Trigonometric Equations

1. **Definition :** - An equation consisting of the trigonometric functions of a variable angle $\theta \in R$ is called a trigonometric equation.
2. $\sin\theta = 0 \Rightarrow \theta = n\pi$, where $n \in z$.
3. $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$, where $n \in z$.
4. $\tan\theta = 0 \Rightarrow \theta = n\pi$ where n is any integer.
5. The solution of $\sin\theta = k$ ($|k| \le 1$) lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ is called the principle solution of the equation.
6. The solution of $\cos\theta = k$ ($|k| \le 1$) lying between 0 and π is called the principal solution of the equation.
7. That solution of $\tan\theta = k$, lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ is called the principal solution of the equation.
8. The general value of θ satisfying $\cos\theta = k$ ($|k| \le 1$) is given by $\theta = 2n\pi \pm \alpha$ where $n \in Z$.
9. The general value of θ satisfying $\sin\theta = k$ ($|k| \le 1$) is given by $\theta = n\pi + (-1)^n\alpha$ where $n \in Z$.
10. The general value of θ satisfying $\tan\theta = k$ is given by $\theta = n\pi + \alpha$ where $n \in Z$. (in each of the above cases, α is the principal solution).
11. If $\sin\theta = k$, $\tan\theta = k$ are given equations, then the general value of θ is given by $\theta = 2n\pi + \alpha$ where α is that solution lying between 0 and 2π.
12. The equation $a\cos\theta + b\sin\theta = c$ will have no solution or will be inconsistent if $|c| > \sqrt{a^2 + b^2}$
13. $\cos n\pi = (-1)^n$, $\sin n\pi = 0$.
14. If $\sin^2\theta = \sin^2\alpha$ or $\cos^2\theta = \cos^2\alpha$ or $\tan^2\theta = \tan^2\alpha$, then $\theta = n\pi \pm \alpha$; $n \in Z$.

PROBLEMS

Very Short answer Questions

1. **Solve the following equations**

(i) $\cos 2\theta = \frac{\sqrt{5}+1}{4}\ \theta \in [0, 2\pi]$

(ii) $\tan^2\theta = 1\ \theta \in [-\pi, \pi]$

(iii) $\sin 3\theta = \frac{\sqrt{3}}{2}\ \theta \in [-\pi . \pi]$

(iv) $\cos^2\theta = \frac{3}{4}\ \theta \in (0, \pi)$

Solution :

(i) $\cos 2\theta = \frac{\sqrt{5}+1}{4} \Rightarrow 2\theta = \frac{\pi}{5}, \frac{9\pi}{5}, \frac{11\pi}{5}, \frac{19\pi}{5}$

$\therefore \theta \in [0, 2\pi] \therefore \theta \in [0, 2\pi] \Rightarrow 2\theta \in [0, 4\pi]$

$\therefore \theta = \frac{\pi}{10}, \frac{9\pi}{10}, \frac{11\pi}{10}, \frac{19\pi}{10}$

(ii) $\tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \qquad \therefore \theta = \pm\pi/4 : \pm 3\pi/4$

$\therefore\ \theta\ \frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

(iii) $\sin 3\theta = \frac{\sqrt{3}}{2} \Rightarrow 3\theta = \frac{-5\pi}{3}, \frac{-4\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

$\therefore\ \theta = \frac{-5\pi}{9}, \frac{-4\pi}{9}, \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$

(iv) $\cos^2\theta = \frac{3}{4} \Rightarrow\ \cos\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

2. Find the general solution of the following equation

(i) $2\sin^2\theta = 3\cos\theta$ (ii) $\sin^2\theta - \cos\theta = \frac{1}{4}$

(iii) $5\cos^2\theta + 7\sin^2\theta = 6$

(iv) $3\sin^4 x + \cos^4 x = 1$

Solution :

(i) $2\sin^2\theta = 3\cos\theta \Rightarrow 2\left(1-\cos^2\theta\right) = 3\cos\theta \Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0 \Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\therefore \cos\theta = \frac{1}{2} \qquad \cos\theta = -2$ not possible $[\because \cos\theta \geq -1]$

$\theta = 2n\pi \pm \pi/3$

(ii) $\sin^2\theta - \cos\theta = \frac{1}{4} \Rightarrow 4\sin^2\theta - 4\cos\theta = 1$

$\therefore 4\left(1-\cos^2\theta\right) - 4\cos\theta = 1 \Rightarrow\ 4\cos^2\theta + 4\cos\theta - 3 = 0$

$4\cos^2\theta + 6\cos\theta - 2\cos\theta - 3 = 0 \Rightarrow (2\cos\theta - 1)(2\cos\theta + 3) = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = -\frac{3}{2}$ not possible

$\theta = 2n\pi \pm \pi/3$

(iii) $5\cos^2\theta + 7\sin^2\theta = 6 \Rightarrow 5\cos^2\theta + 7 - 7\cos^2\theta = 6$

$$-2\cos^2\theta = -1 \Rightarrow \cos^2\theta = \frac{1}{2}$$

$$\cos^2 = \cos^2\frac{\pi}{4} \Rightarrow \theta = n\pi \pm \pi/4$$

(iv) $3\sin^4 x + \cos^4 x = 1$

$3(\sin^2 x)^2 + (\cos^2 x)^2 = 1 \Rightarrow 3\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2 = 1$

$3\{1 + \cos^2 2x - 2\cos 2x\} + 1 + \cos^2 2x + 2\cos 2x = 4$

$3 + 3\cos^2 2x - 6\cos 2x + 1 + \cos^2 2x + 2\cos 2x = 4$

$4\cos^2 2x - 6\cos 2x + 1 + \cos^2 2x + 2\cos 2x = 4$

$4\cos^2 2x - 4\cos 2x = 0 \Rightarrow 4\cos 2x\{\cos 2x - 1\} = 0$

$\cos 2x = 0 \quad : \cos 2x = 1$

$1 - 2\sin^2 x = 0 \quad 1 - 2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2} \quad \sin^2 x = 0$

$x = n\pi \pm \pi/4 \quad x = n\pi \quad n \in z$

3. **Find the general solution of the following equations**

(i) $\sin\theta = \frac{\sqrt{3}}{2} \quad \cos\theta = -\frac{1}{2}$

The principal value of θ satisfying the two given equation $= \alpha = \pi - \frac{2\pi}{3} = 2\pi/3$

$\therefore$ General solution $\theta = 2n\pi + 2\pi/3 \quad n \in z$

(ii) $\tan x = -\frac{1}{\sqrt{3}} \quad \sec x = \frac{2}{\sqrt{3}}$

Principal value of $x = -\pi/6/6$

$\therefore$ General solution of $x = 2n\pi - \frac{\pi}{6}$

(iii) $\operatorname{cosec}\theta = -2 \quad \cot\theta = -\sqrt{3}$

Principal value of $\theta = -\pi/6$

$\therefore$ General solution $\theta = 2n\pi - \pi/6$

4. If x is acute and $\sin\left(x+10^0\right)=\cos\left(3x-68^0\right)$ then find x

Sol. Given $\sin\left(x+10^0\right)=\cos\left(3x-68^0\right)$

$\Rightarrow \sin(x+10)=\sin\left(90^0+3x-68\right)$

$=\sin\left(22^0+3x\right)$

$\therefore x+10=n\pi+(-1)^n\left(22^0+3x\right)$

If n is even, then

$x+10=n\pi+\left(22^0+3x\right)$

$\Rightarrow x=\dfrac{n\pi-12}{2}$ *which* is not acute

If n is odd, then

$x+10=(2k+1)\pi-\left(22^0+3x\right), n=2k+1$

$\Rightarrow x=(2k+1)\dfrac{\pi}{4}-8^0$

If K=0, then $x=37^0$.

5. **If Sin $(270^0 - X)$ = COS 292^0 then find x in $(0, 360^0)$**

Sol. Sin $(270^0 - X)$ = COS 292

$\Rightarrow$ - cos x = cos180+112)

$\Rightarrow$ - cos x = - cos 112

$\Rightarrow$ cos x = cos 112

$\Rightarrow$ $x = 112^0$ or $x = 360-112 = 248^0$

6. **If $x < 90^0$ and $\sin(x+28^0) = \cos(3x - 78^0)$, then find x.**

Sol. $\sin\left(x+28^0\right)=\cos\left(3x-78^0\right)$

$=\sin\left(90^0-3x+78\right)=\sin\left(168^0-3x\right)$

$\Rightarrow x+28=168-3x+2n\pi$

or $x+28=180^0-(168-3x)+2n\pi$

$\Rightarrow 4x=140+2n\pi$ *or* $2x = 16-2n\pi$

$\Rightarrow x=35+n\dfrac{\pi}{2}$ or $8-n\pi$

$\Rightarrow$ $x=35^0$ or 8^0

Short Answer Questions

1. Solve the following equations

(i) $6\tan^2 x - 2\cos^2 x = \cos 2x$ (ii) $4\cos^2\theta + \sqrt{3} = \left(2\sqrt{3}+1\right)\cos\theta$

(iii) $1+\sin 2x = \left(\sin 3x - \cos 3x\right)^2$ (iv) $2\sin^2 x + \sin^2 2x = 2$

Solution :

(i) $6\tan^2 x = 4\cos^2 x - 1$

$6\sin^2 x = \cos^2 x\left(4\cos^2 x - 1\right) \Rightarrow 6\left(1-\cos^2 x\right) = 4\cos^4 x - \cos^2 x$

$4\cos^4 x + 5\cos^2 x - 6 = 0 \Rightarrow 4\cos^4 x + 8\cos^2 x - 3\cos^2 x - 6 = 0$

$4\cos^2 x\left\{\cos^2 x + 2\right\} - 3\left\{\cos^2 x + 2\right\} = 0$

$\cos^2 x = \dfrac{3}{4} \Rightarrow \cos^2 x = \cos^2\dfrac{\pi}{6}$

$x = n\pi \pm \pi/6$

Solution :

(ii) $4\cos^2\theta + \sqrt{3} - 2\sqrt{3}\cos\theta - 2\cos\theta = 0$

$2\cos\theta\left\{2\cos\theta - 1\right\} - \sqrt{3}\left\{2\cos\theta - 1\right\} = 0$

$\left\{2\cos\theta - 1\right\}\left(2\cos\theta - \sqrt{3}\right) = 0 \Rightarrow \cos\theta = \dfrac{1}{2} : \cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 2n\pi \pm \pi/3 \quad or \quad \theta = 2n\pi \pm \pi/6 \quad n \in z$

Solution :

(iii) $1+\sin 2x = \left(\sin 3x - \cos x\right)^2 \Rightarrow 1+\sin 2x = \sin^2 3x + \cos^2 3x - \sin 6x$

$1+\sin 2x = 1 - \sin 6x \Rightarrow \sin 6x + \sin 2x = 0$

$2\sin\left(\dfrac{6x+2x}{2}\right)\cos\left(\dfrac{6x-2x}{2}\right) = 0 \Rightarrow \sin 4x\,.\cos 2x = 0$

$\sin 4x = 0 \quad or \quad \cos 2x = 0$

$4x = n\pi \quad or\ 2x = \left(2n+1\right)\pi/2$

$x = n\pi/4 \qquad x = \left(2n+1\right)\pi/4 \quad n \in z$

Solution :

(iv) $2\sin^2 x + \sin^2 2x = 2$

$2\sin^2 x + 4\sin^2 x\cos^2 x - 2 = 0 \Rightarrow 4\sin^2 x\cos^2 x - 2\cos^2 x = 0$

$2\,co\,x^2 x\left\{2\sin^2 x - 1\right\} = 0$

$\cos^2 x = 0 \qquad \sin^2 x = \dfrac{1}{2}$

$x = \left(2n+1\right)\dfrac{\pi}{2}, x = n\pi \pm \dfrac{\pi}{4}, n \in z$

2. If $x + y = 2\pi/3$ and $\sin x + \sin y = \frac{3}{2}$ find x and y

Solution:

$$x + y = 2\pi/3 \Rightarrow 2\pi/3 - x$$

$$\sin x + \sin y = \frac{3}{2} \Rightarrow \sin x + \sin\left(\frac{2\pi}{3} - x\right) = 3/2$$

$$\sin x + \sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x = 3/2$$

$$\sin x + \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{3}{2}$$

$$\frac{\sqrt{3}}{2}\cos x + \frac{3}{2}\sin x = \frac{3}{2}$$

$$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{3}}{2}$$

$$\cos(x - \pi/3) = \frac{\sqrt{3}}{2}$$

$$x - \pi/3 = 2n\pi \pm \frac{\pi}{6}$$

$$x = 2n\pi + \pi/2 \quad : x = 2n\pi + \pi/6$$

$$x + y = 2\pi/3$$

$$\Rightarrow y = \frac{2\pi}{3} - 2n\pi - \pi/2 \qquad or \qquad y = 2\pi/3 - 2n\pi - \pi/6$$

$$y = \pi/6 - 2n\pi \text{ or } y = \pi/2 - 2n\pi$$

3. Solving the following and write the general solutions

(i) $\sin 7\theta + \sin 4\theta + \sin\theta = 0$ (ii) $\sin 3\theta - \sin\theta = 4\cos^2\theta - 2$

(iii) $\sin x + \sqrt{3}\cos x = \sqrt{2}$ (iv) $\sin 4x - \sin 2x = 2$

Solution :

(i) $\sin 7\theta + \sin\theta + \sin 4\theta = 0 \Rightarrow \quad 2\sin 4\theta \,.\, \cos 3\theta + \sin 4\theta = 0$

$$\sin 4\theta\{2\cos 3\theta + 1\} = 0 \Rightarrow \sin 4\theta = 0 \qquad \cos 3\theta = -\frac{1}{2}$$

$$4\theta = n\pi \qquad\qquad 3\theta = 2n\pi \pm 2\pi/3$$

$$\theta = \frac{n\pi}{4} \qquad\qquad \theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9} \; n \in z$$

Solution :

(ii) $\sin 3\theta - \sin\theta = 4\cos^2\theta - 2$

$$2\cos 2\theta \,.\, \sin\theta - 2\left(2\cos^2\theta - 1\right) = 0$$

$$2\cos 2\theta\sin\theta - 2\cos 2\theta = 0 \Rightarrow 2\cos 2\theta\{\sin\theta - 1\} = 0$$

$$\cos 2\theta = 0 \quad : \quad \sin\theta = 1$$

$$2\theta = (2n+1)\pi/2 \qquad\qquad \theta = n\pi + (-1)^n\,\pi/2$$

$\theta = (2n+1)\pi/4 \quad : \quad \theta = n\pi + (-1)^n \pi/2 \;\; n \in z$

Solution :

(iii) $\sin x + \sqrt{3}\cos x = \sqrt{2} \Rightarrow \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{\sqrt{2}}$

{by dividing both sides with $\sqrt{a^2+b^2}$ i.e., $\sqrt{1^2+(\sqrt{3})^2}$ i.e., 2}

$\sin x \sin\pi/6 + \cos x\cos\pi/6 = \cos\pi/4$

$\cos(x-\pi/6) = \cos\pi/4$

$x - \pi/6 = 2n\pi \pm \pi/4 \Rightarrow x = 2n\pi + \frac{5\pi}{12} \quad or \;\; x : 2n\pi - \pi/12 \quad n \in z$

Solution :

(iv) $\sec 4x - \sec 2x = 2 \Rightarrow \frac{1}{\cos 4x} - \frac{1}{\cos 2x} = 2$

$\cos 2x - \cos 4x = 2\cos 4x\cos 2x$

$\cancel{\cos}\, 2x - \cos 4x = \cos 6x + \cancel{\cos}\, 2x$

$\Rightarrow \cos 6x + \cos 4x = 0 \Rightarrow 2\cos 5x \,.\, x\cos x = 0$

$\cos 5x = 0 \quad or \; \cos x = 0$

$5x = (2n+1)\pi/2 \qquad x = (2n+1)\pi/2$

$x = (2n+1)\frac{\pi}{10} \quad : x = (2n+1)\frac{\pi}{2} \qquad n \in z$

(or) $x = \frac{2n\pi}{5} \pm \pi/10 \quad x = 2n\pi \pm \pi/2$

4. **Solve the following equations**

(i) $\tan\theta + \sec\theta = \sqrt{3} \;\; 0 \le \theta \le 2\pi$

(ii) $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$

(iii) $\cot^2 x - (\sqrt{3}+1)\cot x + \sqrt{3} = 0 \;\; (0 < x < \pi/2)$

(iv) $\sec x \cos 5x + 1 = 0 \qquad 0 < x < 2\pi$

Solution:

(i) $\tan\theta + \sec\theta = \sqrt{3} \Rightarrow \frac{1+\sin\theta}{\cos\theta} = \sqrt{3}\cos\theta - \sin\theta = 1$

Dividing both sides with $\sqrt{(\sqrt{3})^2 + (-1)^2}$ i.e., with 2

$\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = \frac{1}{2} \Rightarrow \cos\theta\cos\pi/6 - \sin\theta\sin\pi/6 = \frac{1}{2}$

$\cos(\theta + \pi/6) = \cos\pi/3$

$\theta + \pi/6 = 2n\pi \pm \frac{\pi}{3}$

$\theta + \pi/6 = 2n\pi \pm \frac{\pi}{3} \qquad (or) \quad \theta + \pi/6 = 2n\pi - \pi/3$

$\theta = 2n\pi + \pi/6 \quad or \quad \theta = 2n\pi - \pi/2 \quad n \in z$

$\because\ 0 \le \theta \le 2\pi \quad \theta = \pi/6,\ 3\pi/2$

But for $\theta = 3\pi/2$ $\tan\theta, \sec\theta$ are not defined

$\therefore\ \theta = \pi/6$

Solution :

(ii) $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$

$2\cos\frac{5x}{2}.\cos\frac{x}{2} = 2\sin x.\cos\frac{x}{2}$

$2\cos\frac{x}{2}\left\{\cos\frac{5x}{2} - \sin x\right\} = 0$

$2\cos\frac{x}{2} = 0 \quad \cos\frac{5x}{2} = \sin x$

$\cos\frac{x}{2} = 0 \quad \cos\frac{5x}{2} = \cos\left(\frac{\pi}{2} - x\right)$

$\frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi \quad \frac{5x}{2} = 2n\pi \pm (\pi/2 - x)$

$\Rightarrow \frac{5x}{2} = 2n\pi + \pi/2 - x$ (or) $\frac{5x}{2} = 2n\pi - \pi/2 + x$

$\Rightarrow 7x/2 = 2n\pi + \pi/2$ (or) $\frac{3x}{2} = 2n\pi - \pi/2$

$\Rightarrow x = \frac{2}{7}(2n\pi + \pi/2) \quad or\ x = \frac{2}{3}\{2n\pi - \pi/2\}$

$n = 0 \Rightarrow x = \pi/7, n = 0: \quad x = -\frac{\pi}{3}$ does not in the given domain.

$n = 1 \quad x = \frac{5\pi}{7} \qquad n = 1 \Rightarrow x = \pi$

$n = 2 \quad x = 9\pi/7$

$n = 3 \quad x = 13\pi/7$

Solution :

(iii) $\cot^2 x - \sqrt{3}\cot x - \cot x + \sqrt{3} = 0$

$(\cot x - \sqrt{3})(\cot x - 1) = 0$

$\tan x = \frac{1}{\sqrt{3}} \quad or \quad \tan x = 1$

$x = \pi/6 \text{ or } x = \pi/4$

Solution :

(iv) $\sec x \cos 5x + 1 = 0 \quad 0 < x < 2\pi$

$\frac{\cos 5x}{\cos x} + 1 = 0 \Rightarrow \quad \frac{\cos 5x + \cos x}{\cos x} = 0$

$\cos 5x + \cos x = 0 \quad \Rightarrow \quad 2\cos 3x \cos 2x = 0$

$\cos 3x = 0 \qquad \cos 2x = 0$

$= \pi/6, \pi/4, 3\pi/4, 5\pi/6, 5\pi/4, 7\pi/6, 11\pi/6, 7\pi/4$

5. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ **then prove that** $\cos(\theta - \pi/4) = \pm\dfrac{1}{2\sqrt{2}}$

Solution:

$\tan(\pi\cos\theta) = \tan\left(\dfrac{\pi}{2} - \pi\sin\theta\right) \Rightarrow \pi\cos\theta = n\pi + \pi/2 - \pi\sin\theta$

$\cancel{\pi}\{\cos\theta + \sin\theta\} = \cancel{\pi}\left\{n + \dfrac{1}{2}\right\}$

$\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta = \dfrac{2n+1}{2\sqrt{2}}$

$\cos(\theta - \pi/4) = \dfrac{2n+1}{\sqrt{2}}$

For n = 0 $\cos(\theta - \pi/4) = \dfrac{1}{\sqrt{2}}$

for n = 1 $\cos(\theta - \pi/4) = \dfrac{3}{\sqrt{2}} > 1$

Not possible

$\therefore$ for n = 2, 3……… $\cos(\theta - \pi/4)$

Has no value

For n = -1 $\cos(\theta - \pi/4) = -\dfrac{1}{\sqrt{2}}$ for n = -2, -3, -4,……..

$\cos(\theta - \pi/4) < -1$ Which is not possible

$\therefore \cos(\theta - \pi/4) = \pm\dfrac{1}{\sqrt{2}}$

6. **Find the range of** θ **if** $\cos\theta + \sin\theta$ **is positive**

Solution: $\cos\theta + \sin\theta > 0 \quad \Rightarrow \dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta > 0$

$\sin(\theta + \pi/4) > 0$

$0 < \theta + \pi/4 < \pi$

$-\dfrac{\pi}{4} < \theta < 3\dfrac{\pi}{4}$

$2n\pi - \dfrac{\pi}{4} < \theta < 2n\pi + 3\dfrac{\pi}{4}$

$\theta = \bigcup\left(2n\pi - \dfrac{\pi}{4}, 2n\pi + 3\dfrac{\pi}{4}\right), n \in z$

Long Answer Questions

1. **Solve** $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$

Solution:

$\sin x + \sin 3x + \sin 2x = \cos x + \cos 3x + \cos 2x$

$2\sin 2x .\cos x + \sin 2x = 2\cos 2x. \cos x + \cos 2x$

$2 \sin 2x \{2\cos x + 1\} - 2 \cos 2x (2\cos x + 1) = 0$

$2\cos x + 1 = 0 \quad \sin 2x = \cos 2x$

$\cos x = \frac{-1}{2} \qquad \tan 2x = 1$

$x = 2n\pi \pm 2\pi/3 \qquad x = \frac{n\pi}{2} + \pi/8 \quad n \in z$

12. **If** $\sin 3x + \sin x + 2\cos x = \sin 2x + 2\cos^2 x$ **find the general solution**

Solution:

$\sin 3x + \sin x + 2\cos = \sin 2x + 2\cos^2 x$

$(2\sin 2x + 2\cos x) = 2\sin \cos x + 2\cos^2 x$

$2\sin 2x \cos x - 2\sin x \cos x + 2\cos x - 2\cos^2 x = 0$

$2 \cos x \{\sin 2x - \sin x + 1 - \cos x\} = 0$

$\cos x = 0 \quad 2\cos\frac{3x}{2} . \sin\frac{x}{2} + 2\sin^2\frac{x}{2} = 0$

$\cos x = 0 \quad 2\sin\frac{x}{2}\left\{\cos\frac{3x}{2} + \sin\frac{x}{2}\right\} = 0$

$\sin\frac{x}{2} = 0 \qquad \cos\frac{3x}{2} = -\sin\frac{x}{2}$

$n = (2n+1)\pi/2 \qquad \frac{x}{2} = n\pi \qquad \cos\frac{3x}{2} = \cos(\pi/2 + x/2)$

$x = 2n\pi \qquad \frac{3x}{2} = 2n\pi \pm (\pi/2 + x/2)$

$\frac{3x}{2} - \frac{x}{2} = 2n\pi + \pi/2 \ : \frac{3x}{2} + \frac{x}{2} = 2n\pi - \frac{\pi}{2}$

$x = (2n+1)\pi/2 ; \quad x = 2n\pi \ ; x = 2n\pi + \pi/2 \quad ; \quad x = n\pi - \pi/4 \ n \in z$

3. **If** α, β **are solutions of the equation** $a\cos\theta + b\sin\theta = c$ **where a, b, c** $\in$ **R and if** $a^2 + b^2 > 0 \ \cos\alpha \neq \cos\beta$ *and* $\sin\alpha \neq \sin\beta$ **then show that**

(i) $\sin\alpha + \sin\beta = \frac{2bc}{a^2 + b^2}$ **(ii)** $\cos\alpha + \cos\beta = \frac{2ac}{a^2 + b^2}$

(iii) $\cos\alpha\cos\beta = \frac{c^2 - b^2}{a^2 + b^2}$ **(iv)_** $\sin\alpha\sin\beta = \frac{c^2 - a^2}{a^2 + b^2}$

Solution:

$a\cos\theta + b\sin\theta = c \Rightarrow (b\sin\theta)^2 = (c - a\cos\theta)^2$

$b^2\{1-\cos^2\theta\} = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

$(a^2+b^2)\cos^2\theta - 2ac\cos\theta = (c^2-b^2) = 0$

Since α, β are solutions $\cos\alpha, \cos\beta$ are roots of above equation

$\therefore$ sum of roots $= \cos\alpha + \cos\beta = \dfrac{2ac}{a^2+b^2}$ Hence (ii) is proved

Product of roots $= \cos\alpha\cos\beta = \dfrac{c^2-b^2}{a^2+b^2}$ (iii) is proved

$a\cos\theta = c - b\sin\theta \Rightarrow (a\cos\theta)^2 = (c-b\sin\theta)^2$

$a^2\{1-\sin^2\theta\} = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$(a^2+b^2)\sin^2\theta - 2bc\sin\theta + (c^2-a^2) = 0$

Since α, β are solution $\sin\alpha, \sin\beta$ are roots

$\sin\alpha + \sin\beta = \dfrac{2bc}{a^2+b^2}$ $\sin\alpha\sin\beta = \dfrac{c^2-a^2}{a^2+b^2}$

4. **Find the common roots of the equation** $\cos 2x + \sin 2x = \cot x$ **and** $2\cos^2 + \cos^2 2x = 1$

Solution:

$2\cos^2 x + \cos^2 2x - 1 = 0 \quad \Rightarrow 2\cos^2 x - 1 + \cos^2 2x = 0$

$\cos 2x + \cos^2 2x = 0 \Rightarrow \cos 2x\{\cos 2x + 1\} = 0$

$\cos 2x = 0\,;\ \cos 2x = -1$

$\cos 2x + \sin 2x = \dfrac{\cos x}{\sin x}$

$\cos 2x.\sin x + \sin 2x.\sin x = \cos x$

$2\cos 2x\sin x + 2\sin 2x\sin x = 2\cos x$

$\sin 3x - \sin x + \cos - \cos 3x = 2\cos x$

$\sin 3x - \cos 3x = \cos x + \sin x$

Squaring on both sides

$1 - \sin 6x = 1 + \sin 2x$

$\sin 2x + \sin 6x = 0$

$2\sin 4x\,.\cos 2x = 0 \Rightarrow \cos 2x = 0 \quad \sin 4x = 0$

$\therefore$ Common solution is $\cos 2x = 0 \Rightarrow 2x = (2n+1)\pi/2$

5. **Solve the equation** $\sqrt{6-\cos x + 7\sin^2 x} + \cos x = 0$

Solution:

$\sqrt{6-\cos x + 7\sin^2 x} = -\cos x$

Squaring on both sides

$6 - \cos x + 7\sin^2 x = \cos^2 x$

$6 - \cos x + 7 - 7\cos^2 x = \cos^2 x$

$8\cos^2 x + \cos x - 13 = 0$

$\cos x = \frac{-1 \pm \sqrt{1+104}}{16}$ This is not possible

$\because \sin x \ \cos x > 1 \quad (or)$

$\cos x < -1$ in this $\cos x$

Hence equation has no solution

6. **If** $|\tan x| = \tan x + \frac{1}{\cos x}$ **and** $x \in [0, 2\pi]$ **find the values of** x

Solution:

$\cos(1)$ suppose $\tan x > 0$

$\therefore \tan x = \tan x + \frac{1}{\cos x} \Rightarrow \frac{1}{\cos x} = 0$ not possible

Case (ii) Suppose $\tan x < 0$

$\therefore -\tan x = \tan x + \frac{1}{\cos x} \Rightarrow -2\tan x = \frac{1}{\cos x}$

$-2\sin x = 1 \Rightarrow \sin x = -\frac{1}{2}$

x Lies in (iii) or (iv) quadrant

But $\tan x < 0$

$\therefore x = -\pi/6 \quad (or)\, 11\pi/6$

But $x \in [0, 2\pi]$

$\therefore x = 11\pi/6$

2. **Solve** $\cos 3\theta = \sin 2\theta$

Ans: $\left.\begin{aligned} \theta &= (4n+1)\pi/10 \\ (or)\ \theta &= (4n-1)\pi/10 \end{aligned}\right\} n \in z$

3. **Solve** $7\sin^2\theta + 3\cos^2\theta = 4$

Ans: $\theta = n\pi \pm \pi/6 \quad n \in z$

4. **Solve** $2\cos^2\theta - \sqrt{3}\sin\theta + 1 = 0$

Ans: $\theta = n\pi + (-1)^n \pi/3 \quad n \in z$

5. **Find all values of x in** $(-\pi, \pi)$ **satisfying the equation**

$81 + \cos x + \cos^2 x + \ldots\ldots. \infty = 3^4$

Ans: $\pi/3$ or $-\pi/3$

6. **Solve** $\tan\theta + 3\cot\theta = 5\sec\theta$

Ans: $\theta = n\pi + (-1)^n \pi/6$

7. **Solve** $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Ans: $\theta = n\pi + \pi/4 \quad n \in z$

8. **Solve** $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$

Ans: $x = 2n\pi + \frac{5\pi}{12} \quad or$

$x = 2n\pi + \pi/12$

9. **If** θ_1, θ_2 **are solutions of the equation** $a\cos 2\theta + b\sin 2\theta = c$ **then find the value of**

(i) $\tan\theta_1 + \tan\theta_2$ **(ii)** $\tan\theta_1 \tan\theta_2$

10. **Solve** $4\sin x \sin 2x \sin 4x = \sin 3x$

Ans: $x = \frac{n\pi}{3} \pm \frac{\pi}{9}$

11. **If** $0 < \theta < \pi$ **then solve** $\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$

Ans: $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

12. **Solve** $\sin 2x - \cos 2x = \sin x - \cos x$

Ans: $x = \{2n\pi / n \in z\} \; u \; \{2n\pi - \pi/6 / n \in z\}$