PERIODICITY AND EXTREME VALUES

VSAQ'S

Find the periods for the given 1-4 functions.

1. cos (3x + 5) + 7

Sol. f(x) = cos(3x + 5) + 7

$$\text{Period} = \frac{\text{period of cos x}}{[\text{coefficient of x}]} = \frac{2\pi}{3}$$

2. tan 5x

Sol. f(x) = tan 5x

$$\text{Period} = \frac{\pi}{5}$$

3. |sin x|

Sol. f(x) = |sin x|

Period = π

$$\left[\because |\sin(\pi + x)| = \left((-\sin x) = (\sin x)\right)\right]$$

4. tan (x + 4x + 9x + … + n^2x) (n any positive integer).

Sol. f(x) = tan (x + 4x + 9x + … + n^2x)

$$= \tan(1 + 4 + 9 + ... + n^2)x$$

$$= \tan(1^2 + 2^2 + 3^2 + ... + n^2)x$$

$$= \tan\left[\frac{n(n+1)(2n+1)}{6}\right]x$$

$$\text{Period} = \frac{\pi}{\frac{n(n+1)(2n+1)}{6}} = \frac{6\pi}{n(n+1)(2n+1)}$$

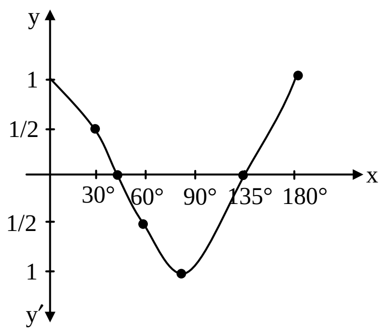
5. Find a sine function whose period is 2/3.

Sol. $f(x) = \sin\left(\frac{2\pi}{2/3}\right)x = \sin(3\pi x)$

6. Draw cos 2x in the interval [0, π].

Sol.

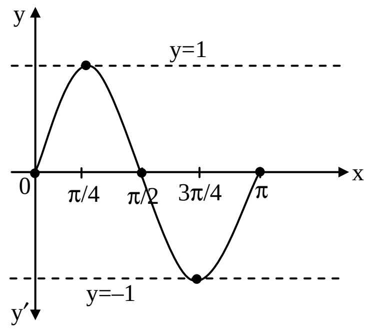
x	0	30°	45°	60°	90°	135°	180°
cos 2x	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	–1	0	1



7. Draw sin 2x in the interval (0, π).

Sol.

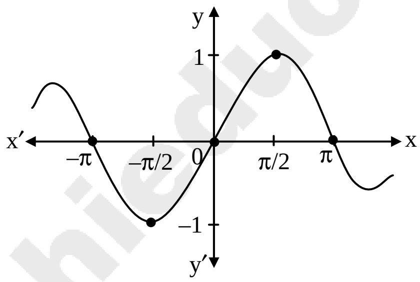
x	0	30°	60°	90°	135°	180°
sin 2x	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0	–1	0



8. Draw sin x in the interval [–π, +π].

Sol.

x	–180° (–π)	–90° (–π/2)	0 (0)	90° (π/2)	180° (π)
sin x	0	–1	0	1	0



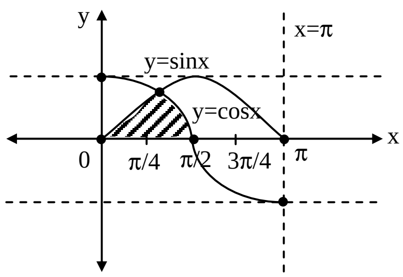
9. Sketch the region enclosed by y = sin x, y = cos x and x-axis in the interval [0, π].

Sol. y = sin x

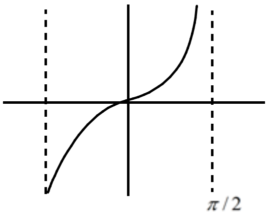
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	(π)
y	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0

y = cos x

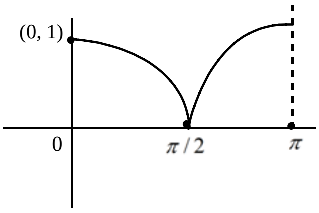
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	(π)
y	1	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{2}$	–1



10. Draw the graph of $y = \tan x$ **in** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



11. Draw the graph of $y = \cos^2 x$ **in** $(0, \pi)^{-\pi/2}$



12. Find the period of the function $f(x) = 2\sin\frac{\pi x}{4} + 3\cos\frac{\pi x}{3}$

Period of $\sin\frac{\pi x}{4}$ is $\frac{2\pi}{\pi/4} = 8$

Period of $\cos\frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3} = 6$

Period of given function is L.C.M of 8, 6

$\therefore$ Period is 24

13. **If** $a \le \cos\theta + 3\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) + 6 \le b$**, then find the largest value of a and smallest value of b**.

Sol. f = $\cos\theta + 3\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) + 6$

$= \cos\theta + 3\sqrt{2}\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right] + 6$

$= \cos\theta + 3\sqrt{2}\left[\frac{\sin\theta}{\sqrt{2}} + \frac{\cos\theta}{\sqrt{2}}\right] + 6$

$= \cos\theta + \frac{3\sqrt{2}}{\sqrt{2}}[\sin\theta + \cos\theta] + 6$

$= \cos\theta + 3\sin\theta + 3\cos\theta + 6$

$= 3\sin\theta + 4\cos\theta + 6$

Minimum value = $C - \sqrt{a^2 + b^2}$

$= 6 - \sqrt{3^2 + 4^2} = 6 - \sqrt{25} = 6 - 5 = 1$

Maximum value = $C + \sqrt{a^2 + b^2}$

$= 6 + 5 = 11$

$\therefore$ a = 1, b = 11.

14. Find the periods for the following function $\cos^4 x$.

Sol. Let $f(x) = \cos^4 x = (\cos^2 x)^2$

$$= \left[\frac{1+\cos 2x}{2}\right]^2$$

$$= \frac{1+2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1}{4}\left[1+2\cos 2x+\frac{1+\cos 4x}{2}\right]$$

$$= \frac{1}{8}[1+4\cos 2x+1+\cos 4x]$$

$$= \frac{1}{8}[3+4\cos 2x+\cos 4x]$$

Period at $\cos 2x = \frac{2\pi}{2} = \pi$

Period at $\cos 4x = \frac{2\pi}{4} = \frac{\pi}{2}$

L.C.M. of $\left(\pi, \frac{\pi}{2}\right) = \pi$

$\therefore$ Period of $f(x) = \pi$.

15. $\frac{5\sin x+3\cos x}{4\sin 2x+5\cos x}$

Sol. Let $f(x) = \frac{5\sin x+3\cos x}{4\sin 2x+5\cos x}$

Period of $\sin x = 2\pi$

Period of $\cos x = 2\pi$

Period of $\sin 2x = \frac{2\pi}{2} = \pi$

Period of $\cos 2x = \frac{2\pi}{2} = \pi$

Period of $f(x) =$

L.C.M. of $\{2\pi, 2\pi, \pi, \pi\} = 2\pi$

16. . Find the maximum and minimum values of $4\sin^2 x+5\cos^2 x$

Solution:

$$4\sin^2 x+5\cos^2 x = 4\left(1-\cos^2 x\right)+5\cos^2 x$$

$$= \cos^2 x+4$$

Maximum value of $\cos^2 x = 1$

$\therefore$ Maximum value of $\cos^2 x+4$ is $1+4=5$

Minimum value of $\cos^2 x = 0$

$\therefore$ Minimum value of $\cos^2 x+4$ is $0+4=4$

17. Find the minimum and maximum value of 3 cos x + 4 sin x.

Sol. Let f(x) = 3 cos x + 4 sin x

Maximum value of $f = C + \sqrt{a^2 + b^2}$

$= 0 + \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Minimum value of $f = C - \sqrt{a^2 + b^2}$

$= -\sqrt{4^2 + 3^2} = -\sqrt{16 + 9} = -\sqrt{25} = -5$

18. Find the range of 7 cos x – 24 sin x + 5.

Sol. Minimum value of $f = C - \sqrt{a^2 + b^2}$

$= 5 - \sqrt{(-24)^2 + 7^2}$

$= 5 - \sqrt{576 + 49}$

$= 5 - \sqrt{625} = 5 - 25 = -20$

Maximum value of $f = C + \sqrt{a^2 + b^2}$

$= 5 + \sqrt{625} = 5 + 25 = 30$