Trigonometry

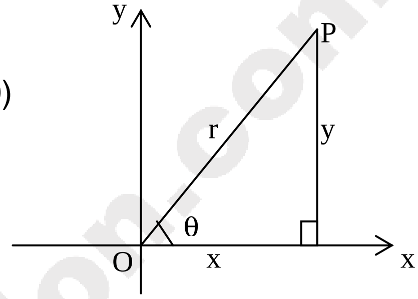
1.TRIGONOMETRIC RATIOS

1. If a ray $\overrightarrow{OP}$ makes an angle θ with the positive direction of X-axis then

i) Sin $\theta = \frac{y}{r}$ ii) cos $\theta = \frac{x}{r}$

iii) tan $\theta = \frac{y}{x} (x \neq 0)$ iv) cot $\theta = \frac{x}{y}$ $(y \neq 0)$

v) sec $\theta = \frac{r}{x}$ $(x \neq 0)$ vi) cosec $\theta = \frac{r}{y}$ $(y \neq 0)$



2. Relations :

i) $\sin\theta \operatorname{cosec}\theta = 1$
ii) $\cos\theta \sec\theta = 1$
iii) $\tan\theta \cot\theta = 1$
iv) $\sin^2\theta + \cos^2\theta = 1$
v) $1 + \tan^2\theta = \sec^2\theta$ $\rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1.$

$\rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} = 1$

vi) $1 + \cot^2\theta = \operatorname{cosec}^2\theta \rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

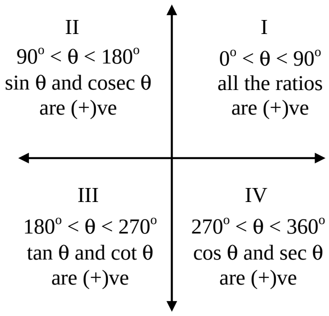
$\rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

vii) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \,.\, \operatorname{cosec}^2\theta$
viii) $\tan^2\theta - \sin^2\theta = \tan^2\theta \,.\, \sin^2\theta$;
$\cot^2\theta - \cos^2\theta = \cot^2\theta \,.\, \cos^2\theta$
ix) $\sin^2\theta + \cos^4\theta = 1 - \sin^2\theta\cos^2\theta$
$= \sin^4\theta + \cos^2\theta$
x) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
xi) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$
xii) $\sin^2 x + \operatorname{cosec}^2 x \geq 2$
xiii) $\cos^2 x + \sec^2 x \geq 2$
xiv) $\tan^2 x + \cot^2 x \geq 2$.

3. Values of trigonometric ratios of certain angles

angle → / ↓ ratio	0^o	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined
cot	undefined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
cosec	undefined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	undefined

4. Signs of Trigonometric ratios : If θ lies in I, II, III, IV quadrants then the signs of trigonometric ratios are as follows.



Note : i) 0^o, 90^o, 180^o, 270^o. 360^o, 450^o, ….. etc. are called quadrant angles.

ii) With "ALL SILVER TEA CUPS" symbol we can remember the signs of trigonometric ratios.

5. Coterminal angles : If two angles differ by an integral multiples of 360^o then two angles are called coterminal angles.
Thus 30^o, 390^o, 750^o, 330^o etc., are coterminal angles.

Fn	$90 \mp \theta$	$180 \mp \theta$	$270 \mp \theta$	$360 \mp \theta$
$\sin\theta$	$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$
$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$	$\cos\theta$
$\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$
$\text{cosec}\,\theta$	$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$
$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$	$\sec\theta$
$\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$

6. Complementary Angles : Two Angles A, B are said to complementary $\Rightarrow A + B = 90^o$

7. Supplementary angles : Two angles A, B are said to be supplementary
$\Rightarrow A + B = 180^o$.

PROBLEMS

VSAQ'S

Convert the following into simplest form :

1. $\tan(\theta - 14\pi)$

Sol. $-\tan(14\pi - \theta)$

$= \tan[7 \cdot 2\pi - \theta] = -[-\tan\theta] = \tan\theta$

2. **cosec** $(5\pi + \theta)$

Sol. $\text{cosec}\,(5\pi + \theta) = -\text{cosec}\,\theta$

$(\because \text{cosec}(n\pi + \theta) = (-1)^n \text{cosec}\theta)$

3. **Find the value of** $\cos\left(-\frac{7\pi}{2}\right)$

Sol. $\cos\left(-\frac{7\pi}{2}\right) = \cos\frac{7\pi}{2} = 0$

$(\because \cos(2n+1)\frac{\pi}{2} = 0)$

4. **Find the value of cot(315°)**

Sol. $\cot(315°) = -\cot 315°$

$= -[\cot 360° - 45°]$

$= -[-\cot 45°]$

$= \cot 45° = 1$

5. **Evaluate**

$\cos^2 45° + \cos^2 135° + \cos^2 225° + \cos^2 315°$

Sol. $\cos 45° = \frac{1}{\sqrt{2}}, \cos 135° = \cos(180° - 45°)$

$$= \cos 45° \qquad = -\frac{1}{\sqrt{2}}$$

$$\cos 225° = \cos(180° + 45°)$$

$$= \cos 45° \qquad = -\frac{1}{\sqrt{2}}$$

$$\cos 135° = \cos(360° - 45°)$$

$$= \cos 45° \qquad = \frac{1}{\sqrt{2}}$$

Given expression

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

6. cos 225° – sin 225° + tan 495° – cot 495°

Sol. cos 225° = cos(180° + 45°)

$$= \cos 45° = -\frac{1}{\sqrt{2}}$$

$$\sin 225° = \sin(180° + 45°)$$

$$= -\sin 45° = \frac{-1}{\sqrt{2}}$$

$$\tan 495° = \tan[5(90°) + 45]$$

$$= -\cot 45° = -1$$

$$\cot 495° = \cot[5(90°) + 45°]$$

$$= -\tan 45° = -1$$

G.E. $$= \frac{-1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right) - 1 + 1$$

$$= \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 + 1 = 0$$

7. If cos θ = t (0 < t < 1) and θ does not lie in the first quadrant, find the value of (a) sinθ (b) tan θ.

Sol. cos θ = t ⇒ lies in IV quadrant.

$$x^2 = AC^2 - BC^2 = 1 - t^2$$

$$x = \sqrt{1 - t^2}$$

a) $\sin\theta = \frac{AB}{AC} = -\sqrt{1 - t^2}$

b) $\tan\theta = \frac{AB}{BC} = -\frac{\sqrt{1 - t^2}}{t}$

8. If $\sin\theta = -\frac{1}{3}$ and θ does not lie in third quadrant, find the values of

(i) $\cos\theta$ **(ii)** $\cot\theta$

Solution:

$\sin\theta = -\frac{1}{3}$ θ does not lie in III Q

$\therefore \theta$ lies in IV Quadrant

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = 2\sqrt{2}$

9. Find the value of $\sin 330^0 . \cos 120^0 + \cos 210^0 . \sin 300^0$

Solution:

$\sin 330^0 = \sin\left(360^0 - 30^0\right) = -\sin 30^0 = -\frac{1}{2}$

$\cos 120^0 = -\frac{1}{2} \cos 210^0 = -\frac{\sqrt{3}}{2}; \sin 300^0 = \sin\left(360^0 - 60^0\right) = -\frac{\sqrt{3}}{2}$

$\sin 330^0 \cos 120^0 + \cos 210^0 . \sin 300^0 = -\frac{1}{2} \times -\frac{1}{2} + \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = 1$

10. If cosec θ + cot θ = $\frac{1}{3}$, find cos θ and determine the quadrant in which θ lies.

Sol. $\text{cosec}^2\,\theta - \cot^2\theta = 1$

(cosec θ + cot θ) (cosec θ – cot θ) = 1

cosec θ – cot θ = 3 …(1)

and cosec θ + cot θ = $\frac{1}{3}$ …(2)

From (1) + (2)

cosec θ – cot θ = 3

cosec θ + cot θ = $\frac{1}{3}$

2cosec θ = $3 + \frac{1}{3} = \frac{10}{3}$

$\cos \text{ec}\theta = \frac{10}{6}$

$\sin\theta = \frac{6}{16}$

From (1) – (2)

cosec θ – cot θ = 3

cosec θ + cot θ = $\frac{1}{3}$

$$-2\cot\theta = 3 - \frac{1}{3} = \frac{8}{3}$$

$$\cot\theta = -\frac{8}{2\times 3} = \frac{-8}{6}$$

$$\therefore \cos\theta = \cot\theta \cdot \sin\theta = \frac{-8}{6} \times \frac{6}{10} = \frac{-4}{5}$$

Thus sin θ is +ve and cos θ is –ve

Then θ lies in II quadrant.

11. If sec θ + tan θ = 5, find the quadrant in which θ lies and find the value of sin θ.

Sol. sec θ + tan θ = 5 …(1)

We know, $\sec^2\theta - \tan^2\theta = 1$

(sec θ + tan θ)(sec θ – tan θ) = 1

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta - \tan\theta = \frac{1}{5} \quad ...(2)$$

Adding (1), (2)

$$\sec\theta + \tan\theta = 5$$

$$\sec\theta - \tan\theta = \frac{1}{5}$$

$$2\sec\theta = 5 + \frac{1}{5}$$

$$2\sec\theta = \frac{25+1}{5}$$

$$\sec\theta = \frac{26}{5} \times \frac{1}{2}$$

$$\sec\theta = \frac{13}{5}$$

$$\tan\theta = \frac{12}{5},\ \sin\theta = \frac{12}{13}$$

∴ sin θ · sec θ and tan θ are positives

∴ θ lies in I quadrant.

12. If $\cos A = \cos B = -\frac{1}{2}$: **A does not lie in second quadrant B does not lie in third quadrant find the value of** $\frac{4\sin B - 4\tan A}{\tan B + \sin A}$

Solution:

$\cos A = \cos B = -\frac{1}{2}$; A does not lie in II Q

$\therefore$ A lies in III Q

$$\sin A = -\frac{\sqrt{3}}{2}$$

$\cos B = -\frac{1}{2}$ B does not lie in III Q

$\therefore$ B lies in II Q

$\sin B = \frac{\sqrt{3}}{2}$ $\tan A = \sqrt{3}$ $\tan B = -\sqrt{3}$

$$\frac{4\sin B - 3\tan A}{\tan B + \sin A} = \frac{4\times\frac{\sqrt{3}}{2} - 3\times\sqrt{3}}{-\sqrt{3} - \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{\frac{-3\sqrt{3}}{2}} = \frac{2}{3}$$

13. $$\frac{\sin(3\pi - A)\cos\left(A - \frac{\pi}{2}\right)\tan\left(\frac{3\pi}{2} - A\right)}{\operatorname{cosec}\left(\frac{13\pi}{2} + A\right)\sec(3\pi + A)\cot\left(A - \frac{\pi}{2}\right)} = -\cos^4 A$$

Sol. $\sin(3\pi - A) = \sin A$

$$\cos\left(A - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - A\right) = \sin A$$

$$\tan\left(\frac{3\pi}{2} - A\right) = \cot A$$

$$\operatorname{cosec}\left(\frac{13\pi}{2} + A\right) = -\sec A$$

$$\sec(3\pi + A) = -\sec A$$

$$\cot\left(A - \frac{\pi}{2}\right) = -\cot\left(\frac{\pi}{2} - A\right) = -\tan A$$

$$\text{L.H.S.} = \frac{\sin A \cdot \sin A \cdot \cot A}{-\sec A \cdot -\sec A \cdot -\tan A}$$

$$= \frac{\sin^2 A \times \frac{\cos A}{\sin A}}{-\frac{1}{\cos^2 A} \times \frac{\sin A}{\cos A}}$$

$$= \sin A \cos A \times \frac{\cos^3 A}{-\sin A} = -\cos^4 A$$

14. $\cot\left(\frac{\pi}{20}\right)\cot\left(\frac{3\pi}{20}\right)\cot\left(\frac{5\pi}{20}\right)\cot\left(\frac{7\pi}{20}\right)\cot\left(\frac{9\pi}{20}\right)=1$

Sol. $\cot\left(\frac{\pi}{20}\right)=\cot 9°=\frac{1}{\tan 9°}$

$\cot\left(\frac{3\pi}{20}\right)=\cot 27°=\frac{1}{\tan 27°}$

$\cot\left(\frac{5\pi}{20}\right)=\cot 45°=1$

$\cot\left(\frac{7\pi}{20}\right)=\cot 63°=\cot(90°-27°)=\tan 27°$

$\cot\left(\frac{9\pi}{20}\right)=\cot 81°=\cot(90°-9°)=\tan 9°$

$\therefore \cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

$=\frac{1}{\tan 9°}\frac{1}{\tan 27°}\cdot 1\cdot \tan 27°\cdot \tan 9°=1$

15. Simplify $\dfrac{\sin\left(-\frac{11\pi}{3}\right)\tan\left(\frac{35\pi}{6}\right)\sec\left(-\frac{7\pi}{3}\right)}{\cos\left(\frac{5\pi}{4}\right)\operatorname{cosec}\left(\frac{7\pi}{4}\right)\cos\left(\frac{17\pi}{6}\right)}$.

Sol. $\sin\left(-\frac{11\pi}{3}\right)=-\sin\frac{11\pi}{3}=-\sin 660°$

$=-\sin(2\cdot 360°-60°)$

$=\sin 60°=\frac{\sqrt{3}}{2}$

$\tan\left(\frac{35\pi}{6}\right)=\tan 1050°=\tan(3\cdot 360°-30°)\quad =-\tan 30°=-\frac{1}{\sqrt{3}}$

$\sec\left(-\frac{7\pi}{3}\right)=\sec(-420°)=\sec 420°\quad =\sec(360°+60°)=\sec 60°=2$

$\cos\left(\frac{5\pi}{4}\right)=\cos(225°)=\cos(180°+45°)=\cos 45°=-\frac{1}{\sqrt{2}}$

$\csc\left(\frac{7\pi}{4}\right)=\csc 315°=\csc(360°-45°)=-\csc 45°=-\sqrt{2}$

$\cos\left(\frac{17\pi}{6}\right)=\cos 510°=\cos(360°+150°)$

$=\cos 100°=\cos(180°-30°)=-\cos 30°=-\frac{\sqrt{3}}{2}$

$$\text{L.H.S.} = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)(2)}{\left(\frac{-1}{\sqrt{2}}\right)(-\sqrt{2})\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{-1}{\frac{-\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \text{R.H.S.}$$

16. If tan 20° = p, prove that $\frac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°} = \frac{1-p^2}{1+p^2}$**.**

Sol. $\frac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°}$

$$= \frac{\tan(360° + 250°) + \tan(360° + 340°)}{\tan(360° + 200°) - \tan(360° + 110°)}$$

$$= \frac{\tan 250° - \tan 340°}{\tan 200° - \tan 110°}$$

$$= \frac{\tan(270° - 20°) - \tan(360° - 20°)}{\tan(180° + 20°) + \tan(90° + 20°)}$$

$$= \frac{\cot 20° - \tan 20°}{\tan 20° + \cot 20°}$$

$$= \frac{\frac{1}{\tan 20°} - \tan 20°}{\tan 20° + \frac{1}{\tan 20°}}$$

$$= \frac{\frac{1}{p} - p}{p + \frac{1}{p}} = \frac{\frac{1-p^2}{p}}{\frac{p^2+1}{p}} = \frac{1-p^2}{p^2+1}$$

$$\therefore \frac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°} = \frac{1-p^2}{1+p^2}$$

17. If α, β are complementary angles such that b sin α = a, then find the value of (sin α cos β – cos α sin β).

Sol. α and β are complementary angles.

$\alpha + \beta = 90°$

$\beta = 90° - \alpha$

$$b \sin\alpha = a \Rightarrow \sin\alpha = \frac{a}{b}$$

$$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{a}{b}\right)^2 = \frac{b^2 - a^2}{b^2}$$

$\cos\alpha = \frac{\sqrt{b^2 - a^2}}{b}$

Since $\beta = 90° - \alpha$

$\sin\beta = \sin(90 - \alpha) = \cos\alpha$

$\therefore \cos\alpha = \sin\beta$

and $\alpha = 90° - \beta$

$\sin\alpha = \sin(90° - \beta) = \cos\beta$

$\therefore \cos\beta = \sin\alpha = \frac{a}{b}$

$\therefore \cos\beta = \frac{a}{b}$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{a}{b} \times \frac{a}{b} - \frac{\sqrt{b^2 - a^2}}{b} \times \frac{\sqrt{b^2 - a^2}}{b}$

$= \frac{a^2}{b^2} - \frac{(b^2 - a^2)}{b^2} = \frac{a^2 - b^2 + a^2}{b^2} = \frac{2a^2 - b^2}{b^2}$

18. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Solution:

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ -------- (1) squaring and adding (1) & (2)

Let $\cos\theta - \sin\theta = x$ -------(2)

$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = \left(\sqrt{2}\cos\theta\right)^2 + x^2$

$\Rightarrow 2 - 2\cos^2\theta = x^2 \Rightarrow x = \sqrt{2}\sin\theta$ $\quad\quad \therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

19. If 8 tan A = 15 and 25 sin B = –7 and neither A nor B is in the fourth quadrant, then show that sin A cos B + cos A sin B = $-\frac{304}{425}$.

Sol. 8 tan A = –15

$\tan A = -\frac{15}{8}$

A lies in II quadrant

$\sin A = \frac{15}{17}, \cos A = \frac{-8}{17}$

$25\sin B = -7$

$\sin B = -\frac{7}{25}$

B lies in III quadrant

$\sin B = -\frac{7}{25}, \cos B = \frac{-24}{25}$

L.H.S. : sin A cos B + cos A sin B

$$=\frac{15}{17}\times\frac{-24}{25}+\frac{-8}{17}\times\frac{-7}{25}=\frac{-3\times 24}{17\times 5}+\frac{-8\times -7}{17\times 25}$$

$$=\frac{-360}{425}+\frac{56}{425}=\frac{-304}{425}$$

20. Prove that $\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}=2$

Solution:

$$\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}$$

$$\sin^2\frac{\pi}{10}+\sin^2\left(\frac{\pi}{2}-\frac{\pi}{10}\right)+\sin^2\left(\frac{\pi}{2}+\frac{\pi}{10}\right)+\sin^2\left(\pi-\frac{\pi}{10}\right)$$

$$\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}\ +\cos^2\frac{\pi}{10}+\sin^2\frac{\pi}{10}=2\left\{\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}\right\}=2$$

$$=\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}=1$$

21. If $\tan 20^0=\lambda$ **then show that** $\frac{\tan 160^0-\tan 110^0}{1+\tan 160^0\tan 110^0}=\frac{1-\lambda^2}{2\lambda}$

22. If $\sec\theta+\tan\theta=\frac{2}{3}$ **find the value of** $\sin\theta$ **and determine the quadrant in which** θ **lies**

Solution:

$$\sec\theta+\tan=\frac{2}{3}$$

$$\sec\theta+\tan\theta=\frac{3}{2}$$

$$2\sec\theta=\frac{2}{3}+\frac{3}{2}\Rightarrow\sec\theta=13/12$$

$$\sin\theta=\frac{\tan\theta}{\sec\theta}=\frac{-15}{13}$$

$\therefore\theta$ lies in IV Quadrant

23. If A, B, C, D are angles of a cyclic quadrilateral then prove that

i) sin A – sin C = sin D – sin B

ii) cos A + cos B + cos C + cos D = 0

Sol. A, B, C, D are the angles of a cyclic quadrilateral.

A + C = 180°; B + D = 180°

A = 180° – C, B = 180° – D

i) sin A – sin C = sin D – sin B

sin A + sin B = sin C + sin D

we have

A = 180° – C ⇒ Sin A = sin C …(1)

B = 180° – D ⇒ sin B = sin D …(2)

Adding (1) and (2)

Sin A + sin B = sin C + sin D

ii) cos A = cos (180° – C) = – cos C

⇒ cos A + cos C = 0 …(1)

cos B = cos (180° – D) = – cos D

$\Rightarrow$ cos B + cos D = 0 ...(2)

Adding (1) and (2)

cos A + cos B + cos C + cos D = 0.

24. If a cos θ – b sin θ = c, then show that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ **.**

Sol. a cos θ – b sin θ = c ...(1)

Let a sin θ + b cos θ = k ...(2)

Squaring and adding

25. If 3 sin A + 5 cos A = 5, then show that 5 sin A – 3 cos A = ±3.

Sol. 3 sin A + 5 cos A = 5

Let 5 sin A – 3 cos A = k

Squaring and adding

$(3\sin A + 5\cos A)^2 + (5\sin A - 3\cos A)^2 = 25 + k^2$

$9\sin^2 A + 25\cos^2 A + 30\sin A\cos A + 25\sin^2 A + 9\cos^2 A - 30\sin A\cos A = 25 + k^2$

$34\sin^2 A + 34\cos^2 A = 25 + k^2$

$34(\sin^2 A + \cos^2 A) = 25 + k^2$

$34(1) = 25 + k^2$

$34 = 25 + k^2$

$k^2 = 34 - 26 = 9$

$k = \pm 3$

26. Prove that $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$.

Sol. We have $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{[\tan\theta + \sec\theta][1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$

$= \tan\theta + \sec\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$

$\therefore \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

27. Prove that $(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta)=2$**.**

Sol. L.H.S. : $(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta)$

$$=\left[1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right]\left[1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right]$$

$$=\left[\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right]\left[\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right]$$

$$=\frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cos\theta}=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}=2\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=2=\text{R.H.S.}$$

Eliminate θ from the following equations.

28. x = a cos^{3}θ ; y = b sin^3 θ

Sol. x = a cos^{3}θ ; y = b sin^3 θ

$$\frac{x}{a}=\cos^3\theta\Rightarrow\cos\theta=\left(\frac{x}{a}\right)^{1/3}$$

$$\frac{y}{b}=\sin^3\theta\Rightarrow\sin\theta=\left(\frac{y}{b}\right)^{1/3}$$

We have $\cos^2\theta+\sin^2\theta=1$

$$\Rightarrow\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$$

29. x = a cos^4 θ = b sin^4 θ.

Sol. $x=a\cos^4\theta\Rightarrow\cos^4\theta=\frac{x}{a}$

$$\Rightarrow\cos^2\theta=\sqrt{\frac{x}{a}}$$

$$y=b\sin^4\theta\Rightarrow\sin^4\theta=\frac{y}{b}$$

$$\Rightarrow\sin^2\theta=\sqrt{\frac{y}{b}}$$

$$\cos^2\theta+\sin^2\theta=1$$

$$\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1\Rightarrow\left(\frac{x}{a}\right)^{1/2}+\left(\frac{y}{b}\right)^{1/2}=1$$

30. x = a(sec θ + tan θ) ; y = b (sec θ – tan θ)

Sol. x = a(sec θ + tan θ) ; y = b (sec θ – tan θ)

xy = ab(sec^2 θ – tan^2 θ) = ab (1) = ab

xy = ab.

31. Prove that $\dfrac{\cos(\pi - A)\cot\left(\dfrac{\pi}{2}+A\right)\cos(-A)}{\tan(\pi+A)\tan\left(3\dfrac{\pi}{2}+A\right)\sin(2\pi-A)} = \cos A$

SAQ'S

32. Prove that $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)=13$

Sol. Consider $(\sin\theta-\cos\theta)^2=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta=1-2\sin\theta\cos\theta$

$(\sin\theta-\cos\theta)^4=[(\sin\theta-\cos\theta)^2]^2=[1-2\sin\theta\cos\theta]^2=1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta$

$\therefore(\sin\theta-\cos\theta)^4=1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta$

$(\sin\theta+\cos\theta)^2=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$=1+2\sin\theta\cos\theta$

$\therefore(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

$\sin^6\theta+\cos^6\theta=(\sin^2\theta)^3+(\cos^2\theta)^3$

$=(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$=1-3\sin^2\theta\cos^2\theta\ \ (\because\sin^2\theta+\cos^2\theta=1)$

$\therefore\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

$\therefore 3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$

$=\ 3[1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta]+6[1+2\sin\theta\cos\theta]+4[1-3\sin^2\theta\cos^2\theta]$

$=\ 3+12\sin^2\theta\cos^2\theta-12\sin\theta\cos\theta+6+12\sin\theta\cos\theta+4-12\sin^2\theta\cos^2\theta$

$=3+6+4=13$

33. Show that $\cos^4\alpha+2\cos^2\alpha\left[1-\dfrac{1}{\sec^2\alpha}\right]=1-\sin^4\alpha$.

Sol. L.H.S. : $\cos^4\alpha+2\cos^2\alpha\left[1-\dfrac{1}{\sec^2\alpha}\right]$

$=\cos^4\alpha+2\cos^2\alpha\sin^2\alpha$

$(\because 1-\cos^2\alpha=\sin^2\alpha)$

$=\cos^2\alpha[\cos^2\alpha+2\sin^2\alpha]$

$=(1-\sin^2\alpha)[(1-\sin^2\alpha)+2\sin^2\alpha]$

$=(1-\sin^2\alpha)(1+\sin^2\alpha)$

$=1-\sin^4\alpha=$ R.H.S.

34. Prove that $\dfrac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2}=\dfrac{1-\cos\theta}{1+\cos\theta}$.

Sol. Consider

$1+\sin\theta-\cos\theta=(1-\cos\theta)+\sin\theta$

$$=2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}=2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]$$

Again consider

$1+\sin\theta+\cos\theta=(1+\cos\theta)+\sin\theta$

$$=2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$=2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]$$

$$\text{L.H.S.}:\left[\frac{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]}{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}\right]^2$$

$$=\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}=\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\frac{1-\cos\theta}{1+\cos\theta}=\text{R.H.S.}$$

35. If $\dfrac{2\sin\theta}{1+\cos\theta+\sin\theta}=x$**, find the value of** $\dfrac{1-\cos\theta+\sin\theta}{1+\sin\theta}$.

Sol. $\dfrac{2\sin\theta}{1+\cos\theta+\sin\theta}=x$

$$\Rightarrow\frac{2\cdot\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}=x$$

$$\Rightarrow\frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}=x$$

$$\Rightarrow\frac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}=x\qquad\text{...(1)}$$

$$\frac{1-\cos\theta+\sin\theta}{1+\sin\theta} = \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]}{\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]} = \frac{2\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}} = x \ (\because \text{from}(1))$$