TRANSFORMATIONS

1. $\sin C + \sin D = 2\sin\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$.

2. $\sin C - \sin D = 2\cos\frac{C+D}{2} \cdot \sin\frac{C-D}{2}$.

3. $\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$.

4. $\cos C - \cos D = 2\sin\frac{C+D}{2} \cdot \sin\frac{D-C}{2}$.

5. $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

6. $2\cos A \sin B = \sin(A + B) - \sin(A - B)$

7. $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

8. $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

   (or)

   $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$.

9. $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan(\frac{A+B}{2})$.

10. If $\sin A + \sin B = x$, and $\cos A + \cos B = y$. Then

    i) $\tan(\frac{A+B}{2}) = \frac{x}{y}$

    ii) $\sin(A + B) = \frac{2xy}{y^2 + x^2}$

    iii) $\cos(A + B) = \frac{y^2 - x^2}{y^2 + x^2}$

    iv) $\tan(A + B) = \frac{2xy}{y^2 - x^2}$

VSAQ'S

1. Prove that $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$

Solution:

$$\sin 50^0 - \sin 70^0 + \sin 10^0 = 2\cos\left(\frac{50^0 + 70^0}{2}\right)\sin\left(\frac{50^0 - 70^0}{2}\right) + \sin 10^0$$

$$= 2\cos 60^0 \sin\left(-10^0\right) + \sin 10^0$$

$$= -2 \times \frac{1}{2}\sin 10^0 + \sin 10^0 = -\sin 10^0 + \sin 10^0 = 0$$

2. Prove that $\frac{\sin 70^0 - \cos 40^0}{\cos 50^0 - \sin 20^0} = \frac{1}{\sqrt{3}}$

$$\frac{\sin 70^0 - \cos 40^0}{\cos 50^0 - \sin 20^0} = \frac{\sin 70^0 - \sin 50^0}{\cos 50^0 - \cos 70^0} = \frac{2\cos 60^0.\sin 10^0}{2\sin 60^0.\sin 10^0} = \frac{1}{\sqrt{3}}$$

3. Prove that cos 55° + cos 65° + cos 175° = 0

Sol. cos 55° + cos 65° + cos 175°

$$= 2\cos\left(\frac{55° + 65°}{2}\right)\cos\left(\frac{55° - 65°}{2}\right)$$

$$+\cos(180° - 5°)$$

$$= 2\cos 60° \cos(-5°) - \cos 5°$$

$$= 2 \times \frac{1}{2}\cos 5° - \cos 5° = 0$$

4. Prove that $\cos 20° \cos 40° - \sin 5° \sin 25° = \frac{\sqrt{3}+1}{4}$.

Sol. $\cos 20° \cos 40° - \sin 5° \sin 25°$

$$= \frac{1}{2}[2\cos 40° \cos 20° - 2\sin 25° \sin 5°]$$

$$= \frac{1}{2}[\cos(40° + 20°) + \cos(40° - 20°)$$

$$+\cos(25° + 5°) - \cos(25° - 5°)]$$

$$= \frac{1}{2}[\cos 60° + \cos 20° + \cos 30° - \cos 20°]$$

$$= \frac{1}{2}[\cos 60° + \cos 30°]$$

$$= \frac{1}{2}\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}+1}{4}$$

5. **Prove that** $4\left\{\cos 66^0 + \sin 84^0\right\} = \sqrt{3} + \sqrt{5}$

Solution:

$$4\left\{\cos 66^0 + \sin 80^0\right\} = 4\left\{\cos 66^0 + \cos 66^0\right\} \left\{\because \sin 84^0 = \cos 66^0\right\}$$

$$= 4\left\{2\cos 36^0 . \cos 30^0\right\} = 8\left(\frac{\sqrt{5}+1}{4}\right)\frac{\sqrt{3}}{2} = \sqrt{3} + \sqrt{15}$$

6. **Prove that** $\cos 48^0 \cos 12^0 = \frac{3+\sqrt{5}}{8}$

Solution:

$$\cos 48^0 \cos 12^0 = \frac{1}{2}\left\{2\cos 48^0 + \cos 12^0\right\} = \frac{1}{2}\left\{\cos 60^0 + \cos 36^0\right\}$$

$$\frac{1}{2}\left\{\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right\} = \frac{2+\sqrt{5}+1}{8} = \frac{\sqrt{5}+3}{8}$$

7. **Prove that** $\sin^2(\alpha-45°)+\sin^2(\alpha+15°) - \sin^2(\alpha-15°) = \frac{1}{2}$.

Sol. $\sin^2(\alpha-45°)+\sin^2(\alpha+15°)-\sin^2(\alpha-15°)$

$$= \sin^2(\alpha-45°)+\sin^2(\alpha+15°+\alpha-15°)\cdot$$
$$\sin(\alpha+15-\alpha+15)$$

$$= \sin^2(\alpha-45°)+\sin 2\alpha\cdot\sin 30°$$

$$= \frac{1-\cos(2\alpha-90°)}{2}+\frac{\sin 2\alpha}{2}$$

$$= \frac{1-\sin 2\alpha+\sin 2\alpha}{2} = \frac{1}{2}$$

8. **Prove that** $\cos\theta + \cos\left(120^0 + \theta\right) + \cos\left(240^0 + \theta\right) = 0$

Solution:

$$\cos\theta + \cos\left(120^0 - \theta\right) + \cos\left(240^0 - \theta\right) = \cos\theta + 2\cos\left(\frac{120^0\theta + 240^0 + \theta}{2}\right)$$

$$\cos\left(\frac{120^0 + \theta - 240^0\theta}{2}\right)$$

$$= \cos\theta + 2\cos\left(180^0 + \theta\right) - \cos\left(60^0\right) = \cos\theta - 2\cos\theta \times \frac{1}{2}$$

$$= \cos\theta - \cos\theta = 0$$

9. **If** $\sin x+\sin y=\frac{1}{4}$ **and** $\cos x+\cos y=\frac{1}{3}$**, then show that**

(i) $\tan\left(\frac{x+y}{2}\right)=\frac{3}{4}$**, (ii)** $\cot(x+y)=\frac{7}{24}$.

Sol. i) $\sin x+\sin y=\frac{1}{4}$ ...(1)

$\cos x+\cos y=\frac{1}{3}$ ...(2)

$(1)\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{1}{4}$...(3)

$(2)\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{1}{3}$...(4)

Dividing $\frac{(3)}{(4)}$, we get

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{1}{4}\times\frac{3}{1}$$

$$\tan\left(\frac{x+y}{2}\right)=\frac{3}{4}$$

ii) Let $\tan\left(\frac{x+y}{2}\right)=\frac{3}{4}=t$

$$\tan(x+y)=\frac{2t}{1-t^2}=\frac{2\left(\frac{3}{4}\right)}{1-\frac{9}{16}}=\frac{24}{7}$$

$$\therefore \cot(x+y)=\frac{1}{\tan(x+y)}=\frac{7}{24}$$

10. **If** $\sin x+\sin y=a$ **and** $\cos x+\cos y=b$ **find the values of**

(i) $\tan\left(\frac{x+y}{2}\right)$ **(ii)** $\sin\left(\frac{x-y}{2}\right)$

Solution:

$$\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{a}{b}\Rightarrow\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{a}{b}$$

$$\tan\left(\frac{x+y}{2}\right)=\frac{a}{b}$$

$$(\sin x+\sin y)^2+(\cos x+\cos y)^2=a^2+b^2$$

$$\sin^2 x+\sin^2 y+2\sin x\sin y+\cos^2 x+\cos^2 y+2\cos x\cos y=a^2+b^2$$

$$\left(\sin^2 x+\cos^2 x\right)+\left(\sin^2 y+\cos^2 y\right)+2\left\{\cos x\cos y+\sin x+\sin y\right\}=a^2+b^2$$

$$1+1+2\cos(x-y)=a^2+b^2 \Rightarrow \cos(x-y)=\frac{a^2+b^2-2}{2}$$

$$\sin^2\left(\frac{x-y}{2}\right)=\sqrt{1\frac{-\cos(x-y)}{2}}=\pm\sqrt{\left(4-a^2-b^2\right)/4}$$

11. Prove that $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}=\tan\dfrac{A}{2}\cot\dfrac{B}{2}$

Solution:

$$\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}=\frac{\{1-\cos(A+B)-\{\cos A-\cos B\}\}}{\{1-\cos(A+B)+\{\cos A-\cos B\}\}}$$

$$\frac{2\sin^2\left(\frac{A+B}{2}\right)+2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\sin^2\left(\frac{A+B}{2}\right)-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}=\frac{2\sin\left(\frac{A+B}{2}\right)\left\{\sin\left(\frac{A+B}{2}\right)+\sin^2\left(\frac{A+B}{2}\right)\right\}}{2\sin\left(\frac{A+B}{2}\right)\left\{\sin\left(\frac{A+B}{2}\right)-\sin\left(\frac{A-B}{2}\right)\right\}}$$

$$\frac{2\sin\frac{A}{2}\cos\frac{B}{2}}{2\cos\frac{A}{2}\sin\frac{B}{2}}=\tan\frac{A}{2}\cot\frac{B}{2}$$

SAQ'S

12. If neither (A – 15°) nor (A – 75°) is an integral multiple of 180°, prove that $\cot(15^\circ - A) + \tan(15^\circ + A) = \dfrac{4\cos 2A}{1-2\sin 2A}$.

Sol. $\cot(15^\circ - A) + \tan(15^\circ + A) = \dfrac{\cos(15^\circ - A)}{\sin(15^\circ - A)} + \dfrac{\sin(15^\circ + A)}{\cos(15^\circ + A)}$

$$= \frac{\cos(15^\circ - A)\cos(15^\circ + A) + \sin(15^\circ + A)\sin(15^\circ - A)}{\sin(15^\circ - A)\cos(15^\circ + A)}$$

$$= \frac{2\left[(\cos^2 A - \sin^2 15^\circ) + \sin^2(15^\circ - \sin^2 A)\right]}{\left[2\cos(15^\circ + A)\sin(15^\circ - A)\right]}$$

$$= \frac{2(\cos^2 A - \sin^2 A)}{\sin(15^\circ + A + 15^\circ - A) - \sin(15^\circ + A - 15^\circ + A)}$$

$$= \frac{2(\cos^2 A - \sin^2 A)}{\sin 30^\circ - \sin 2A}$$

$$= \frac{2\cos 2A}{\frac{1}{2} - \sin 2A} = \frac{4\cos 2A}{1 - 2\sin 2A}$$

13. Prove that $4\cos 12^\circ \cos 48^\circ \cos 72^\circ = \cos 36^\circ$.

Sol. $4\cos 12^\circ \cos 48^\circ \cos 72^\circ$

$= (2\cos 48^\circ \cos 12^\circ)(2\cos 72^\circ)$

$= [\cos(48+12) + \cos(48-12)]2\cos 72^\circ$

$= [\cos 60^\circ + \cos 36^\circ]2\cos 72^\circ$

$= 2\cos 60^\circ \cos 72^\circ + 2\cos 36^\circ \cos 72^\circ$

$= 2\times\frac{1}{2}\cos 72^\circ + \cos(72^\circ + 36^\circ) + \cos(72^\circ - 36^\circ)$

$= \cos 72^\circ + \cos 108^\circ + \cos 36^\circ$

$= \cos(90^\circ - 18^\circ) + \cos(90^\circ + 18^\circ) + \cos 36^\circ$

$= \sin 18^\circ - \sin 18^\circ + \cos 36^\circ$

$= \cos 36^\circ$

14. Prove that $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$

Sol. $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ$

$$= 2\sin\left(\frac{10^\circ + 20^\circ}{2}\right)\cos\left(\frac{10^\circ - 20^\circ}{2}\right) + 2\sin\left(\frac{40^\circ + 50^\circ}{2}\right)\cos\left(\frac{40^\circ - 50^\circ}{2}\right)$$

$= 2\sin 15^\circ \cos 5^\circ + 2\sin 45^\circ \cos 5^\circ$

$= 2\cos 5^\circ[\sin 15^\circ + \cos 45^\circ]$

$$= 2\cos 5^\circ\left[2\sin\left(\frac{15^\circ + 45^\circ}{2}\right)\cos\left(\frac{15^\circ - 45^\circ}{2}\right)\right]$$

$= 2\cos 5°[2\sin 30° \cos 15°]$

$= 4\cos 5° \cdot \frac{1}{2}\cos 15° = 2\cos 15° \cos 5°$

$= \cos(15+5) + \cos(15-5) = \cos 20° + \cos 10°$

$= \cos(90°-70°) + \cos(90°-10°)$

$= \sin 70° + \sin 80°$

15. If none of the denominators is zero, prove that

$$\left[\frac{\cos A + \cos B}{\sin A - \sin B}\right]^n + \left[\frac{\sin A + \sin B}{\cos A - \cos B}\right]^n = \begin{cases} 2\cdot\cot^n\left(\frac{A-B}{2}\right), & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}.$$

Sol. $$\left[\frac{\cos A + \cos B}{\sin A - \sin B}\right]^n + \left[\frac{\sin A + \sin B}{\cos A - \cos B}\right]^n = \left[\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right]^n + \left[\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right]$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n \cot^n\left(\frac{A-B}{2}\right) = 0$$

if n is odd, since $(-1)^n = -1$

$$= 2\cot^n\left(\frac{A-B}{2}\right)$$ if n is even, since $(-1)^n = 1$

16. If $\cos x + \cos y = \frac{4}{5}$ *and* $\cos x - \cos y = \frac{2}{7}$ **then find the value of**

$$14\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right)$$

Solution:

$$\frac{\cos x + \cos y}{\cos x - \cos y} = \frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{5}\right)} \Rightarrow \frac{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)} = \frac{\cancel{4}^2}{5} \times \frac{7}{\cancel{2}}$$

$$\frac{-\cot\left(\frac{x+y}{2}\right)}{\tan\left(\frac{x-y}{2}\right)} = \frac{14}{5} \Rightarrow -5\cot\left(\frac{x-y}{2}\right) = 14\tan\left(\frac{x-y}{2}\right)$$

$$14\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right) = 0$$

17. If $\sec(\theta+\alpha) + \sec(\theta-\alpha) = 2\sec\theta$ *and* $\cos\alpha \neq 1$ **then show that**

$$\cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$$

Solution:

$$\sec(\theta+\alpha) + \sec(\theta-\alpha) = 2\sec\theta$$

$$\frac{1}{\cos(\theta+\alpha)} + \frac{1}{\cos(\theta-\alpha)} = \frac{2}{\cos\theta} \Rightarrow \frac{\cos(\theta-\alpha) + \cos(\theta+\alpha)}{\cos(\theta-\alpha)\cos(\theta+\alpha)} = \frac{2}{\cos\theta}$$

$$\left(\cancel{2}\cos\theta\cos\alpha\right)\cos\theta = \cancel{2}\left\{\cos^2\theta - \sin^2\alpha\right\}$$

$$\cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha \Rightarrow \sin^2\alpha = \cos^2\theta\left(1-\cos\alpha\right)$$

$$\cos^2\theta = \frac{\cancel{(1-\cos\alpha)}\,(1+\cos\alpha)}{\cancel{(1-\cos\alpha)}} \Rightarrow \cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$$

18. If sin A = sin B and cos A = cos B, then prove that A = 2nπ + B for some integer n.

Sol. sin A = sin B and cos A = cos B

$$\Rightarrow \sin A - \sin B = 0 \text{ and } \cos A - \cos B = 0$$

$$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0 \text{ and}$$

$$-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0$$

$$\Rightarrow \sin\left(\frac{A-B}{2}\right) = 0 \text{ and } \sin\left(\frac{A-B}{2}\right) = 0$$

$$\Rightarrow \frac{A-B}{2} = n\pi$$

$$\Rightarrow A - B = 2n\pi \Rightarrow A = 2n\pi + B \ (n \in Z)$$

19. If cos nα ≠ 0 and $\cos\frac{\alpha}{2} \neq 0$, then show that

$$\frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha} = \tan\frac{\alpha}{2}$$

Sol. Let $\cos n\alpha \neq 0$ and $\cos\frac{\alpha}{2} \neq 0$ then

$$\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha \quad = \cos(n\alpha+\alpha) + \cos(n\alpha-\alpha) + 2\cos n\alpha$$

$$= 2\cos n\alpha\cos\alpha + 2\cos n\alpha$$

$$= 2\cos n\alpha[1+\cos\alpha]$$

$$= 4\cos^2\frac{\alpha}{2}\cos n\alpha \neq 0$$

$$\sin(n+1)\alpha - \sin(n-1)\alpha = \sin(n\alpha+\alpha) - \sin(n\alpha-\alpha)$$

$$= 2\cos n\alpha\sin\alpha$$

$$= 4\cos n\alpha\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\therefore \frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha}$$

$$= \frac{4\cos n\alpha\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\cos^2\frac{\alpha}{2}\cos n\alpha} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \tan\frac{\alpha}{2}$$

20. If none of x, y, z is an odd multiple of π/2 and if sin(y + z – x), sin(z + x – y), sin(x + y – z) are in A.P., then prove that tan x, tan y, tan z are also in A.P.

Sol. sin(y + z – x), sin(z + x – y), sin(x + y – z) are in A.P.

$$\Rightarrow \sin(z+x-y)-\sin(y+z-x)=\sin(x+y-z)-\sin(z+x-y)$$

$$\Rightarrow 2\cos\left(\frac{z+x-y+y+z-x}{2}\right)\sin\left(\frac{z+x-y-y-z+x}{2}\right)$$

$$=2\cos\left(\frac{x+y-z+z+x-y}{2}\right)\sin\left(\frac{x+y-z-z-x+y}{2}\right)$$

$$\Rightarrow 2\cos z\sin(x-y)=2\cos x\sin(y-z)$$

$$\Rightarrow 2\cos z[\sin x\cos y-\cos x\sin y]=2\cos x[\sin y\cos z-\cos y\sin z]$$

Dividing with cosx cosy cosz, we get

$$\Rightarrow \frac{2\cos z[\sin x\cos y-\cos x\sin y]}{\cos x\cos y\cos z}=\frac{2\cos x[\sin y\cos z-\cos y\sin z]}{\cos x\cos y\cos z}$$

$$\Rightarrow \frac{\sin x\cos y}{\cos x\cos y}-\frac{\cos x\sin y}{\cos x\cos y}=\frac{\sin y\cos z}{\cos y\cos z}-\frac{\cos y\sin z}{\cos y\cos z}$$

$$\Rightarrow \frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}=\frac{\sin y}{\cos y}-\frac{\sin z}{\cos z}$$

$$\Rightarrow \tan x-\tan y=\tan y-\tan z$$

$$\Rightarrow \tan x+\tan z=2\tan y$$

⇒ tan x, tan y, tan z are in A.P.

21. If $x\cos\theta=y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$ for some $\theta\in R$ then show that

$xy + yz + zx = 0$

Solution:

Let $x\cos\theta=y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)=k$

$\cos\theta=\frac{k}{x}$ $\cos\left(\theta+\frac{2\pi}{3}\right)=\frac{k}{y}$: $\cos\left(\theta+\frac{4\pi}{3}\right)=\frac{k}{z}$

$\frac{k}{x}+\frac{k}{y}+\frac{k}{2}=\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)$

$\frac{k}{x}+\frac{k}{4}+\frac{k}{2}=0$ { Refer the problem (1) in short answer question)

$xy + yz + zx = 0$

22. If neither A or A + B is an odd multiple of π/2 and if m sin B = n sin(2A + B) then prove that (m+n) tan A = (m–n) tan (A+B).

Sol. Given m sin B = n sin(2A + B)

$$\frac{m}{n}=\frac{\sin(2A+B)}{\sin B}$$

By componendo and dividendo, we get

$$\frac{m+n}{m-n}=\frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}$$

$$=\frac{2\sin(A+B)\cos A}{2\cos(A+B)\sin A}$$

$$\frac{m+n}{m-n}=\tan(A+B)\cot A$$

$$\frac{(m+n)}{\cot A}=(m-n)\tan(A+B)$$

$$(m+n)\tan A=(m-n)\tan(A+B)$$

23. If $\tan(A+B)=\lambda\tan(A-B)$ **then show that** $(\lambda+1)\sin 2B=(\lambda-1)\sin 2A$

Solution:

$$\frac{\tan(A+B)}{\tan(A-B)}=\frac{\lambda}{1}\Rightarrow\frac{\sin(A+B)}{\cos(A+B)}\times\frac{\cos(A-B)}{\sin(A+B)}=\frac{\lambda}{1}$$

Using componedo and dividendo

$$\frac{\sin(A+B)\cos(A-B)+\cos(A+B).\sin(A-B)}{\sin(A+B)\cos(A-B)-\cos(A+B)\sin(A-B)}=\frac{\lambda+1}{\lambda-1}$$

$$\frac{\sin 2A}{\sin 2B}=\frac{\lambda+1}{\lambda-1}\Rightarrow(\lambda-1)\sin 2A=(\lambda+1)\sin 2B$$

LAQ'S

24. If A, B, C are the angles of a triangle then prove that

(i) $\sin 2A-\sin 2B+\sin 2C=4\cos A\sin B\cos C$

(ii) $\cos 2A-\cos 2B+\cos 2C=1-4\sin A\cos B\sin C$

Solution :

(i) $A+B+C=180^0\Rightarrow A+B=180^0-C \quad \sin(A+B)=\sin C$

$C=180^0-(A+B) \quad \sin C=\sin(A+B)$

$\cos(A+B)=-\cos C$

$$\sin 2A-\sin 2B+\sin 2C=2\cos(A+B)\sin(A-B)+2\sin C\cos C$$

$$=-2\cos C\sin(A+B)+2\sin C\cos C$$

$$=+2\cos C\left[-\sin(A-B)+\sin(A+B)\right]$$

$$=2\cos C\left[2\cos A\sin B\right]=4\cos A\sin B\cos C$$

(ii) $\cos 2A-\cos 2B+\cos 2C=-2\sin(A+B).\sin(A-B)+1-2\sin^2 C$

$$=1-2\sin C.\sin(A-B)-2\sin^2 C$$

$$= 1 - 2\sin C\{\sin(A-B) + \sin C\}$$

$$= 1 - 2\sin C\{\sin(A+B) + \sin(A+B)\}$$

$$= 1 - 2\sin C\{2\sin A\cos B\} = 1 - 2\sin A\cos B\sin C$$

25. If A, B, C are angles of a triangle then prove that

(i) $\sin A + \sin B - \sin C = 4\sin A/2\,\sin B/2\,\cos C/2$

(ii) $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Solution :

(i) $\sin A + \sin B - \sin C = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$$= \sin\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$$

$$= 2\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right\}$$

$$= 2\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$$

$$= 2\cos\frac{C}{2}\left\{2\sin\frac{A}{2}\sin\frac{B}{2}\right\}$$

$$= 4\sin A/2\,\sin B/2\,\cos C/2$$

Solution

(ii) $\cos A + \cos B - coC = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \cos C$

$$= 2\cos\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - \left\{1 - 2\sin^2\frac{C}{2}\right\}$$

$$= 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 1 + 2\sin^2\frac{C}{2}$$

$$-1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \sin\frac{C}{2}\right\}$$

$$= -1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \sin\left(90^0 - \frac{A+B}{\mathbf{2}}\right)\right\}$$

$$= -1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right\}$$

$$= -1 + 2\sin\frac{C}{2}\left\{2\cos\frac{A}{2}\cos\frac{B}{2}\right\}$$

$$= -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

26. If A, B, C are the angles of a triangle then prove that

(i) $\sin^2 A+\sin^2 B-\sin^2 C=2\sin A\sin B\cos C$

(ii) $\cos^2 A+\cos^2 B-\cos^2 C=1-2\sin A\sin B\cos C$

Solution:

(i) $A+B+C=180^0$

$\sin^2 A+\sin^2 B-\sin^2 C=\sin^2 A+\sin(B+C)\sin(B-C)$

$\{\because \sin^2 B-\sin^2 C=\sin(B+C)\sin(B-C)\}$

$=\sin^2 A+\sin(180^0-A)\sin(B-C)$

$=\sin^2 A+\sin A\sin(B-C)$

$=\sin A\{\sin A+\sin(B-C)\}$

$=\sin A\{\sin(180^0-B+C)+\sin(B-C)\}$

$=\sin A\{\sin(B+C)+\sin(B-C)\}$

$=\sin A\{2\sin B\cos C\}=2\sin A\sin B\cos C$

Solution :

(ii) $\cos^2 A+\cos^2 B-\cos^2 C=\cos^2 A=\{\cos^2 C-\cos^2 B\}$

$=\cos^2 A-\sin(B+C)\sin(B-C)$

$=1-\sin^2 A-\sin(180^0-A)\sin(B-C)$

$=1-\sin^2 A-\sin A\sin(B-C)$

$=1-\sin A\{\sin A+\sin(B-C)\}$

$=1-\sin A\{\sin(180^0-B+C)+\sin(B-C)\}$

$=1-\sin A\{\sin(B+C)+\sin(B-C)\}$

$=1-\sin A\{2\sin B\cos C\}$

$=1-2\sin A\sin B\cos C$

27. If $A+B+C=180^0$ then prove that

(i) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\left\{1+\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right\}$

(ii) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Here $A+B+C=180^0=A/2+B/2+C/2=90^0$

Solution:

(i) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=\cos^2\frac{A}{2}+1-\sin^2 B/2+\cos^2 C/2$

$=1+\left\{\cos^2\frac{A}{2}-\sin^2\frac{B}{2}\right\}+\cos^2\frac{C}{2}$

$=1+\cos\left(\frac{A}{2}+\frac{B}{2}\right)\cos\left(\frac{A}{2}-\frac{B}{2}\right)+\cos^2\frac{C}{2}$

$$= 1 + \cos\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + 1 - \sin^2\frac{C}{2}$$

$$= 2 + \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right\}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\left(90^0 - \frac{A+B}{2}\right)\right\}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$$

$$2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 2 + 2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

(ii) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = \cos^2\frac{A}{2} - \left\{\cos^2\frac{C}{2} - \cos^2\frac{B}{2}\right\}$

$$= \cos^2\frac{A}{2} - \sin\left(\frac{B+C}{2}\right).\sin\left(\frac{B-C}{2}\right)$$

$$= \cos^2\frac{A}{2} - \sin\left(90^0 - \frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)$$

$$= \cos^2\frac{A}{2} - \cos\frac{A}{2}\sin^2\left(\frac{B-C}{2}\right)$$

$$= \cos\frac{A}{2}\left\{\cos\frac{A}{2} - \sin\frac{B-C}{2}\right\}$$

$$= \cos\frac{A}{2}\left\{\cos\left(90^0 - \frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right)\right\}$$

$$= \cos\frac{A}{2}\left\{\sin\left(\frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right)\right\} = 2\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

28. In a triangle ABC prove that

(i) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$

(ii) $\cos\frac{A}{2} + \cos\frac{B}{2} - \cos\frac{C}{2} = 4\cos\left(\frac{\pi+A}{4}\right)\cos\left(\frac{\pi+B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$

(iii) $\sin\frac{A}{2} + \sin\frac{B}{2} - \sin\frac{C}{2} = -1 + 4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$

Solution:

(i) Given $A + B + C = \pi$

R.H..S $4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right) = 2\left\{\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\right\}\left\{2\cos\left(\frac{\pi-C}{4}\right)\right\}$

$$= \left\{\cos\left(\frac{\pi - A + \pi - B}{\ }\right) + \cos\left(\frac{\pi - A - \pi + B}{4}\right)\right\}\left\{2\cos\left(\frac{\pi - C}{4}\right)\right\}$$

$$\{\because 2\cos A\cos B = \cos(A+B) + \cos(A+B)\}$$

$$= \left\{\cos\left\{\frac{\pi}{2} - \left(\frac{A+B}{4}\right)\right\} + \cos\left(\frac{A-B}{4}\right)\right\} 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= 2\cos\frac{\pi - C}{4}\sin\left(\frac{A+B}{4}\right) + 2\cos\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A-B}{4}\right) \because \cos\left(\frac{\pi}{2} - \frac{A+B}{4}\right) = \sin\left(\frac{A+B}{4}\right)$$

$$= \sin\left(\frac{\pi - C + A + B}{4}\right) - \sin\left(\frac{\pi - C - A - B}{4}\right) + \cos\left(\frac{\pi - C + A - B}{4}\right) + \cos\left(\frac{\pi - C - A + B}{4}\right)$$

$$\left\{\begin{array}{l}\because 2\cos A\sin B = \sin(A+B) - \sin(A-B) \\ 2\cos A\cos B = \cos(A+B) + \cos(A-B)\end{array}\right\}$$

$$\therefore \sin\left(\frac{\pi - C + \pi - C}{4}\right) - \sin\left\{\frac{A + B + C - C - A - B}{4}\right\} + \cos\left(\frac{\cancel{A} + \cancel{B} + \cancel{C} - \cancel{C} + A - B}{4}\right)$$

$$+ \cos\left\{\frac{\cancel{A} + \cancel{B} + \cancel{C} - \cancel{C} - \cancel{A} + \cancel{B}}{4}\right\} \left\{\begin{array}{l}\because \pi = A + B + C \\ ad\ A + B = \pi - C\end{array}\right\}$$

$$= \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) + \cos\frac{A}{2} + \cos\frac{B}{2}$$

$$= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$$

Solution :

(ii) R.H.S

$$= 4\cos\left(\frac{\pi + A}{4}\right)\cos\left(\frac{\pi + B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left\{2\cos\left(\frac{\pi + A}{4}\right)\cos\left(\frac{\pi + B}{4}\right)\right\} 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left[\cos\left(\frac{\pi + A + \pi + B}{4}\right) + \cos\left(\frac{\cancel{\pi} + A - \cancel{\pi} - B}{4}\right)\right] 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left[\cos\left(\frac{\pi}{2} + \frac{A+B}{4}\right) + \cos\left(\frac{A-B}{4}\right)\right] 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= -2\cos\left(\frac{\pi - C}{4}\right)\sin\left(\frac{A+B}{4}\right) + 2\cos\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A-B}{4}\right)$$

$$= -2\cos\left(\frac{\pi - C}{4}\right)\sin\left(\frac{\pi - C}{4}\right) + \cos\left(\frac{\pi - C + A - B}{4}\right) + \cos\left(\frac{\pi - C - A + B}{4}\right)$$

$$= -\sin 2\left(\frac{\pi - C}{4}\right) + \cos\left(\frac{\cancel{A} + \cancel{B} + \cancel{C} - A - \cancel{B}}{4}\right) + \cos\left(\frac{\cancel{A} + B + \cancel{C} - \cancel{C} - \cancel{A} + B}{4}\right)$$

$$= -\sin\left(\frac{\pi}{2} - \frac{C}{2}\right) + \cos\frac{A}{2} + \cos\frac{B}{2}$$

$$= \cos\frac{A}{2} + \cos\frac{B}{2} - \cos\frac{C}{2}$$

Solution :

(iii) R.H.S

$$= -1 + 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right).\sin\left(\frac{\pi - C}{4}\right)$$

$$= -1 + \left\{2\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\right\} 2\sin\left(\frac{\pi - C}{4}\right)$$

$$= -1 + \left\{\cos\left(\frac{\pi}{2} - \left(\frac{A + B}{4}\right)\right) + \cos\frac{A - B}{4}\right\} 2\sin\left(\frac{\pi - C}{4}\right)$$

$$= -1 + 2\sin\left(\frac{A + B}{4}\right)\sin\left(\frac{\pi - C}{4}\right) + 2\sin\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A - B}{4}\right)$$

$$= -1 + 2\sin\left(\frac{\pi - C}{4}\right) + \sin\left(\frac{A + B + C - C + A - B}{4}\right) + \sin\left\{\frac{A + B + C - C - A + B}{4}\right\}$$

$$= -\left\{1 - 2\sin^2\frac{\pi - C}{4}\right\} + \sin\frac{A}{2} + \sin\frac{B}{2}$$

$$= -\cos\left(\frac{\pi - C}{2}\right) + \sin\frac{A}{2} + \sin\frac{B}{2}$$

$$= \sin\frac{A}{2} + \sin\frac{B}{2} - \sin\frac{C}{2}$$

29. If $A + B + C = 90^0$ then prove that

$$\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A \sin B \sin C$$

Solution :

$$\cos 2A + \cos 3B + \cos 2C = 2\cos(A + B)\cos(A - B) + \cos 2C$$

$$= 2\cos(90^0 - C)\cos(A - B) + \cos 2C \quad \{\because A + B = 90^0 - C\}$$

$$= 2\sin C\cos(A - B) + 1 - 2\sin^2 C$$

$$= 1 + 2\sin C\{\cos(A - B) - \sin C\}$$

$$= 1 + 2\sin\{\cos(A - B) - \sin(90^0 - A + B)\}$$

$$= 1 + 2\sin C\{\cos(A - B) - \cos(A + B)\}$$

$$= 1 + 2\sin C\{2\sin A\sin B\}$$

$$= 1 + 4\sin A\sin B\sin C$$

30. If $A + B + C = 270^0$ then prove that

(i) $\cos^2 A + \cos^2 B - \cos^2 C = -2\cos A\cos B\sin C$

(ii) $\sin 2A + \sin 2B - \sin 2C = -4\sin A\sin B\cos C$

Solution:

(i) $\cos^2 A + \cos^2 B - \cos^2 C = \cos^2 A - \{\cos^2 C - \cos^2 B\}$

$$= \cos^2 A - \sin(B + C)\sin(B - C)$$

$$= \cos^2 A - \sin(270^0 - A)\sin(B - C)$$

$$= \cos^2 A + \cos A\sin(B - C)$$

$= \cos A\{\cos A + \sin(B - C)\}$

$= \cos A\{\cos(270^0 - \overline{B + C}) + \sin(B - C)\}$

$\cos A\{-\sin(B - C) + \sin(B - C)\}$

$-\cos A(2\cos B \sin C) = -2\cos A\cos B\sin C$

(ii) $\sin 2A + \sin 2B - \sin 2C = -4\sin A\sin B\cos C$

$2\sin(A + B)\cos(A - B) - \sin 2C$

$2\sin(270^0 - C)\cos(A - B) - \sin 2C$

$-2\cos C\cos(A - B) - 2\sin C\cos C$

$-2\cos[\cos(A - B) + \sin(220^0 - A + B)]$

$-2\cos C[\cos(A - B) - \cos(A + B)]$

$-4\sin A\sin B\cos C$

31. If $A + B + C = 0^0$ then prove that

(i) $\sin 2A + \sin 2B + \sin 2C = -4\sin A\sin B\sin C$

(ii) $\sin A + \sin B - \sin C = -4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Solution :

(i) $A + B + C = 0^0 \Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 0^0 \Rightarrow \frac{A}{2} + \frac{B}{2} = -\frac{C}{2}$

$\sin 2A + \sin 2B + \sin 2C = 2\sin(A + B)\cos(A - B) + \sin 2C$

$= 2\sin(-C)\cos(A - B) + \sin 2C$

$= -2\sin C\cos(A - B) + 2\sin C\cos C$

$= -2\sin C[\cos(A - B) - \cos C]$

$= -2\sin C[\cos(A - B) - \cos(A + B)]$ $\left[\begin{array}{l} \because C = -(A + B) \\ \cos C = \cos(A + B) \end{array}\right]$

$= -2\sin C\{2\sin A\sin B\}$

$= -4\sin A\sin B\sin C$

Solution :

(ii) $\sin A + \sin B - \sin C = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right) = \sin C$

$= 2\sin\left(-\frac{C}{2}\right)\cos\left(\frac{A - B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= -2\sin\frac{C}{2}\left[\cos\left(\frac{A - B}{2}\right) + \cos\frac{C}{2}\right]$

$= -2\sin\frac{C}{2}\left[\cos\left(\frac{A - B}{2}\right) + \cos\left(\frac{A + B}{2}\right)\right]$

$= -2\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$

$$= -4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

32. **If** $A+B+C+D=360^0$ **then prove that**

(i) $\sin A - \sin B + \sin C - \sin D = -4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$

(ii) $\cos 2A + \cos 2B + \cos 2C + \cos 2D = 4\cos(A+B)\cos(A+C)\cos(A+D)$

Solution:

(i) $A+B+C+D=360^0 \Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}+\frac{D}{2}=180^0$

$$\therefore \frac{A+B}{2}=180^0-\left(\frac{C+D}{2}\right)$$

$$\sin A-\sin B+\sin C-\sin D=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)+2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)+2\cos\left\{180^0+\overline{\frac{A+B}{2}}\right\}\sin\left(\frac{C-D}{2}\right)$$

$$=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)-2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{\sin\left(\frac{A-B}{2}\right)-\sin\left(\frac{C-D}{2}\right)\right\}$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\left(\frac{A+B+C-D}{4}\right).\sin\left(\frac{A-B-C+D}{4}\right)\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C-360^0+A+C}{4}\right\}\sin\left\{\frac{A+D-360^0+A+D}{4}\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2}-90^0\right\}\sin\left\{\frac{A+D}{2}-90^0\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2}-90^0\right\}\sin\left\{\frac{A+D}{2}-90^0\right\}$$

$$-4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$$

(ii) $\cos 2A+\cos 2B+\cos 2C+\cos 2D=2\cos(A+B)\cos(A-B)+2\cos(C+D).\cos(C-D)$

$$=2\cos(A+B)\cos(A-B)+2\cos\left(360^0-\overline{A+B}\right).\cos(C-D)$$

$$=2\cos(A+B)\cos(A-B)+2\cos(A+B)\cos(C-D)$$

$$=2\cos(A+B)\{\cos(A-B)+\cos(C-D)\}$$

$$=2\cos(A+B)\left\{+2\cos\left(\frac{A+B+C-D}{2}\right)\cos\left(\frac{A-B-C+D}{2}\right)\right\}$$

$$4\cos(A+B)\cos\left\{\frac{(B+D)-(A+C)}{2}\right\}\cos\left\{\frac{(B+C)-(A+D)}{2}\right\}$$

$$4\cos(A+B)\cos\left\{\frac{360^0-2(A+C)}{2}\right\}\cos\left\{\frac{360^0-2(A+D)}{2}\right\}$$

$4\cos(A+B)\cos\left[180^0-\overline{A+C}\right]\cos\left(180^0-\overline{A+D}\right)$

$\{4\cos(A+B)\}\{-\cos(A+D)\}\{-\cos(A+D)\}$

$4\cos(A+B)\cos(A+C)\cos(A+D)$

33. **If** $A+B+C=2S$ **then prove that**

(i) $\sin(s-A)+\sin(s-B)+\sin C=4\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{S-B}{2}\right)\sin\frac{C}{2}$

(ii) $\cos(s-A)+\cos(s-B)+\cos C=-1+4\cos\left(\frac{s-A}{2}\right)\cos\left(\frac{s-B}{2}\right)\cos\frac{C}{2}$

Solution :

(i) $\sin(s-A)+\sin(s-B)+\sin C$

$$=2\cos\left(\frac{2s-A-B}{2}\right)\cos\left(\frac{B-A}{2}\right)+\sin C$$

$$=2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}$$

$$=2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)+\cos\frac{C}{2}\right\}$$

$$=2\sin\frac{C}{2}\left\{2\cos\left(\frac{A-B+C}{4}\right).\cos\left(\frac{A-B-C}{4}\right)\right\}$$

$$=4\sin\frac{C}{2}\left\{\cos\left(\frac{2s-B-B}{4}\right)\cos\left(\frac{2s-A-A}{4}\right)\right\}$$

$$4\cos\left(\frac{s-A}{2}\right)\cos\left(\frac{s-B}{2}\right)\sin\frac{C}{2}$$

Solution (ii)

$$\cos(s-A)+\cos(s-B)+\cos C$$

$$=2\cos\left(\frac{2s-A-B}{2}\right)\cos\left(\frac{B-A}{2}\right)+\cos C$$

$$=2\cos\frac{C}{2}\cos\left(\frac{B-A}{2}\right)+2\cos^2\frac{C}{2}-1$$

$$=-1+2\cos\frac{C}{2}\left[\cos\left(\frac{B-A}{2}\right)+\cos\frac{C}{2}\right]$$

$$=-1+2\cos\frac{C}{2}\left[2\cos\left(\frac{B-A+C}{4}\right)\cos\left(\frac{B-A-C}{4}\right)\right]$$

$$=-1+4\cos\frac{C}{2}\cos\left(\frac{B+C-A}{4}\right)\cos\left(\frac{A+C-B}{4}\right)$$

$$=-1+4\cos\frac{C}{2}\cos\left\{\frac{2s-A-A}{4}\right\}\cos\left(\frac{2s-B-B}{4}\right)$$

$$=-1+4\cos\frac{C}{2}\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{s-B}{2}\right)$$

$$=-1+4\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{S-B}{2}\right)\cos\left(\frac{S-B}{2}\right)\cos\frac{C}{2}$$

34. If A,B,C are angles of a triangle then prove that

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

SOL.

$A+B+C = 180^0$

$$LHS=\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}$$

$$=\sin^2\frac{A}{2}+\sin\left(\frac{B}{2}+\frac{C}{2}\right).\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=\sin^2\frac{A}{2}+\sin\left(90-\frac{A}{2}\right).\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=1\text{-}\cos^2\frac{A}{2}+\cos\frac{A}{2}.\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=1-\cos\frac{A}{2}\left(\cos\frac{A}{2}-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(\cos\left(90-\left(\frac{B}{2}+\frac{C}{2}\right)\right)-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(\sin\left(\frac{B}{2}+\frac{C}{2}\right)-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(2\cos\frac{B}{2}\sin\frac{C}{2}\right)$$

$$=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=RHS$$

35. If A+B+C =3π/2 , prove that cos2A+cos2B+cos2C =1- 4sinA.sinB.sinC

36. 13. If A,B,C are angles of a triangle, then prove that

$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=\ 1+4\sin\frac{\pi-A}{4}+\sin\frac{\pi-B}{4}+\sin\frac{\pi-C}{4}$$

Try your self.

37. If A, B, C are the angles of a triangle then prove that cos2A+cos2B+cos2C = -4cosAcosBcosC-1

cos2A+cos2B+cos2C =

$$= 2\cos\frac{2A+2B}{2}\cos\frac{2A-2B}{2}+\cos 2C$$

$$= 2\cos(A+B)\cos(A-B)+2\cos^2 C-1$$

$$= 2\cos(\pi-c)\cos(A-B)+2\cos^2 C-1$$

$$= -2\cos C\cos(A-B)+2\cos^2 C-1$$

$$= 2\cos C\left(-\cos(A-B)+\cos C\right)-1$$

$$= 2\cos C\left(-\cos(A-B)+\cos\left(\pi-(A+B)\right)\right)-1$$

$$= 2\cos C\left(-\cos(A-B)-\cos(A+B)\right)-1$$

$$= 2\cos C\left(-2\cos A\cos B\right)-1$$

$$= -4\cos A\cos B\cos C-1$$