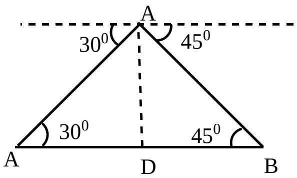
$$h = \frac{1}{\cot\alpha+\cot\beta} = \frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$$

7. From the point on a bridge across a river the angles of depression of the banks on opposite sides of the river are 30^0 *and* 45^0 respectively. If the bridge is at a height of 4 meters from its banks find the width of the river?

Solution : -

Let A, B represent points on the bank on opposite sides of the river so that AB is the width of the river. Let P be a point on the bridge at a height of 4 meters

i.e., PD = 4 mt

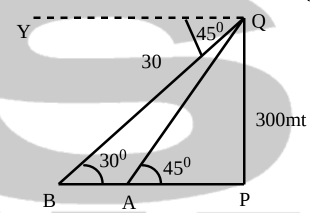


In triangle PDA $\tan 30^0 = \frac{4}{AD} \Rightarrow AD = 4\sqrt{3}$

In triangle PDB $\tan 45^0 = \frac{h}{DB} \Rightarrow \frac{4}{DB} = 1 \Rightarrow DB = 4$

$$AB = AD + DB = 4\sqrt{3} + 4 = 4\left(\sqrt{3}+1\right)$$

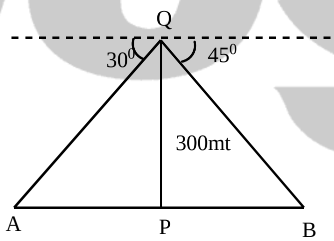
8. From the top of a light house 300 mts high the angle of dipressino made by two boats situated in a horizontal line along the foot of the light house are 30^0 *and* 45^0 find the distance between the two boats (i) when they are on the same side (ii) when they are on either side of the light house?



Solution : - (i)

Refer to problem '2'

And try your self



Solution (ii)

Refer to problem 7

And try your self

9. From the top of a hill 200 meters high the angle of depression of the top and bottom of a pillar on the level ground are 30^0*and* 60^0 respectively. Find the height of the pillar.?

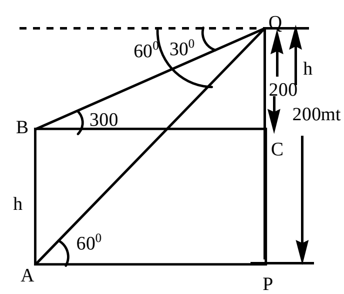
Solution : -

Let PQ be the full of height 200 mt

AB be the pillar of height h

Draw QY parallel to horizontal

Angle of depression of B from Q is 30^0



Angle of depression of A from Q is 60^0

Draw BC parallel to Qy

Since AB = h QC = 200 – h

Let BC = AP = x

In $\Delta^{le}\ QCB \tan 30^0 = \dfrac{200-h}{x}$ -------(1)

In $\Delta^{le}\ QPA \tan 60^0 = \dfrac{100}{x}$ --------(2)

(1) $\div$ (2) we have

$$\frac{\tan 30^0}{\tan 60^0} = \frac{200-h}{100} \Rightarrow \frac{1}{3} = \frac{200-h}{200}$$

$$200 = 600 \ -3h \Rightarrow h = \frac{400}{3}$$

10. A pillar of 10 meter height is mounted on a tower from a point on the level ground, the angles of elevation of the top of the foot of the pillar are 75^0 *and* 45^0 respectively find the height of the tower.?

Solution : -

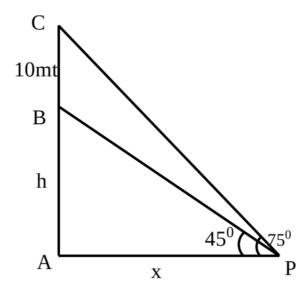
Let AB be the tower and BC be the pillar of height 10mt

Let P be the point of observation

Angles of elevation from P to B and C are respectively 45^0 *and* 75^0

In triangle BAP $\tan 45^0 = \dfrac{h}{x}$ -------(1)

In triangle CAP $\tan 75^0 = \dfrac{h+10}{x}$ -------(2)



$$\frac{\tan 45^0}{\tan 75^0} = \frac{h}{h+10}$$

$$\frac{1}{2+\sqrt{3}} = \frac{h}{h+10}$$

$$h+10 = \left(2+\sqrt{3}\right)h$$

$$10 = \left(1+\sqrt{3}\right)h \Rightarrow h = \frac{10}{\sqrt{3}+1} = 5\left(\sqrt{3}-1\right)$$

11. From the top of a pillar of a height 80 meters the angles of depression of the top and foot of the another pillar are 30^0 *and* 45^0 respectively. Find the distance between the two pillars and the height of the second pillar ?

Solution : -

Let PQ be the pillar of height 80 mts

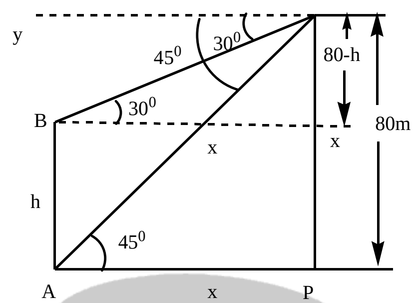
AB be the second pillar of height h

Draw BX parallel to AP

QY parallel to AP

Angles of depression of B and A

From Q are 30^0 *and* 45^0



In triangle QXB $\tan 30^0 = \dfrac{80-h}{x}$ -------(1)

In triangle QPA

$\tan 45^0 = \dfrac{80}{h}$ ----------(2)

(1) ÷ (2) when $\dfrac{\tan 30^0}{\tan 45^0} = \dfrac{80-h}{80}$

$$80\sqrt{3} - h\sqrt{3} = 80 \Rightarrow 80\left(\sqrt{3}-1\right)/\sqrt{3}$$

12. From a point A on the level ground away from the foot of the tower the angle of elevation of the top of the tower is 30^0. From a point at a height of h meters vertically above A the angle of elevation of the foot of the tower is 60^0. Find the height of tower in term of h ?

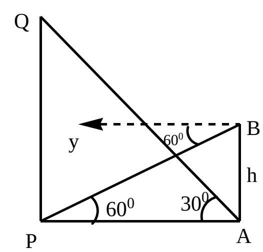
Solution: -

Let PQ be the tower of height ‘x’

A be the first point of observation

B be the point of a height of ‘h’

From the point A



Angle of elevation of Q from A is 30^0

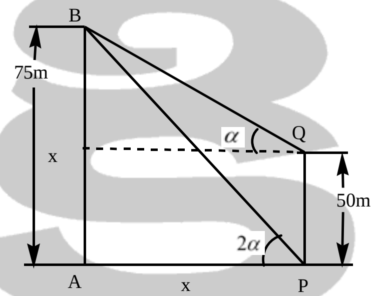
Angle of depression of P from B is 60^0 Let AP = 4

In triangle QPA $\tan 30^0 = \frac{x}{y}$

In triangle BAP $\tan 60^0 = \frac{h}{y}$

$$\frac{\tan 30^0}{\tan 60^0} = \frac{\left(\frac{x}{y}\right)}{\frac{h}{y}} \Rightarrow \frac{1}{3} = \frac{x}{h} \Rightarrow x = \frac{h}{3}$$

13. The angle of elevation of the top of a tower from the foot of the building is twice the angle of elevation from the top of the building. The height of the building is 50 meters and the height of the tower is 75 meters. Find the angle of elevation of the top of the tower from the foot of the building?



Let AB be the tower of height 75 mt

PQ be the building of height 50m

Draw QX parallel to AP

Angles of elevation of B from Q AP

Angle of elevation of B from Q is α

Angle of elevation of B from P is 2α

In triangle BAP $\tan 2\alpha = \frac{75}{x}$ --------------(1)

In triangle BXQ $\tan\alpha = \frac{25}{x}$ -----------------(2)

(1) ÷ (2) we have $\frac{\tan 2\alpha}{\tan\alpha} = 3 \Rightarrow \tan 2\alpha = 3\tan\alpha$

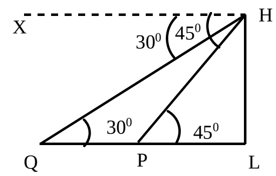
$$\frac{2\tan\alpha}{1-\tan\alpha} = 3\tan\alpha \Rightarrow 2 = 3 - 3\tan^2\alpha$$

$$3\tan^2\alpha = 1 \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^0 \Rightarrow 2\alpha = 60^0$$

14. As observed from the top of a light house of height 75m from the sea level the angles of depressions of two ships are 30^0 *and* 45^0. It one ship is exactly between the two ships?

Solution : -

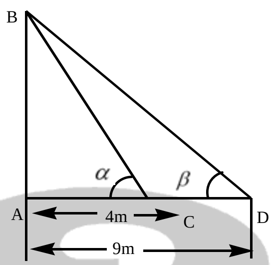


Refer to Problem 2s

And try your self

15. The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complementary prove that the height of the tower is 6m

Solution: -



AB be the tower f height h

Angle of elevation of B from C and D

Are respectively are α, β

Given $\alpha + \beta = 90^0$

$\alpha = 90^0 - \beta$

$\tan\alpha = \tan\left(90^0 - \beta\right)$

$\tan\alpha = \cot\beta \Rightarrow \tan\alpha = \frac{1}{\tan\beta} \Rightarrow \tan\alpha\tan\beta = 1$ ----------------(1)

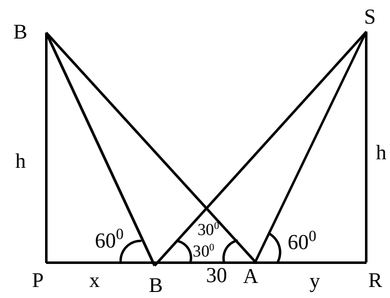
In triangle BAC $\tan\alpha = \frac{h}{4}$

In triangle BAQ $\tan\beta = \frac{h}{4}$

$\tan\alpha\tan\beta = 1 \Rightarrow \frac{h^2}{36} = 1 \Rightarrow h = 6$

16. A and B are two points on the line segment joining the feet of two equal pillars. AB = 30 meters. From A the angles of elevation of top of the pillars are 30^0 *and* 60^0. From B they are 60^0 *and* 30^0 respectively in the same order find the height of the pillars and the distance between them?

Solution : -



In triangle QPB

$\tan 60^0 = \frac{h}{x} \Rightarrow h = x\sqrt{3}$

In $\Delta^{le}\ QPA$

$\tan 30^0 = \frac{h}{x+30}$

$x + 30 = h\sqrt{3}$

But $h = x\sqrt{3}$

$\therefore x + 30 = 3x \Rightarrow x = 15$

$h = 15\sqrt{3}$

In triangle SRY

$\tan 60^0 = \frac{h}{y}$

$\sqrt{3} = \frac{15\sqrt{3}}{y} \Rightarrow y = 15$

Distance between the poles $= x + 30 + y = 60m$

Height of pillar $= 15\sqrt{3}$

16. Over a tower AB of height h mt there is a flat staff BC. AB and BC are making equal angles at a point distance d mt from the foot A of the tower. Show that the height of the flag staff is $h\frac{\left(d^2+h^2\right)}{d^2-h^2}$

Solution : -

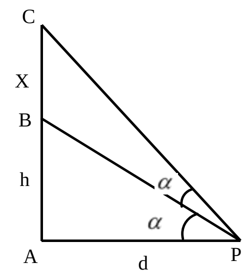
AB is the tower of height h

BC is the flag staff

Let the height of the flat staff be 'x'

P be any point let Ap = d

In triangle BAD



$\tan\alpha = \frac{h}{d}$

In triangle CAD $\tan 2\alpha = \frac{h+x}{d}$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+x}{d}$$

$$\frac{2\frac{h}{d}}{1-\frac{h^2}{d^2}} = \frac{h+x}{d} \Rightarrow \frac{2h}{d}\times\frac{d^2}{d^2-h^2} = \frac{h+x}{d}$$

$$\frac{2hd^2}{d^2-h^2} - h = x \Rightarrow x = \frac{2hd^2-hd^2+h^3}{d^2-h^2}$$

$$x = \frac{hd^2+h^3}{d^2-h^2} \Rightarrow x = \frac{h\left(d^2+h^2\right)}{d^2-h^2}$$

17. ABCD is a monument built vertically on the ground from a point P away from the moment on the level ground the angles of elevation of AB, AC, AD are α, β *and* γ respectively. If AB= a, AC = b , AD = c AP = x and $\alpha+\beta+\gamma=180^0$ then show that $(a+b+c)x^2 = abc$

Solution : -

ABCD is movement

A is on the ground

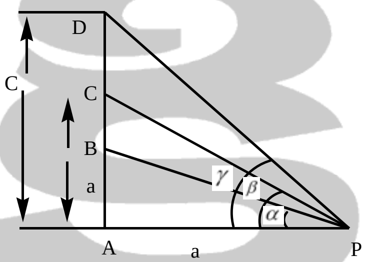
Given AB = a AC = b

AD = C

P be a point of observation

Given AP = x

α, β, γ are the angles of elevation of B, C, D from P



In triangle BAP $\tan\alpha = \frac{a}{x}$

In triangle CAP $\tan\beta = \frac{b}{x}$

In triangle DAP $\tan\gamma = \frac{c}{x}$

Given that $\alpha+\beta+\gamma=180^0 \Rightarrow \alpha+\beta=180^0-\gamma$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = -\tan\gamma \Rightarrow \tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\tan\beta\tan\gamma$$

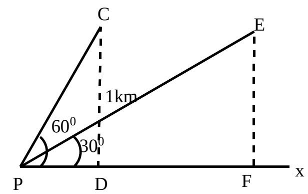
$$\frac{a}{x}+\frac{b}{x}+\frac{c}{x} = \frac{abc}{x^3} \Rightarrow (a+b+c)x^2 = abc$$

18. An aeroplane is moving one kilometers high from west to east horizontally. From a point on the ground the angle of elevation of the aeroplane is 60^0 and after 60 seconds the angles of elevation of the aeroplane is observed as 30^0. Find the speed of the aeroplane in km/hr ?

Solution : -

Let C, E be the positions of aeroplane

P be the point of observation



Angle of elevation of C from P is 60^0

Angle of elevation of E from P is 30^0

Draw CD and EF perpendicular to horizontal line PX

In triangle CDP $\tan 60^0 = \frac{1}{PD}$

$\Rightarrow CD = \frac{1}{\sqrt{3}}$

In triangle EFP $\tan 30^0 = \frac{1}{PF}$

$PF = \sqrt{3}$

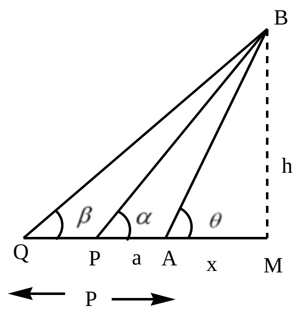
Distance travelled by aeropalne in 10 seconds = CE = DF

$CE = DF = PD - CD = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\therefore$ Speed $= \frac{distance}{time} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{10}{360}\right)} = \frac{2}{\sqrt{3}} \times 360 = \frac{720}{\sqrt{3}} = 240\sqrt{3}\ km/h$

19. **A pillar is leaning towards east and α, β are the angles of elevation of the top of the pillar from points due west of the pillar at distance a and b respectively. Show that the angle between the pillar and the horizontal is** $\tan^{-1}\left\{\frac{b-a}{b\cot\alpha - a\cot\beta}\right\}$

Solution : -



Let AB be the pole inclined towards east

P, Q are the points of observations

At a distance of a and b from A respectively

Angles of elevation from P, Q to B are α, β

Draw BM perpendicular to AQ let $BM = h \quad AM = x$

In triangle AMB $\tan\theta = \frac{h}{x} \Rightarrow x = h\cot\theta \rightarrow (1)$

In triangle PMB $\tan\alpha = \frac{h}{a+x} \Rightarrow a + x = h\cot\alpha \rightarrow (2)$

In triangle QMB $\tan\beta = \frac{h}{b+x} \Rightarrow b + x = h\cot\beta \rightarrow (3)$

(2) ------(1) we have $a + x - x = h\{\cot\alpha - \cot\theta\} \rightarrow (4)$

(3) -------(1) we have $b + x - x = h\{\cot\beta - \cot\theta\} \rightarrow (5)$

(4) $\div$(5) we have $\frac{a}{b} = \frac{\cot\alpha - \cot\theta}{\cot\beta - \cot\theta}$

$a\cot\beta - a\cot\theta = b\cot\alpha - b\cot\theta$

$(b-a)\cot\theta = b\cot\alpha - a\cot\beta$

$\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a} \Rightarrow \tan\theta = \frac{b-a}{b\cot\alpha - a\cot\beta}$

$\theta = \tan^{-1}\left\{\frac{b-a}{b\cot\alpha - a\cot\beta}\right\}$

20. From a point B on the level ground away from the foot of the hill AD, the top of the hill makes an angles of elevation α from the point B, the point C is reached by moving a distance d along a slant/slope which makes an angle ϑ with the horizontal. If B is the angle of elevation of the top of the hill from C, find the height of the hill ?

Solution : -

AD be the hill b is the point of observation

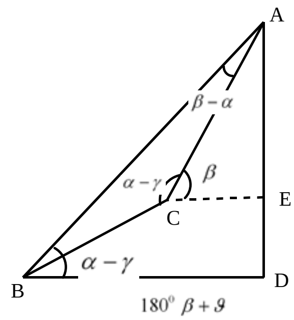
BC is the inclined plane

Draw CE parallel to BD

Angle of elevation of A from B is α

Angle of elevation of A from C is β

Angle of inclination of BC with BD is ϑ



In $\Delta^{le}\ ADB \quad \angle BAD\ 90^0-\alpha$

In $\Delta^{le}\ CEA \quad \angle CAE = 90^0-\beta$

In $\Delta^{le}\ BCA \quad \angle ABC = \alpha-\vartheta$

$\angle BAC = (90-\alpha)-(90-\beta) = \beta-\alpha$

$\angle BAC = 180^0 - \{\angle BAC + \angle ABC\}$

$= 180^0 - \{\alpha - \vartheta + \beta - \alpha\}$

$= 180^0 - (\beta - \vartheta)$

Using since rule

$$\frac{BC}{\sin(\beta-\alpha)} = \frac{AB}{\sin 180^0 - (\beta-\vartheta)}$$

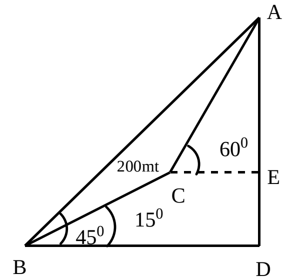
$$\frac{d}{\sin(\beta-\alpha)} = \frac{AB}{\sin(\beta-\vartheta)} \Rightarrow AB = \frac{d\sin(\beta-\vartheta)}{\sin(\beta-\alpha)}$$

In triangle BDA

$$\sin\alpha = \frac{AD}{AB} \Rightarrow AD = AB\sin\alpha$$

$$\therefore AD = \frac{d \sin\alpha \sin(\beta - \vartheta)}{\sin(\beta - \alpha)}$$

21. From a point on the slope of a hill, the angle of elevation of the top of the hill is 45^0. After walking 200 mt from that point on a slope of 15^0 angle with horizontal the same point on the top of the hill makes an angle of elevation 60^0 show that the height of the hill is 100 $\left(\sqrt{6}+\sqrt{6}\right)$

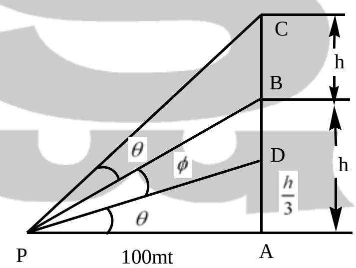


Solution : -

Do the problem again by taking $\alpha = 45^0, \vartheta = 15^0, \beta = 60^0$ and d = 200 m in the above problem

22. A church tower AB standing on a level plane is surrounded by a spire BC of the same height as the tower D is a point in AB such that $AD = \frac{1}{3}$ AD. At a point on the plane 100 mt from the foot of the tower the angles subtended by AD and BC are equal. Find the height of tower ?

Solution :-



In triangle OQP

$$\tan\theta = \frac{h}{300}$$

In triangle BAP

$$\tan(\theta + Q) = \frac{h}{100}$$

In triangle CAP

$$\tan(2\theta + Q) = \frac{2h}{100} = \frac{h}{50}$$

$$\tan(2\theta + Q) = \frac{h}{50} \Rightarrow \tan\{\theta + \theta + Q\} = \frac{h}{50}$$

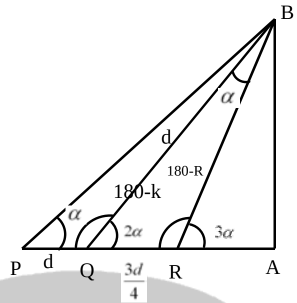
$$\frac{\tan\theta + \tan(\theta + Q)}{1 - \tan\theta\tan(\theta + Q)} = \frac{h}{50}$$

$$\frac{\frac{h}{300} + \frac{h}{100}}{1 - \frac{h^2}{30,000}} = \frac{h}{50} \Rightarrow \frac{400\cancel{h}}{\cancel{30,000}} \times \frac{\cancel{30000}}{30000 - h^2} = \frac{\cancel{h}}{50}$$

$$20,000 = 30,000 - h^2 \Rightarrow h^2 = 10,000 \Rightarrow h = 100mt$$

23. **A man observes a tower AB of height h from a point P on the ground the moves a distance 'd' towards the foot of the tower and finds that the angle of elevation is doubled. He further moves a distance $\frac{3d}{4}$ in the same direction and the angle of elevation is three times that at P. Prove that** $36h^2 = 35d^2$

Solution : In triangle PQB



$$\angle PQB = 180 - 2\alpha$$

$$\angle PBQ = 180 - \{\alpha + 180 - 2\alpha\}$$

$$= \alpha$$

$$\therefore \angle BPQ = \angle PBQ = \alpha$$

$\therefore BQ = BQ = d$ {sides opposite to equal angles are equal}

In triangle BQR $\angle QRB = 180 - 3\alpha$

$$\angle QBR = 180^0 - \{2\alpha + 180 - 3\alpha\} = \alpha$$

Using since rule in this triangle BQR

$$\frac{BQ}{\sin(180 - 3\alpha)} = \frac{QR}{\sin\alpha} \Rightarrow \frac{d}{\frac{3d}{4}} = \frac{\sin 3\alpha}{\sin\alpha}$$

$$\frac{4}{3} = 3 - 4\sin^2\alpha \Rightarrow 4\sin^2\alpha = 3 - \frac{4}{3}$$

$$\sin^2\alpha = \frac{5}{12}$$

$$\cos^2\alpha = \frac{7}{12}$$

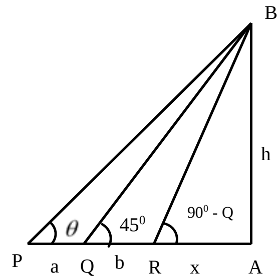
In $\Delta^{le}\ QAB$

$$\sin 2\alpha = \frac{AB}{BQ} \Rightarrow \sin 2\alpha = \frac{h}{d}$$

$$h = 2d\sin\alpha\cos\alpha \Rightarrow h^2 = 4d^2\sin^2\alpha\cos^2\alpha$$

$$h^2 = \cancel{4}d^2 \times \frac{5}{\cancel{12}\ 3} \times \frac{7}{12} \Rightarrow 36h^2 = 35d^2$$

24. **An observer finds that the angle of elevation of the tower is θ, an advancing 'a' meters towards the tower, the elevation is 45^0 and an advancing b meter near the elevation is $90^0-\theta$. Find the height of the tower.?**



Solution : -

AB be the tower of height h

P the first point of observation

From which the angle of elevation is 'θ'

Q be the second point such that PQ = a and angle of elevation from Q is 45^0

R be the third pant such that QR = b and angle of elevation is $90^0-\theta$ let AR = x

In triangle BAR $\tan\left(90^0-\theta\right)=\frac{h}{x}\Rightarrow \cot\theta=\frac{h}{2}\Rightarrow x=h\tan\theta$ -------(1)

In triangle QAB $\tan 45^0=\frac{h}{b+x}\Rightarrow h=b+x$ ---------(2)

In triangle PAB $\tan\theta=\frac{h}{a+b+x}\Rightarrow a+b+x=h\cot\theta$ -------(3)

(2) – (1) we have (3) – (2) we have

$b=h\left(1-\tan\theta\right)$ ------(4) $a=h\left\{\cot\theta-1\right\}$ -------(5)

$\frac{(4)}{(5)}$ we have $\frac{b}{a}=\frac{h(1-\tan\theta)}{h(\cot\theta-1)}$

$$\frac{b}{a}=\frac{\not{h}\left\{1-\tan\theta\right\}}{\not{h}\left\{\frac{1}{\tan\theta}-1\right\}}\Rightarrow\frac{b}{a}=\tan\theta$$

Sub $\tan\theta$ is (3) we have $b=h\left(1-\frac{b}{a}\right)\Rightarrow h=\frac{ab}{a-b}$